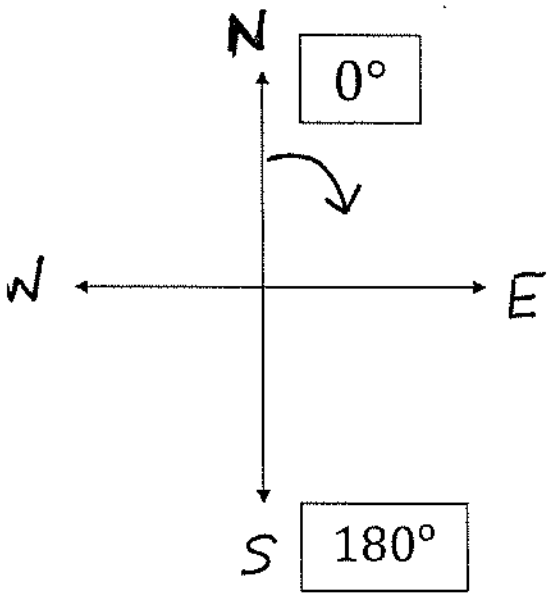
Notes

Pre-Calculus
Sec. 6.6 Vectors
Day 2

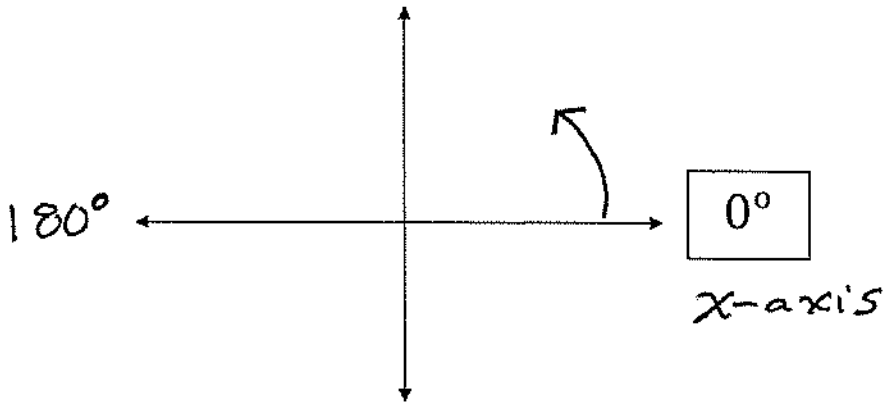
Compass Direction : clockwise

"Bearing"



Trigonometric Direction :counterclockwise

"Direction Angle/ *True Direction"



DEFINITIONS FOR SPECIFIC ANGLES:

1) Direction Angle- the entire angle counterclockwise from the positive x-axis

2) A **bearing** is an angle, measured clockwise from the north direction.

3) A direction bearing is an angle formed by the terminal side and the closest y-axis

$\vec{v} = \|\vec{v}\| \langle \cos\theta, \sin\theta \rangle$

Component Form of a Velocity Vector:

Direction angle

$$v = \langle \| v \| \cos\theta, \| v \| \sin\theta \rangle$$

Speed is the magnitude.

NE, NW, SE, SW (CUT the quadrant in half)

adjective (in that direction)

