

# A2 UNIT 5 REVIEW #1

Name Key

## Polynomial Expressions/Functions

Date \_\_\_\_\_ Pd \_\_\_\_\_

1) FACTOR completely, if possible.

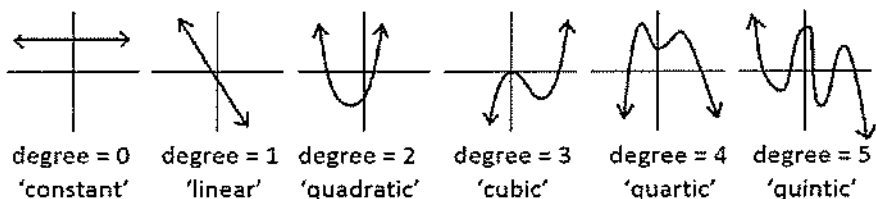
<p>a) <math>4x^3 - 25x^2 + 25x</math></p> <p><math>x(4x-5)(x-5)</math></p> <p>Max # of turning Pts. <u>2</u></p>	<p>b) <math>x^2 + 9</math></p> <p>Prime</p> <p>Max # of turning Pts. <u>1</u></p>	<p>c) <math>2x^3 - 3x^2 + 10x - 15</math></p> <p><math>(2x-3)(x^2+5)</math></p> <p>Max # of turning Pts. <u>3</u></p>	<p>d) <math>8x^3 - 1</math> Cubes (SOAP)</p> <p><math>(2x-1)(4x^2+2x+1)</math></p> <p>Max # of turning Pts. <u>4</u></p>
<p>e) <math>2x^2 - 32</math></p> <p><math>2(x+4)(x-4)</math></p>	<p>f) <math>-5x^2 + 18x - 9</math></p> <p><math>-1(5x-3)(x-3)</math></p>	<p>g) <math>2x^4 + 7x^2 + 6</math></p> <p><math>(2x^2+3)(x^2+2)</math></p>	<p>h) <math>x^5 - x^3 + 64x^2 - 64</math></p> <p><math>(x+1)(x-1)(x+4)(x^2-4x+16)</math></p>

2) Find the indicated characteristics

<p>a. State the end behavior. In terms of infinity</p> <p><math>f(x) = x^3(3x+1)(x-6)</math></p> <p>Sign (a) <u>+</u> Degree <u>5</u></p> <p><math>\downarrow \uparrow</math> (arrows)</p> <p>• End Behavior (arrows)  <math>x \rightarrow -\infty, y \rightarrow -\infty</math>  <math>x \rightarrow \infty, y \rightarrow \infty</math></p> <p>• y-intercept (x=0) <u>0</u></p> <p><math>f(0) = \underline{0}</math></p>	<p>b. State the end behavior. In terms of infinity</p> <p><math>f(x) = x^2 - 3x^4 - 5x^6</math></p> <p>Sign (a) <u>-</u> Degree <u>6</u></p> <p><math>\downarrow \downarrow</math> (arrows)</p> <p>• End Behavior (arrows)  <math>x \rightarrow -\infty, y \rightarrow -\infty</math>  <math>x \rightarrow \infty, y \rightarrow -\infty</math></p> <p>• y-intercept (x=0) <u>0</u></p> <p><math>f(0) = \underline{0}</math></p>	<p>c. Find the zeros &amp; state the multiplicity of each zero &amp; if each Bounces or Crosses at the x-axis</p> <p><math>f(x) = (x+2)^2(x-2)</math></p> <table border="1"> <thead> <tr> <th>z</th> <th>M</th> <th>B/C?</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>2</td> <td>B</td> </tr> <tr> <td>2</td> <td>1</td> <td>C</td> </tr> </tbody> </table>	z	M	B/C?	-2	2	B	2	1	C	<p>d. Find the zeros &amp; state the multiplicity of each zero &amp; if each Bounces or Crosses at the x-axis</p> <p><math>y = 18x - 9x^2 - 2x^3</math></p> <p><math>-x(2x-3)(x+6) = 0</math></p> <table border="1"> <thead> <tr> <th>z</th> <th>M</th> <th>B/C?</th> </tr> </thead> <tbody> <tr> <td>-6</td> <td>1</td> <td>C</td> </tr> <tr> <td>3/2</td> <td>1</td> <td>C</td> </tr> <tr> <td>0</td> <td>1</td> <td>C</td> </tr> </tbody> </table>	z	M	B/C?	-6	1	C	3/2	1	C	0	1	C
z	M	B/C?																						
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- A graph "crosses" the x-axis at a zero if the multiplicity of that zero is odd.
- A graph "bounces" off the x-axis at a zero if the multiplicity of that zero is even.

