Notes

6.6 Vectors in the Plane

Objective: To represent vectors as directed line segments, write the component forms of vectors, perform basic vector operations and represent vectors graphically, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

Vectors consist of change in the x and y direction. These numbers are called the ***components*** of the vector.

Definitions:

• need both to be a vector

Vector: ***Quantities described by a direction and magnitude (size)***

Vectors are represented by an: ***arrowhead*** (shows direction) ↗

Examples of vectors

Velocity represents: ***direction of movement and speed***

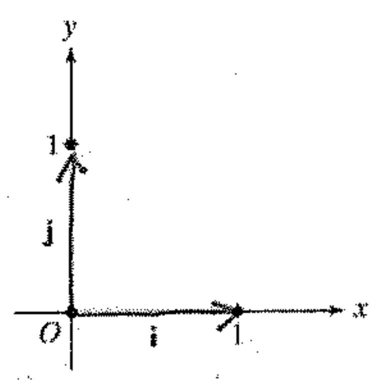
Force represents: ***direction and strength***

← magnitude

In this PreCalculus book vectors are represented as **ai+bj (linear combination) form instead of $\langle a, b \rangle$. Component form

The i and j Unit Vectors

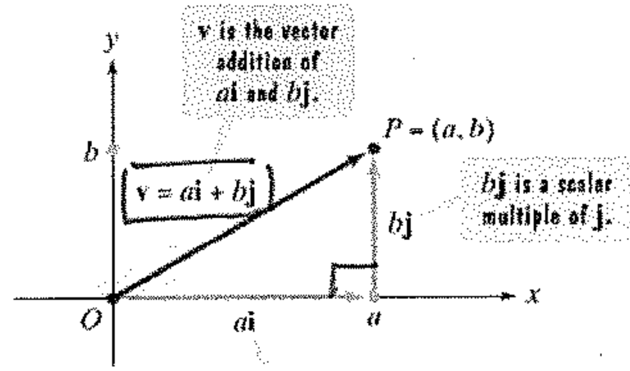
Vector **i** is the unit vector whose direction is along the positive x-axis. Vector **j** is the unit vector whose direction is along the positive y-axis.



$\langle a, b \rangle$ Vector

(a, b) ordered pair

Why are the unit vectors **i** and **j** important? Vectors in the rectangular coordinate system can be represented in terms of **i** and **j**. For example, consider vector **v** with initial point at the origin, (0, 0), and terminal point at $P = (a, b)$. The vector **v** is shown in **Figure 6.55.** We can represent **v** using **i** and **j** as $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$.



"vector" (0,0)

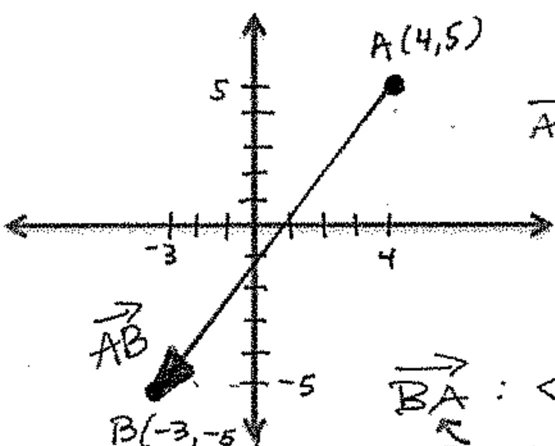
The *directed line segment* whose intial point is the origin is often the most convenient representative of a set of equivalent directed line segments. This representative of a vector v is referred to as : *standard position vector*

Vector notation:

magnitude (absolute value)

Example. Given Point A is $(4,5)$ (Initial) and Point B is $(-3,-5)$ (Terminal). Find $\overrightarrow{AB}$ (vector) and $\overrightarrow{BA}$. Find $\left\|\overrightarrow{AB}\right\|$.