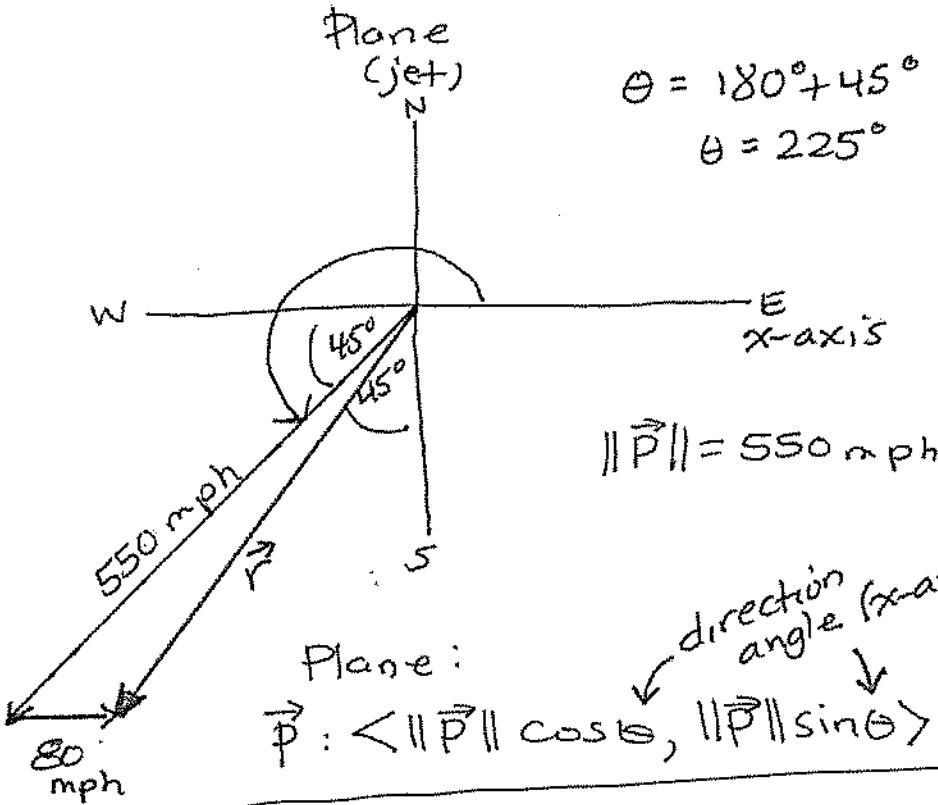
plane — magnitude

Ex. 1) A jet maintains an airspeed of 550 miles per hour in a southwesterly direction. The velocity of the jet stream (wind) is a constant 80 miles per hour *from* the west. Find the actual speed and direction of the aircraft.

SW

opposite direction

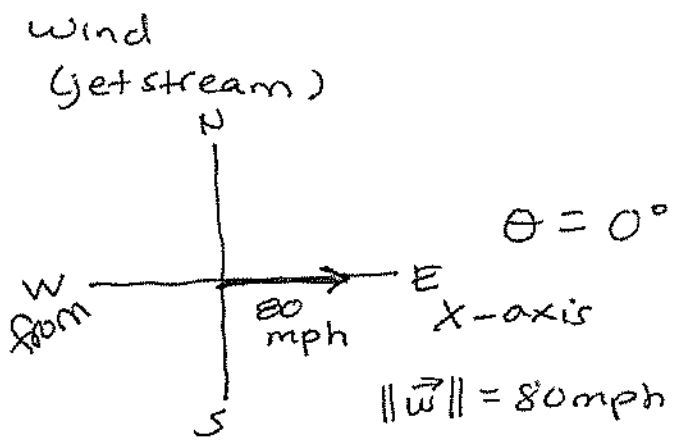
resultant vector



$\theta = 180° + 45°$

$\theta = 225°$

$\|\vec{p}\| = 550$ mph



$\theta = 0°$

$\|\vec{w}\| = 80$ mph

Plane:

direction angle (x-axis)

$\vec{p} : \langle \|\vec{p}\| \cos\theta, \|\vec{p}\| \sin\theta \rangle$

$\vec{p} : \langle 550 \cos 225°, 550 \sin 225° \rangle$

Wind:

$\vec{w} : \langle 80 \cos 0°, 80 \sin 0° \rangle$

continues →

EX 1
Continued

Resultant vector = Plane vector + wind vector

$$\vec{r} = \vec{p} + \vec{w}$$

$$\vec{r} : \langle 550\cos 225° + 80\cos 0°,\ 550\sin 225° + 80\sin 0° \rangle$$

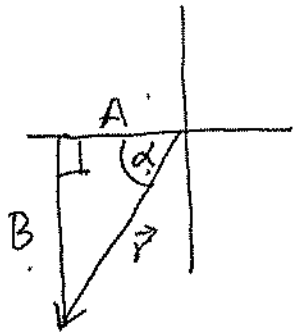
$$\vec{r} : \langle \underset{A}{-308.9087\ldots}, \underset{B}{-388.9087\ldots} \rangle$$

QIII

Store as a positive number under A and B.

Find the reference angle:

$$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$$



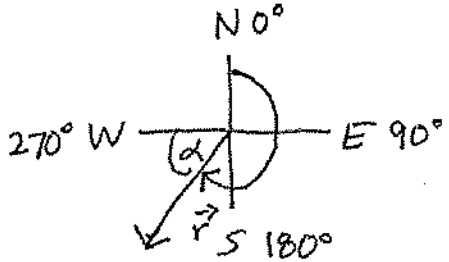
$\alpha = 51.54°$ reference angle.