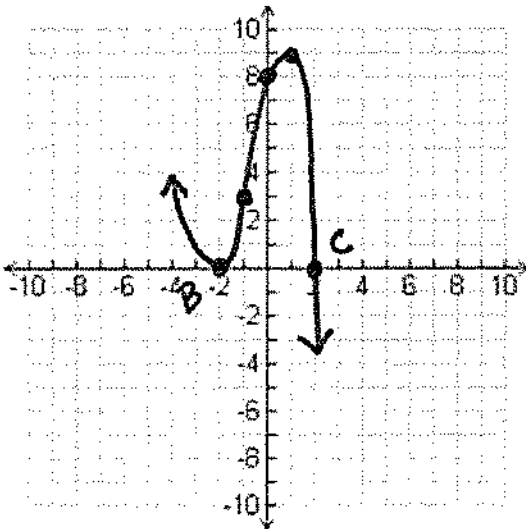
- A polynomial's graph can have AT MOST 1 fewer turning points than its degree.



### 3) Sketch the following polynomial functions

A)  $f(x) = -x^3 - 2x^2 + 4x + 8$

x-int		Z	M	B/C
multiplicities		-2	2	B
y-int	(0,8)	2	1	C
Domain	$\{x x \in \mathbb{R}\}$			
Range	$\{y y \in \mathbb{R}\}$			
Sign on a (leading Coeff)	negative			
Degree (even or odd)	3 (odd)			
Max # of Turning Pts.	2			
End Behavior	$\uparrow x \rightarrow -\infty, y \rightarrow \infty$ $\downarrow x \rightarrow \infty, y \rightarrow -\infty$			

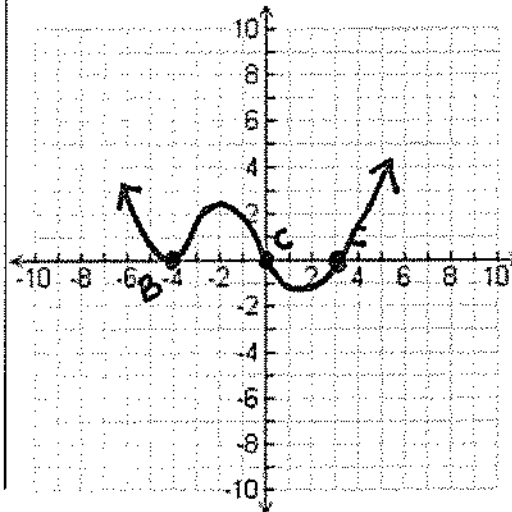


Extra points:

X	Y
-1	3
1	9

B)  $g(x) = x(x-3)(x+4)^2$

x-int		Z	M	B/C
multiplicities		0	1	C
y-int	(0,0)	3	1	C
Domain	$\{x x \in \mathbb{R}\}$	-4	2	B
* Range	skip here			
Sign on a (leading Coeff)	positive			
Degree (even or odd)	4			
Max # of Turning Pts.	3			
End Behavior	$\uparrow\uparrow x \rightarrow -\infty, y \rightarrow \infty$ $x \rightarrow \infty, y \rightarrow \infty$			



Extra points

X	Y
1	-50
-1	36

### Rational Functions/Expressions

4) (a & b) Simplify and state the excluded values. (c & d) Perform the indicated operation and simplify.