initial Terminal



$\overrightarrow{AB} : \langle(-3-4), (-5-5)\rangle$

$\boxed{\overrightarrow{AB} : \langle -7, -10 \rangle}$ component

or $-7i - 10j$ linear combination

$\left\|\overrightarrow{AB}\right\| = \sqrt{(-7)^2 + (-10)^2}$

$= \sqrt{49 + 100}$

$\boxed{\left\|\overrightarrow{AB}\right\| = \sqrt{149}}$

$\overrightarrow{BA} : \langle(4-(-3)), (5-(-5))\rangle$

Terminal $\boxed{\overrightarrow{BA} : \langle 7, 10 \rangle}$

opposite direction gives opposite signs

$\overrightarrow{AB}$ & $\overrightarrow{BA}$ have the same magnitude

To find the component form of a vector you do the following operation:

Terminal point subtract initial point: $\left\langle (x_2 - x_1), (y_2 - y_1) \right\rangle$ ← $\langle a, b \rangle$ component form

T − I

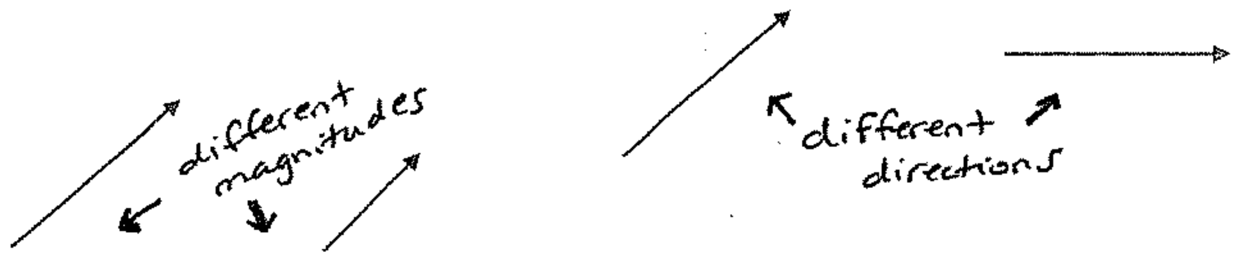
To find the magnitude (aka. Speed/size/length/weight) of a vector (distance formula)

$$\left\|\vec{v}\right\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given : $\langle a, b \rangle$ component form

$\|\vec{v}\| = \sqrt{a^2 + b^2}$ magnitude

Two vectors are equal if they have the same magnitude and direction.



$|\vec{u}| = 7$ $|\vec{w}| = 5$ $|\vec{z}| = 7$ $|\vec{m}| = 7$

Which vectors are equal? $\vec{u}$ and $\vec{z}$

Example #2: Given A = $(5,1)$ and B = $(1,-5)$. Give the component form of $\overrightarrow{AB}$ and $\|\overrightarrow{AB}\|$.

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$\overrightarrow{AB}$: $\langle (1-5), (-5-1) \rangle$

$\boxed{\overrightarrow{AB}: \langle -4, -6 \rangle}$

$\|\overrightarrow{AB}\| = \sqrt{(-4)^2 + (-6)^2}$

$= \sqrt{16+36}$

$= \sqrt{52}$

$\boxed{\|\overrightarrow{AB}\| = 2\sqrt{13}}$

2|52
2|26
13

Example #2b: Given A = $(-9,1)$ and B = $(-2,8)$. Give the component form of $\overrightarrow{BA}$ and $\|\overrightarrow{BA}\|$.

T I

$\overrightarrow{BA}$: $\langle (-9-(-2)), (1-8) \rangle$

$\boxed{\overrightarrow{BA}: \langle -7, -7 \rangle}$

$\|\overrightarrow{BA}\| = \sqrt{(-7)^2 + (-7)^2}$

$= \sqrt{49+49}$

$= \sqrt{98}$

$\boxed{\|\overrightarrow{BA}\| = 7\sqrt{2}}$

2|98
7|49
7

Example #3: If $\mathbf{v} = \langle 1,3 \rangle$ and $\mathbf{u} = \langle -2,-5 \rangle$ find:

a. $\boxed{3}\mathbf{v}$

scalar multiplication (indicates size)