Speed = magnitude

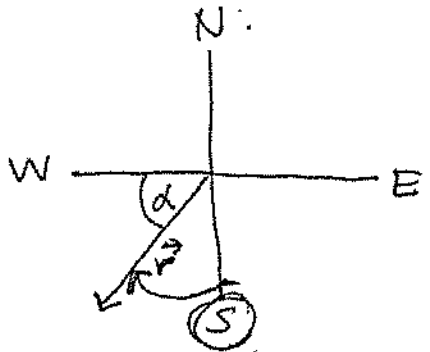
$= \| \vec{r} \|$

$= \sqrt{A^2 + B^2}$

Speed $\approx$ 496.66 mph of the resultant



or



Two ways to give the angle for navigation:

① From North:

$\theta = 270° - \alpha$

$\theta = 218.46°$ from North

or

② $S\ 90° - \alpha\ W$

S 38.46° W

Direction of the

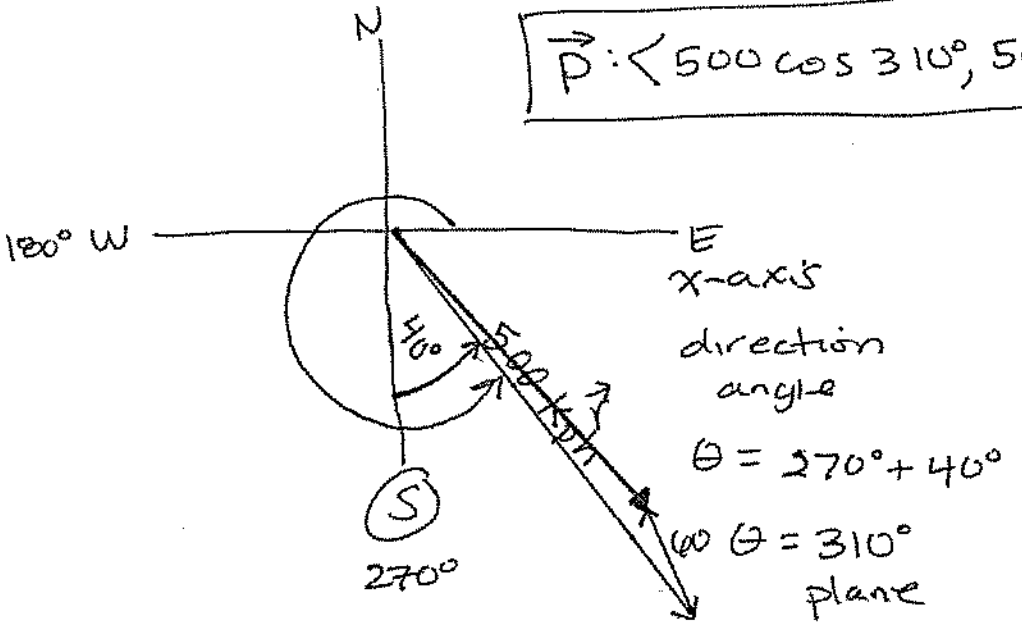
"30° west of North"

"40° east of South"

Ex. 2) An airplane has an airspeed of 500 kilometers per hour bearing S 40° E. The wind velocity is 60 kilometers in the direction N 30° W. Find the resultant vector representing the path of the plane relative to the ground. What is the ground speed of the plane? What is the direction? $\vec{r} = \vec{p} + \vec{w}$

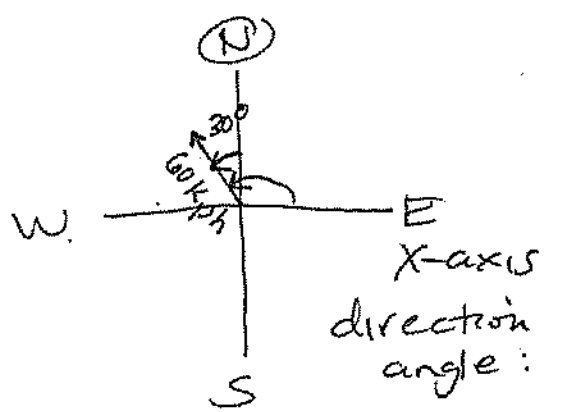
(resultant) (resultant)

Plane:



$\vec{p}: \langle 500\cos 310°, 500\sin 310° \rangle$

Wind:



$\vec{w}: \langle 60\cos 120°, 60\sin 120° \rangle$

resultant: $\vec{r} = \vec{p} + \vec{w}$

$\vec{r}: \langle \underset{A}{291.3938\ldots}, \underset{B}{-331.0606\ldots} \rangle$

Store both as a positive.