a.  $\frac{x^3 - 2x^2 - x + 2}{x^4 - 3x^2 + 2}$

$$\frac{x-2}{x^2-2}$$

$$x \neq \pm 1, \pm \sqrt{2}$$

b.  $\frac{x^2-9}{x^3+3x^2}$

$$\frac{x-3}{x^2}$$

$$x \neq -3, 0$$

c.  $\frac{4x}{x+1} \cdot \frac{x^2-6x-7}{x^2-7x}$

$$4$$

$$x \neq -1, 0, 7$$

d.  $\frac{x^2-x-12}{4x+12} \div \frac{x^2+x-20}{3x+15}$

$$\frac{3}{4}$$

$$x \neq -5, -3, 4$$

5) Simplify (remember to factor when necessary). State Restrictions.

a.  $\frac{120x^3y}{25xy^5}$

$$\frac{24x^2}{5y^4} \quad \left\{ \begin{array}{l} x \neq 0 \\ y \neq 0 \end{array} \right.$$

b.  $\frac{x^2+9x+20}{2x+8}$

$$\frac{x+5}{2} \quad \left\{ x \neq -4 \right.$$

c.  $\frac{x^2-x-12}{x^2-2x-8}$

$$\frac{x+3}{x+2} \quad \left\{ x \neq -2, 4 \right.$$

6) Multiply or divide (remember to factor when necessary). State Restrictions.

A.  $\frac{5n+15}{4n+8} \cdot \frac{2n+4}{3n+9}$

$$\frac{5}{6} \quad \left\{ n \neq -2, -3 \right.$$

B.  $\frac{x^2-x-12}{x-4} \div \frac{2x+6}{x-5}$

$$\frac{x-5}{2} \quad \left\{ x \neq -3, 4, 5 \right.$$

C.  $\frac{x+3}{10x+20} \cdot \frac{x+2}{x^2+4x+3}$

$$\frac{1}{10(x+1)} \quad \left\{ x \neq -3, -2, -1 \right.$$

RECALL

To Add or Subtract Two Fractions

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}, \quad c \neq 0$$

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}, \quad c \neq 0$$

To Find the Least Common Denominator of Rational Expressions

- Factor each denominator completely. Any factors that occur more than once should be expressed as powers. For example,  $(x-3)(x-3)$  should be expressed as  $(x-3)^2$ .
- List all different factors (other than 1) that appear in any of the denominators. When the same factor appears in more than one denominator, write that factor with the highest power that appears.
- The least common denominator is the product of all the factors listed in step 2.

Adding or Subtracting Rational Expressions

Step 1 Identify a common denominator.

Step 2 Multiply each expression by an appropriate form of 1 so that each term has the common denominator as its denominator.

Step 3 Write each expression using the common denominator.

Step 4 Add or subtract the numerators, combining like terms as needed.

Step 5 Factor as needed.

Step 6 Simplify as needed.

7) Add or subtract these rational expressions. State restrictions.

A.  $\frac{5}{12x} - \frac{3}{4}$

$$\frac{5-9x}{12x} \quad \left\{ x \neq 0 \right.$$

B.  $\frac{5}{x+1} - \frac{3}{x-4}$

$$\frac{2x-23}{(x+1)(x-4)} \quad \left\{ x \neq -1, 4 \right.$$

C.  $\frac{4}{7x-35} + \frac{5}{x-5}$

$$\frac{39}{7(x-5)} \quad \left\{ x \neq 5 \right.$$

8) Solve each equation for x. SHOW WORK!