$3\vec{v}$

$= 3\langle 1,3 \rangle$

$= \boxed{\langle 3, 9 \rangle}$

or $3i + 9j$

b. $\mathbf{v} + \mathbf{u}$

vector addition

$\langle 1,3 \rangle + \langle -2,-5 \rangle$

add x's ; add y's

$\langle (1+(-2)), (3+(-5)) \rangle$

$= \boxed{\langle -1, -2 \rangle}$

or $-i - 2j$

c. $\mathbf{v} - \mathbf{u}$

$\vec{v} + (-\vec{u})$

opposite change the signs

$\langle 1,3 \rangle + \langle +2, +5 \rangle$

$= \boxed{\langle 3, 8 \rangle}$

or $3i + 8j$

d. $2\mathbf{v} + 3\mathbf{u}$

$2\langle 1,3 \rangle + 3\langle -2,-5 \rangle$

$\langle 2,6 \rangle + \langle -6,-15 \rangle$

$= \boxed{\langle -4, -9 \rangle}$

or $-4i - 9j$

DEFINITIONS

Fill in the blanks.

1) A vector can be used to represent a quantity that involves both magnitude and direction.
2) The directed line segment $\overrightarrow{PQ}$ has initial point P and terminal point Q.
3) The magnitude of the directed line segment $\overrightarrow{PQ}$ is denoted by $\|\overrightarrow{PQ}\|$
4) The directed line segment whose initial point is the origin (0,0) is said to be in standard position. Also called the position vector.
5) The two basic vector operations are scalar multiplication and vector addition.
6) The vector u + v is called the resultant of vector addition.

There are 2 methods to choose from when adding vectors.

Method #1 : Estimate the components of each vector and perform the indicated operation to find the resultant vector Use when no numbers are given.

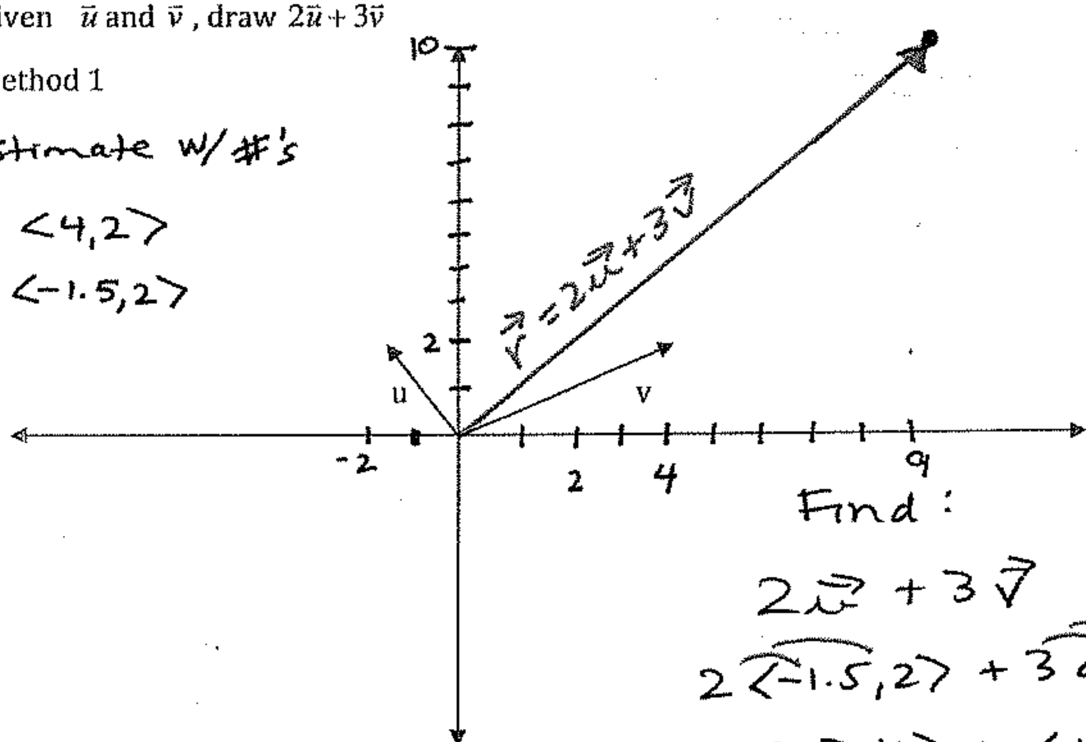
Given $\vec{u}$ and $\vec{v}$, draw $2\vec{u}+3\vec{v}$