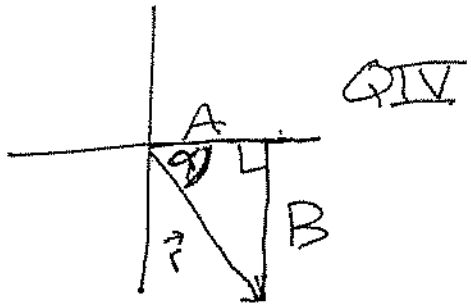
QTV

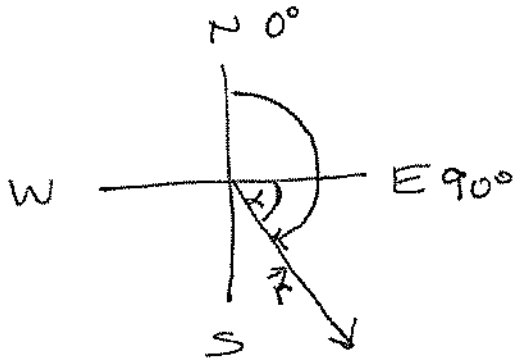
continues →

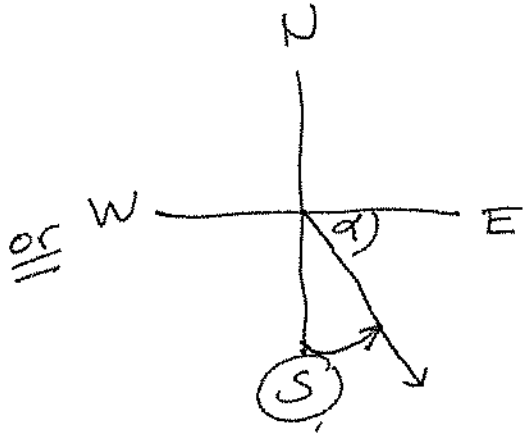
EX. 2
Continued

$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$

$\alpha = 48.65°$







or

Speed = magnitude

$= \|\vec{r}\|$

$= \sqrt{A^2+B^2}$

$\boxed{\text{Speed} \approx 441.03 \text{ kph}}$

From North:

$\theta = 90° + \alpha$

$\theta = \boxed{138.65° \text{ from North}}$ or $\boxed{S\ 41.35°\ E}$

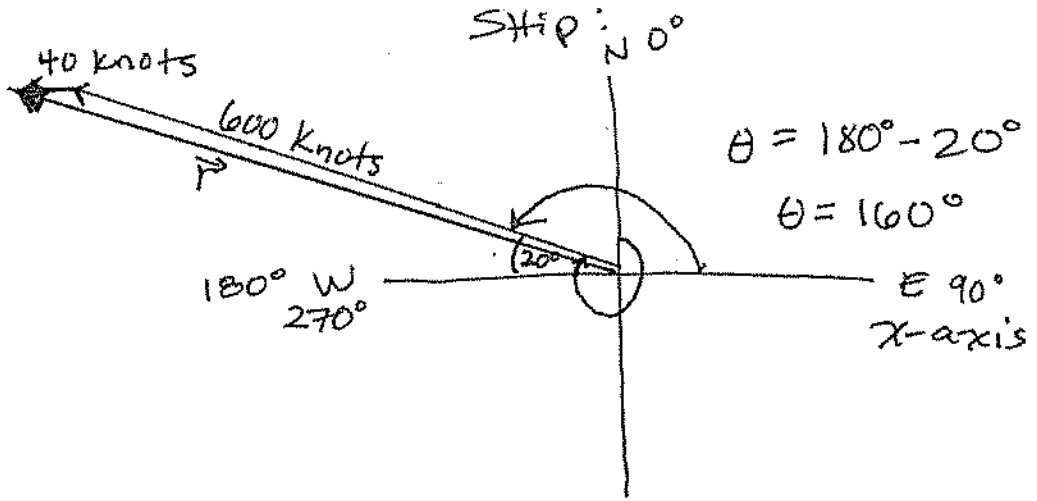
$S\ 90° - \alpha\ E$

"compass"
Implies
from North (navigation)