A.  $\frac{3x}{x+7} - \frac{8}{2(x+7)} = \frac{-22}{x+7}$

$$\left\{ x \neq -7 \right.$$

$$\left\{ x = -6 \right.$$

B.  $\frac{2}{x-6} + \frac{7}{x+2} = \frac{4x+2}{x^2-4x-12}$

$$\left\{ x \neq -2, 6 \right.$$

$$\left\{ x = 8 \right.$$

C.  $\frac{1}{x} + \frac{3}{x-1} = 0$

$$\left\{ x \neq 0, 1 \right.$$

$$\left\{ x = \frac{1}{4} \right.$$

9) Find the x-intercepts

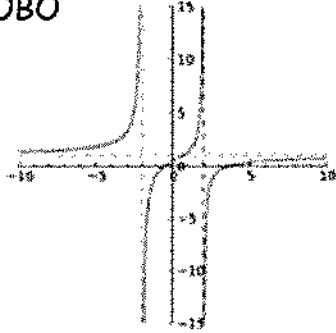
<p>a) <math>y = \frac{7x}{x+5}</math></p> <p><math>(0,0)</math></p>	<p>b) <math>g(x) = \frac{5}{x}</math></p> <p>none</p>	<p>c) <math>y = \frac{x^2+3x}{x^2-16}</math></p> <p><math>(0,0)</math> and <math>(-3,0)</math></p>	<p>d) <math>f(x) = \frac{x^2-x-12}{x^2-2x-8}</math></p> <p><math>(-3,0)</math></p> <p>*do not include the hole</p>
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10) Find The Domain

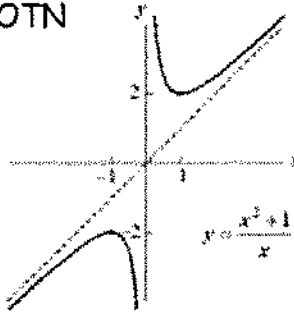
<p>a) <math>f(x) = \frac{x^2+x-12}{6x^2+x-12}</math></p> <p><math>\{x   x \neq -\frac{3}{2}, \frac{4}{3}\}</math></p>	<p>b) <math>g(x) = \frac{x^2-9}{x^3-4x}</math></p> <p><math>\{x   x \neq -2, 0, 2\}</math></p>	<p>c) <math>y = \frac{(x-1)(4x+4)}{(x-3)(x-4)}</math></p> <p><math>\{x   x \neq 3, 4\}</math></p>	<p>d) <math>h(x) = \frac{x-2}{x}</math></p> <p><math>\{x   x \neq 0\}</math></p>
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Three examples of BOBO, BOTN, EATS DC

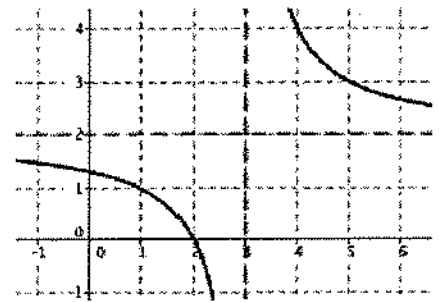
BOBO



BOTN



EATS DC



11) Find the Horizontal Asymptote

<p>a) <math>y = \frac{16x+1}{4x^2-2}</math> BOBO</p> <p><math>y=0</math></p>	<p>b) <math>y = \frac{16x^2+1}{4x^2-2}</math> EATS DC</p> <p><math>y=4</math></p>	<p>c) <math>y = \frac{16x^2+1}{4x-2}</math> BOTN</p> <p>NO HA</p> <p>→ yes, slant (one degree higher)</p>
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12) Find the vertical asymptotes, if any.

<p>a) <math>y = \frac{x}{3x^2-2x-8}</math></p> <p><math>x = -\frac{4}{3}   x = 2</math></p>	<p>b) <math>y = \frac{x+1}{x^2+16x+15}</math></p> <p><math>x = -15</math></p> <p>*do not include the hole</p>	<p>c) <math>y = \frac{x^2}{x^2+9}</math></p> <p>no VA</p>	<p>d) <math>y = \frac{6x}{x-7}</math></p> <p><math>x = 7</math></p>
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13) Find all holes, if any.

