Sec. 6.7: Dot Product

Dot Product: Given 2 vectors in component form,

u = $< x_1 , y_1 >$ and **v** = $< x_2 , y_2 >$, their dot product is:

dot product (solid dot)

$$< x_1 , y_1 > \bullet < x_2 , y_2 > = x_1x_2 + y_1y_2$$

This product is a ***scalar***, rather than a vector.

dot

If **u • v = 0,** the vectors **u** and **v** are ***orthogonal*** (perpendicular).

(angle btwn = 90°)

Ex1: Find $\langle 2, 3 \rangle \overset{\text{dot}}{\bullet} \langle -4, 5 \rangle$

$(2)(-4) + (3)(5)$

$-8 + 15$

$= \boxed{7}$ not orthogonal ($\perp$) vectors

Ex2: Find $\mathbf{u} \overset{\text{dot}}{\bullet} \mathbf{v}$

Given $\mathbf{u} = i - 2j$ and $\mathbf{v} = 6i + 3j$

$\langle 1, -2 \rangle \underset{\text{dot}}{\bullet} \langle 6, 3 \rangle$

$(1)(6) + (-2)(3)$

$6 + (-6)$

$\boxed{0}$ vector $\vec{u}$ and $\vec{v}$ are orthogonal ($\perp$)

2 vectors are parallel if one is the scalar product of the other one.

so..... $\langle \underset{x_1}{2}, \underset{y_1}{1} \rangle$ and $\langle \underset{x_2}{4}, \underset{y_2}{2} \rangle$ factor

or w/ ratios $\frac{x_1}{x_2} = \frac{y_1}{y_2}$

$\underset{\text{scalar}}{2}\underbrace{\langle 2, 1 \rangle}_{\text{same vector}}$ *factor*

$\frac{2}{4} \times \frac{1}{2}$ cross multiply

$\boxed{4 = 4}$ equal ∴ parallel vectors

Ex.3: Are the vectors orthogonal, parallel, or neither?

a) < 4, 12 > and < 12, 36 >

b) < 2, -3 > and < 6, 4 >

a) $\perp$? $\langle 4,12\rangle \cdot \langle 12,36\rangle$

$(4)(12) + (12)(36) \neq 0$

(not $\perp$)

$\parallel$? $\langle 4,12\rangle$ and $\langle 12,36\rangle$

$\underset{\text{scalar product}}{3}\langle 4, 12\rangle$

$\frac{4}{12} \times \frac{12}{36}$

$144 = 144$

Parallel vectors

b) $\perp$? $\langle 2,-3\rangle \cdot \langle 6,4\rangle$

$(2)(6) + (-3)(4)$

$12 + (-12)$

$= \boxed{0}$

∴ Orthogonal vectors ($\perp$)

Ex.4:Find the value of k so that the vectors $5i+kj$ and $2i+3j$

$\langle 5, K \rangle$ $\langle 2, 3 \rangle$

$x_1\ y_1$ $x_2\ y_2$

a) Parallel:

use ratios $\frac{x_1}{x_2} = \frac{y_1}{y_2}$

cross multiply $\frac{5}{2} \times \frac{K}{3}$

$2K = 15$

$\boxed{K = \frac{15}{2}}$ for $\parallel$ vectors

b) Orthogonal:

dot product $= 0$

$\langle 5, K \rangle \cdot \langle 2, 3 \rangle$ set equal to zero

$(5)(2) + (K)(3) = 0$

$10 + 3K = 0$

$3K = -10$

$\boxed{K = -\frac{10}{3}}$ for $\perp$ vectors

Finding Angles Between 2 Vectors

dot product

$$\cos\theta = \frac{\vec{u} \bullet \vec{v}}{|\vec{u}||\vec{v}|} \quad for\ 0° \le \theta \le 180°$$

magnitudes

restriction

$\theta = \arccos x$

Ex.5: **u** = < 1, 3 > and **v** = < −2, 4 >, find the measure of the angle between **u** and **v**.

$\vec{u} \cdot \vec{v} = (1)(-2) + (3)(4)$
$= -2 + 12$
$= 10$

$\vec{u} : |\langle 1, 3 \rangle|$
$= \sqrt{(1)^2 + (3)^2}$
$= \sqrt{10}$

$\vec{v} : |\langle -2, 4 \rangle|$
$= \sqrt{(-2)^2 + 4^2}$
$= \sqrt{20}$

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$

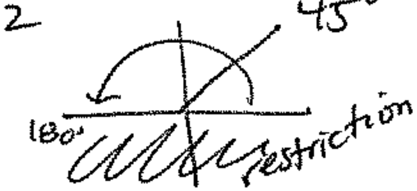
$\cos\theta = \dfrac{10}{(\sqrt{10})(\sqrt{20})}$