$\vec{s}$ $\vec{w}$

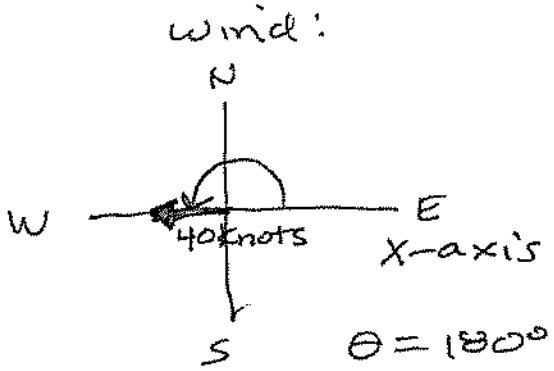
Ex. 3) A ship is 290° at 600 knots, it meets a wind at 40 knots from the east. Find the resultant speed and direction.

opposite



$\theta = 180° - 20°$

$\theta = 160°$



$\theta = 180°$

Ship:

$\vec{s} : \langle 600\cos 160°, 600\sin 160° \rangle$

wind:

$\vec{w} : \langle 40\cos 180°, 40\sin 180° \rangle$

resultant:

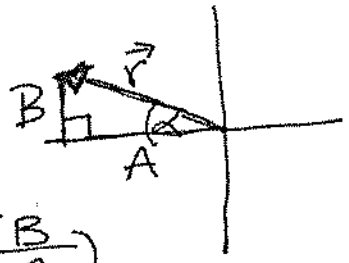
$\vec{r} = \vec{s} + \vec{w}$

$\vec{r} = \langle \overset{A}{-603.8155\ldots}, \overset{B}{205.2120\ldots} \rangle$ QII

Store as positives.

continues →

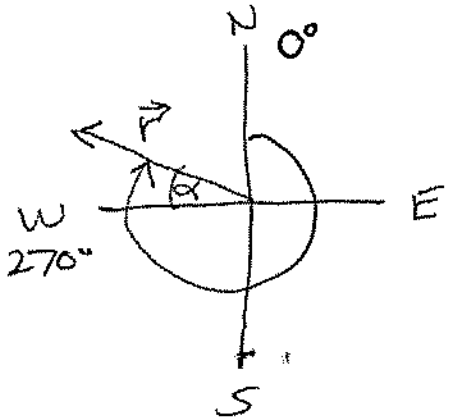
Ex3 continued



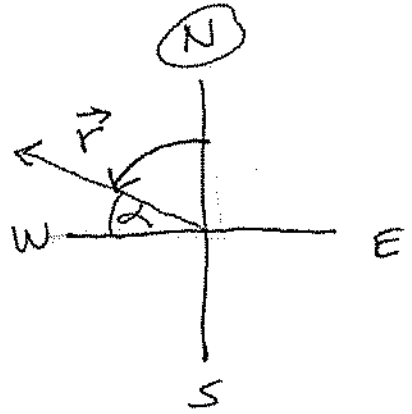
QII

$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$

$\alpha = 18.77°$



or



Speed =

$\|\vec{r}\|$

$= \sqrt{A^2+B^2}$

Speed ≈ 637.73 knots

from north:

$\theta = 270° + \alpha$

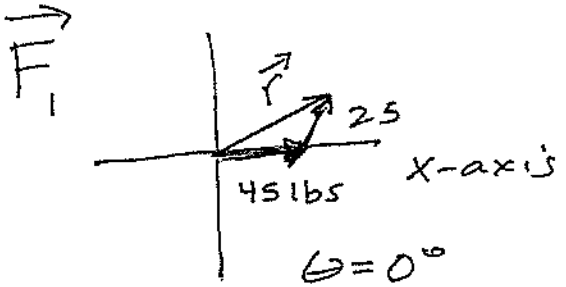
$\theta =$ 288.77° from north

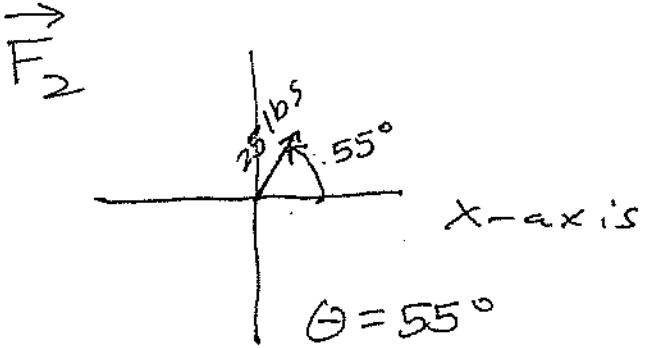
or

N 71.23° W

N $90° - \alpha$ W

Ex. 4) Two forces of 45 pounds and 25 pounds act on a body so that the angle between them is 55°. Assume that the 45 pound force is acting on the positive x-axis. Find the magnitude and direction of the resultant.