a)  $f(x) = \frac{9-x^2}{x+3}$

$(-3, 6)$

b)  $y = \frac{x+5}{5x^2-3x-2}$

none

c)  $y = \frac{5x^2-9x-18}{x^2-3x}$

$(3, 7)$

d)  $y = \frac{x^3+x^2+3x+3}{x^2-1}$

$(-1, -2)$

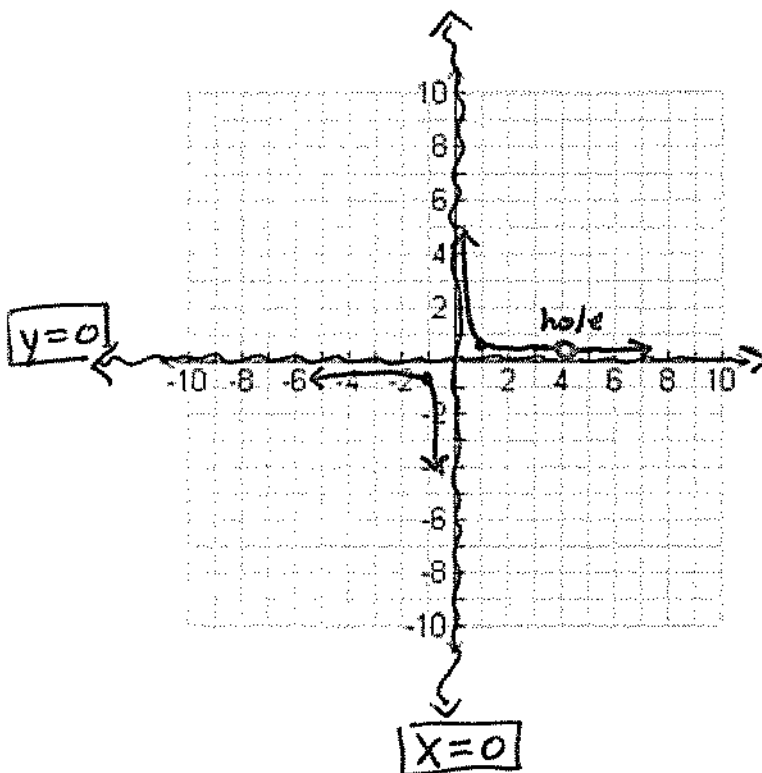
14) Sketch the following rational equations

A)  $y = \frac{x-4}{2x^2-8x}$

BOBO

x-int	none
y-int	none
VA	$x=0$
HA	$y=0$
Holes	$(4, \frac{1}{8})$
Domain	$\{x   x \neq 0, 4\}$
Range	$\{y   y \neq 0, \frac{1}{8}\}$
End Behavior	$x \rightarrow -\infty, y \rightarrow 0$ $x \rightarrow \infty, y \rightarrow 0$