$\cos\theta = \dfrac{10}{\sqrt{200}} \quad \sqrt{200} \to 2,\ 100$

$\cos\theta = \dfrac{\cancel{10}^{\,1}}{\cancel{10}\sqrt{2}}$

$\cos\theta = \dfrac{\sqrt{2}}{2}$

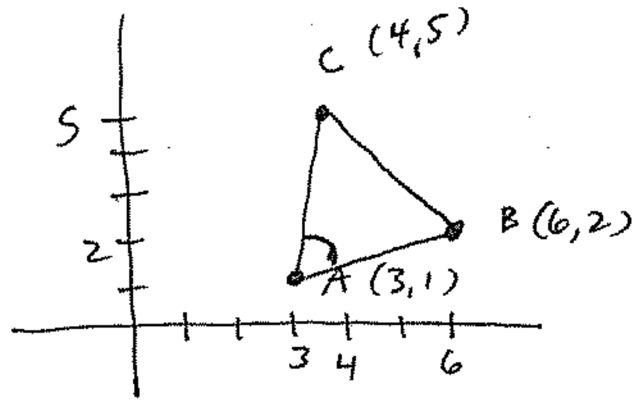
$\boxed{\theta = 45°}$



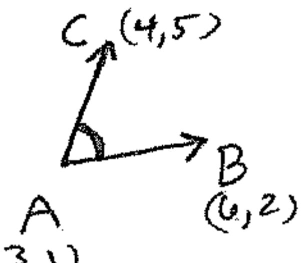
three points (not vectors)

Ex.6: Given A (3, 1), B (6, 2), and C (4, 5) form Δ ABC, find the measure of the interior angles.

Round to the hundreths



① Finding ∡A



draw the vectors out of the angle you are looking for.

Terminal pt minus Initial pt

$\vec{AB}: \langle 6-3,\ 2-1 \rangle$

$\vec{AB}: \langle 3, 1 \rangle$

$\vec{AC}: \langle 4-3,\ 5-1 \rangle$

$\vec{AC}: \langle 1, 4 \rangle$

$$\cos A = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}|\,|\vec{AC}|}$$

$\vec{AB} \cdot \vec{AC}$
$= (3)(1) + (1)(4)$
$= 3 + 4$
$= 7$

$|\vec{AB}| = |\langle 3,1 \rangle|$
$= \sqrt{3^2+1^2}$
$= \sqrt{10}$

$|\vec{AC}| = |\langle 1,4 \rangle|$
$= \sqrt{1^2+4^2} = \sqrt{17}$

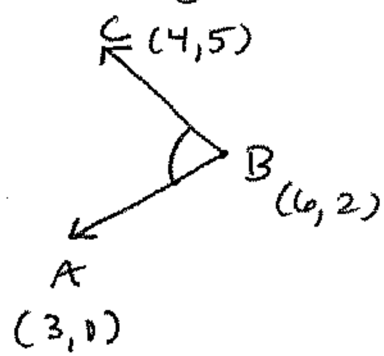
$$\cos A = \frac{7}{(\sqrt{10})(\sqrt{17})}$$

$$\cos A = \frac{7}{\sqrt{170}}$$

$$\measuredangle A = \cos^{-1}\left(\frac{7}{\sqrt{170}}\right)$$

$\measuredangle A \approx 57.53°$

② Finding $\measuredangle B$



$\underline{\underline{\overrightarrow{BA}}} : \langle 3-6,\ 1-2 \rangle = \langle -3, -1 \rangle$

$\underline{\underline{\overrightarrow{BC}}} : \langle 4-6,\ 5-2 \rangle = \langle -2, 3 \rangle$

$$\cos B = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = (-3)(-2) + (-1)(3) = 6 + (-3) = 3$$

$$\|\overrightarrow{BA}\| = |\langle -3, -1 \rangle\| = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10}$$

$$\cos B = \frac{3}{(\sqrt{10})(\sqrt{13})}$$

$$\|\overrightarrow{BC}\| = |\langle -2, 3 \rangle| = \sqrt{(-2)^2 + (3)^2} = \sqrt{13}$$

$$\cos B = \frac{3}{\sqrt{130}}$$

$$\measuredangle B = \cos^{-1}\left(\frac{3}{\sqrt{130}}\right)$$

$$\boxed{\measuredangle B \approx 74.74^\circ}$$

$$\measuredangle C = 180^\circ - (\measuredangle A + \measuredangle B) \quad \boxed{\measuredangle C \approx 47.73^\circ}$$

1) A vector is made up of ______ and ______.

2) If the ______ = 0, then the vectors are ______

3) The arrowhead on a vector indicates ______.

4) To find a vector given 2 pts you use ______ − ______.

5) 2 vectors are ______, if their ratios are ______.

6) In the problem $2\langle 3,-8\rangle$; 2 is called a ______ value.

ANSWERS →

1) A vector is made up of magnitude and direction.

2) If the dot product = 0, then the vectors are orthogonal

3) The arrowhead on a vector indicates direction.

4) To find a vector given 2 pts you use terminal – initial.

5) 2 vectors are parallel, if their ratios are equal.

6) In the problem $2\langle 3,-8\rangle$; 2 is called a scalar value.