$\vec{F}_1 : \langle 45\cos 0°, 45\sin 0° \rangle$

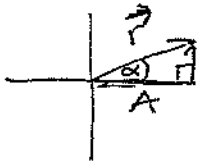
$\vec{F}_2 : \langle 25\cos 55°, 25\sin 55° \rangle$

$$\vec{r} = \vec{F}_1 + \vec{F}_2 + (\ldots \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \ldots \text{etc} \ldots)$$

$$\vec{r} = \langle 59.3394\ldots, 20.4788\ldots \rangle \quad \text{QI}$$

A B

Magnitude $= \|\vec{r}\| = \sqrt{A^2 + B^2}$



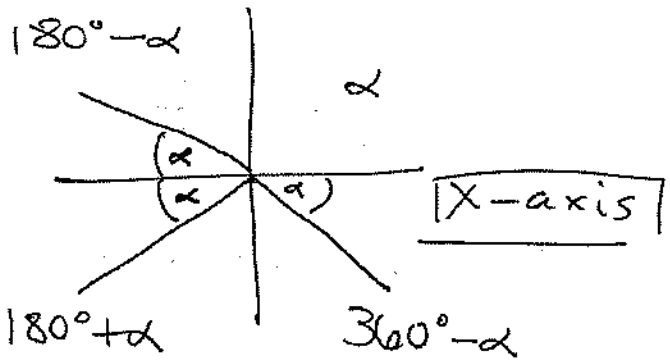
$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$

$\alpha = 19.04°$

Magnitude ≈ 62.77 lbs

19.04° from the positive x-axis

If in another quadrant w/ respect to the Positive x-axis.



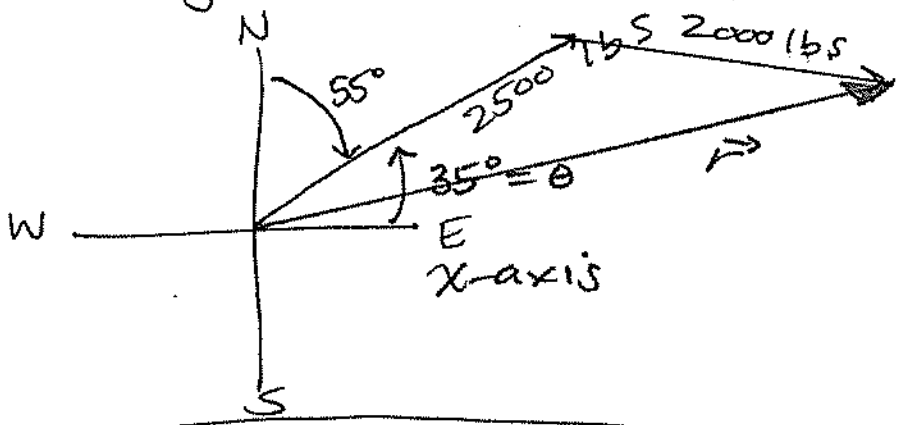
Ex. 5) Two tugboats are pulling on a large ship that has gone aground. One tug pulls with a force of 2500 pounds in a compass direction of 55°. The second tug pulls with a force of 2000 pounds in a compass direction of 95°. Find the magnitude and the compass direction of the resultant force.