X	Y
1	$\frac{1}{2}$
-1	$-\frac{1}{2}$

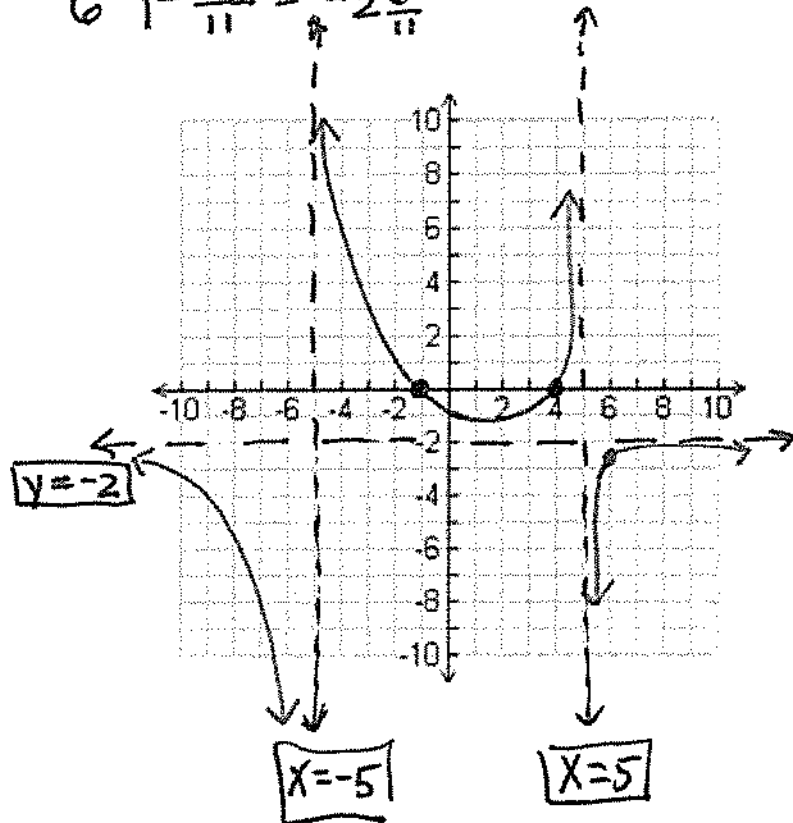


B)  $y = \frac{2x^2-6x-8}{-x^2+25}$

EATS DC

x-int	$(-1, 0); (4, 0)$
y-int	$(0, -\frac{9}{25})$
VA	$x=-5; x=5$
HA	$y=-2$
Holes	none
Domain	$\{x   x \neq -5, 5\}$
Range	skip here
End Behavior	$x \rightarrow -\infty, y \rightarrow -2$ $x \rightarrow \infty, y \rightarrow -2$

X	Y
-6	$-\frac{100}{11}$
6	$-\frac{28}{11} = -2\frac{6}{11}$



15) Find the Least Common Multiple for each of the following pairs of expressions:

a)  $4x^2 - 16$  AND  $6x^2 - 24x + 24$

b)  $x^2 + 5x + 6$  AND  $x^2 + 6x + 8$

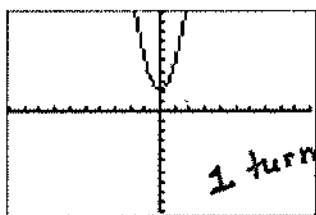
LCM:  $12(x+2)(x-2)^2$

LCM:  $(x+2)(x+3)(x+4)$

16) For each POLYNOMIAL sketch below, state the following:

the number of real zeros, whether the sign of the leading coefficient (a) is positive or negative, the least possible degree and if that value is even or odd.

A)



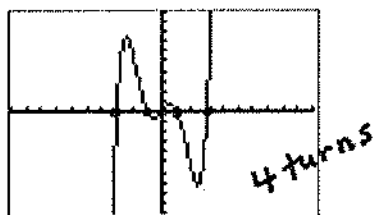
Number of Real Zeros 0

Sign of L.C. (a) +

Least Possible Degree 2

Even or Odd Even

B)



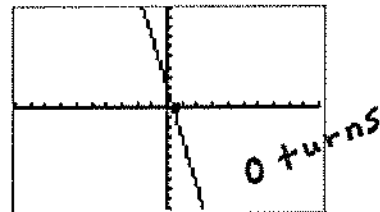
Number of Real Zeros 5

Sign of L.C. (a) +

Least Possible Degree 5

Even or Odd Odd

C)



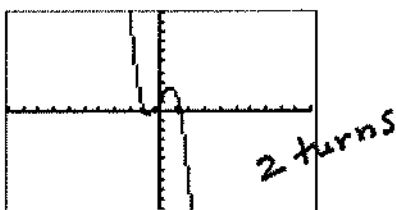
Number of Real Zeros 1

Sign of L.C. (a) -

Least Possible Degree 1

Even or Odd Odd

D)



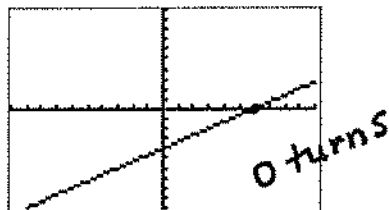
Number of Real Zeros 3

Sign of L.C. (a) -

Least Possible Degree 3

Even or Odd Odd

E)



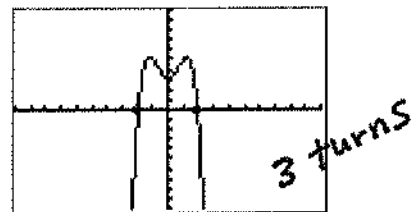
Number of Real Zeros 1

Sign of L.C. (a) +

Least Possible Degree 1

Even or Odd Odd

F)



Number of Real Zeros 2

Sign of L.C. (a) -

Least Possible Degree 4

Even or Odd Even