From North →

* work on next page →

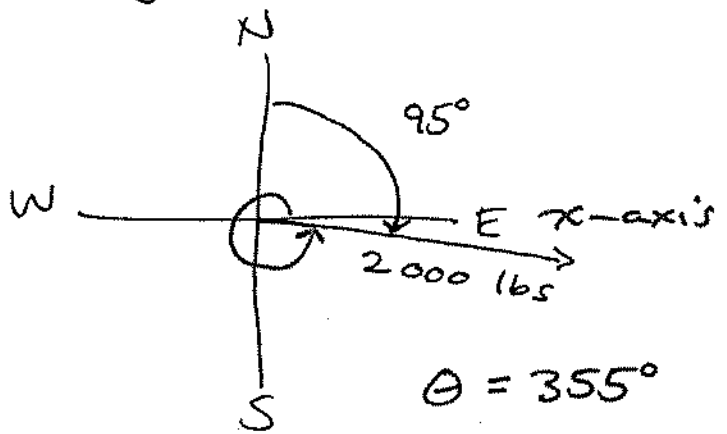
Ex5)

Tugboat #1



$\vec{T}_1 : \langle 2500\cos 35°,\ 2500\sin 35° \rangle$

Tugboat #2

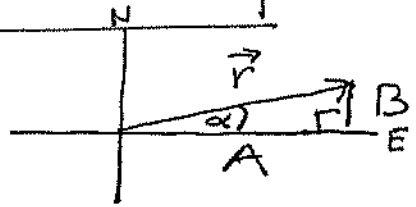


$\theta = 355°$

$\vec{T}_2 : \langle 2000\cos 355°,\ 2000\sin 355° \rangle$

$\vec{r} = \vec{T}_1 + \vec{T}_2$

$\vec{r} = \langle \underset{A}{4040.2695\ldots},\ \underset{B}{1259.6296\ldots} \rangle$ QI



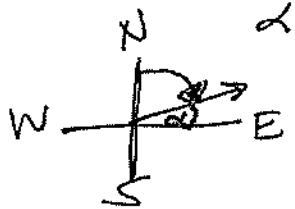
Magnitude =

$\|\vec{r}\| = 4232.071 \text{ lbs}$

$= \sqrt{A^2 + B^2}$

$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$

$\alpha = 17.32°$ $\quad \theta = 90° - \alpha$



72.68° From North

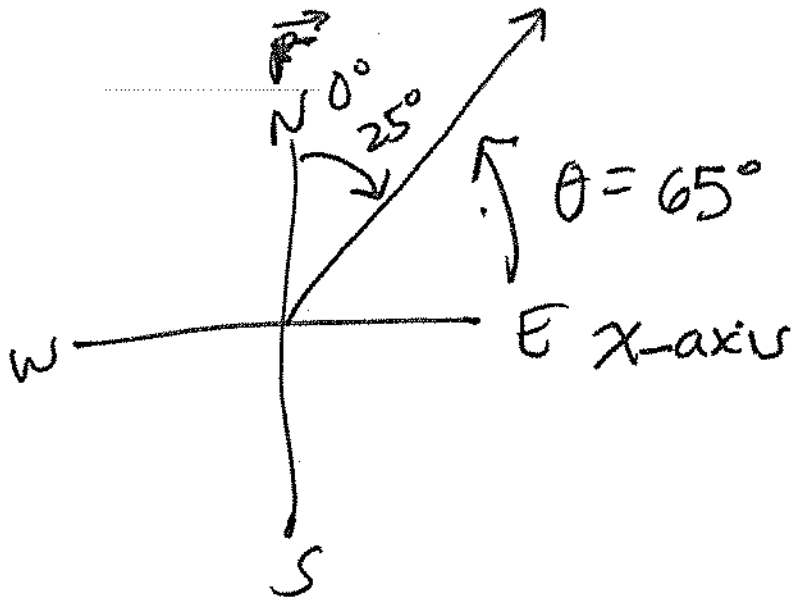
or

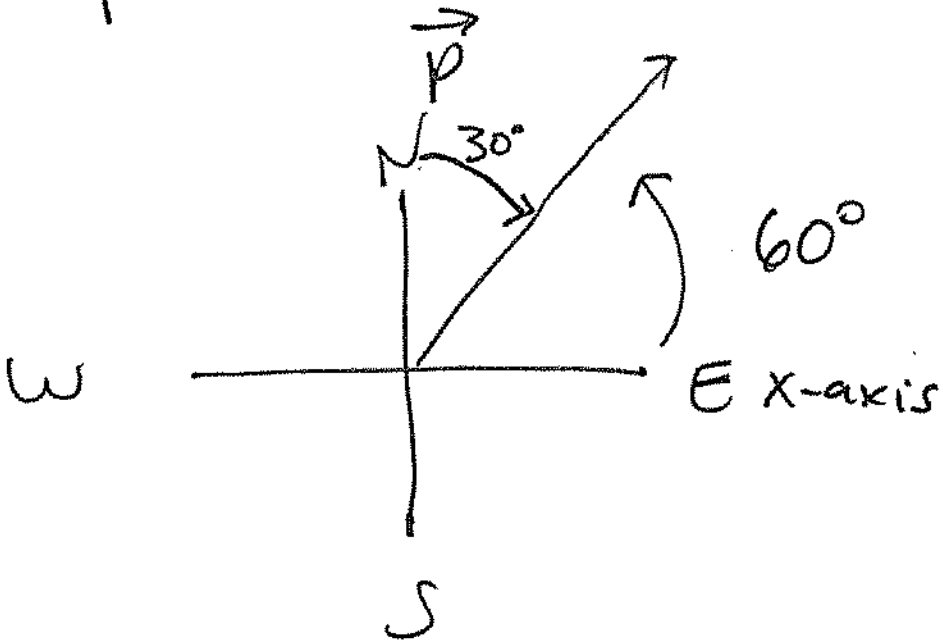
N 72.68° E

Ex. 6) A plane is at N30°E with the speed of 520 mph. It's ground speed is 535 mph and its true course is 25°. Find the speed and direction angle, to the nearest tenth, of the wind.

Ex. 6) A plane is at N30°E with the speed of 520 mph. It's ground speed is 535 mph and its true course is 25°. Find the speed and direction angle, to the nearest tenth, of the wind.

$\vec{w} = \vec{r} - \vec{p}$





$\vec{r}$: $\langle 535\cos 65°, 535\sin 65° \rangle$

$\vec{p}$: $\langle 520\cos 60°, 520\sin 60° \rangle$

$\vec{w} = \vec{r} - \vec{p}$

$\vec{w}$: $\langle -33.8992\ldots, 34.54145\ldots \rangle$
A B

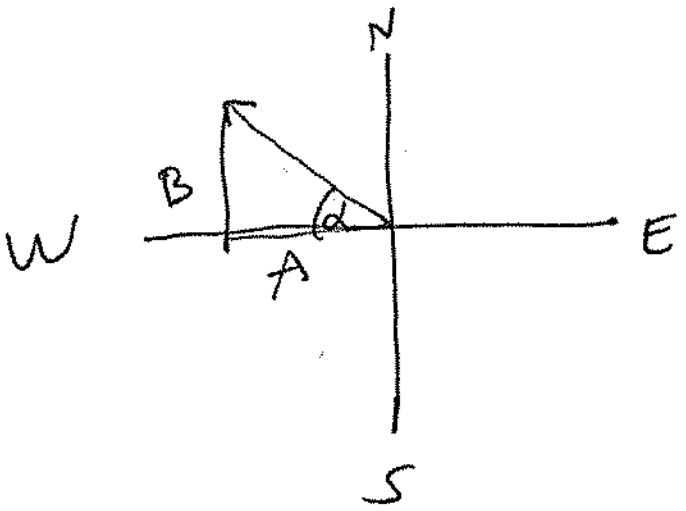
store as positives

QII

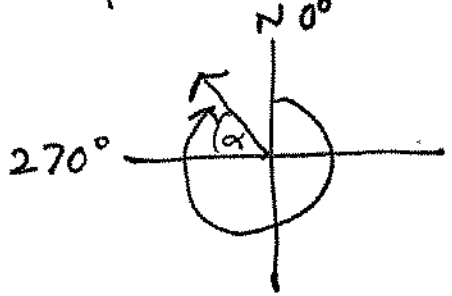
EX6 continued

Wind speed $= \sqrt{A^2+B^2}$

$\approx$ **48.40 mph**



From North:

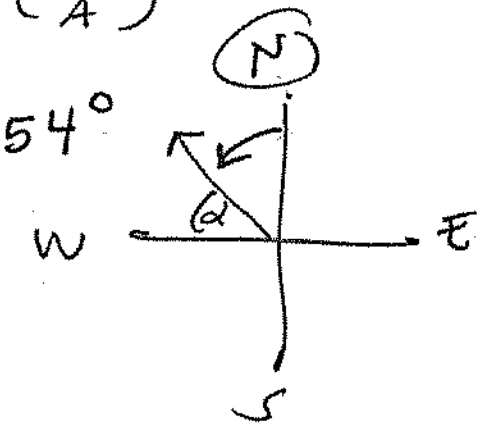


$\theta = 270° + \alpha$

θ = 315.54° from North or

$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$

$\alpha = 45.54°$



N 90° − α W

N 44.46° W