Method 1

Estimate w/ #'s

$\vec{v}: \langle 4,2\rangle$

$\vec{u}: \langle -1.5,2\rangle$



Find:

$2\vec{u}+3\vec{v}$

$2\langle -1.5,2\rangle + 3\langle 4,2\rangle$

$\langle -3,4\rangle + \langle 12,6\rangle$

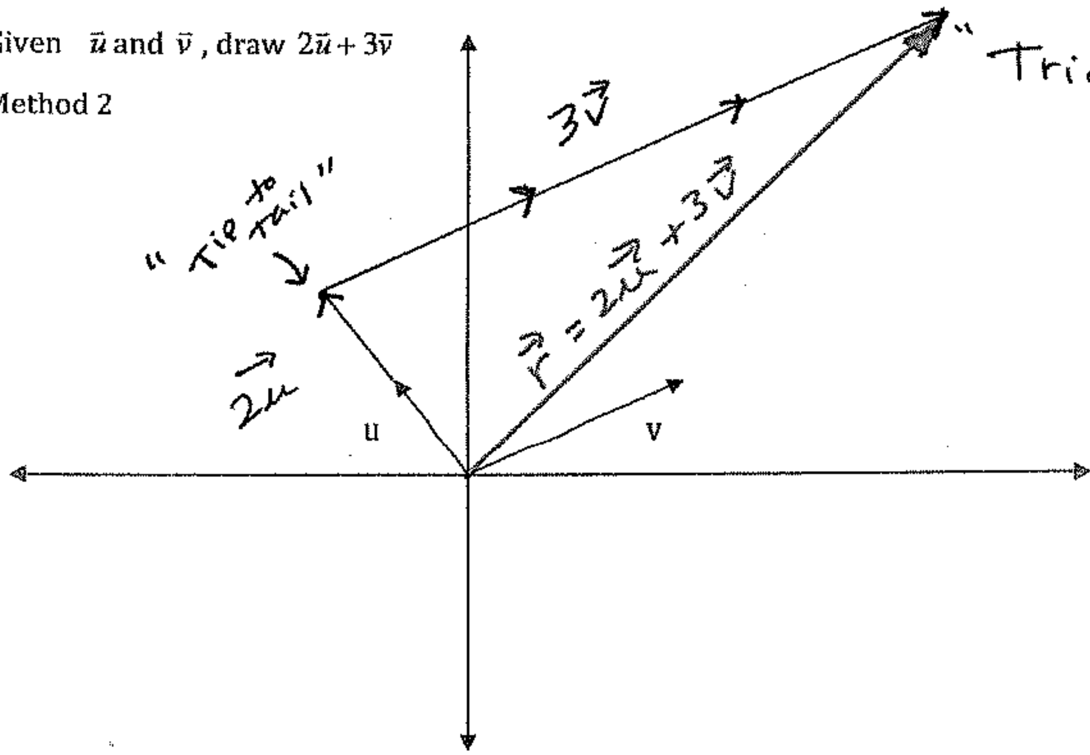
$= \langle 9,10\rangle$

resultant

Method#2: Use a ruler and measure each vector and draw "Tip to Tail" (terminal point of the first vector connects to the initial point of the second vector) to find the resultant vector.

Given $\vec{u}$ and $\vec{v}$, draw $2\vec{u}+3\vec{v}$

Method 2



"Triangular method"

From earlier:

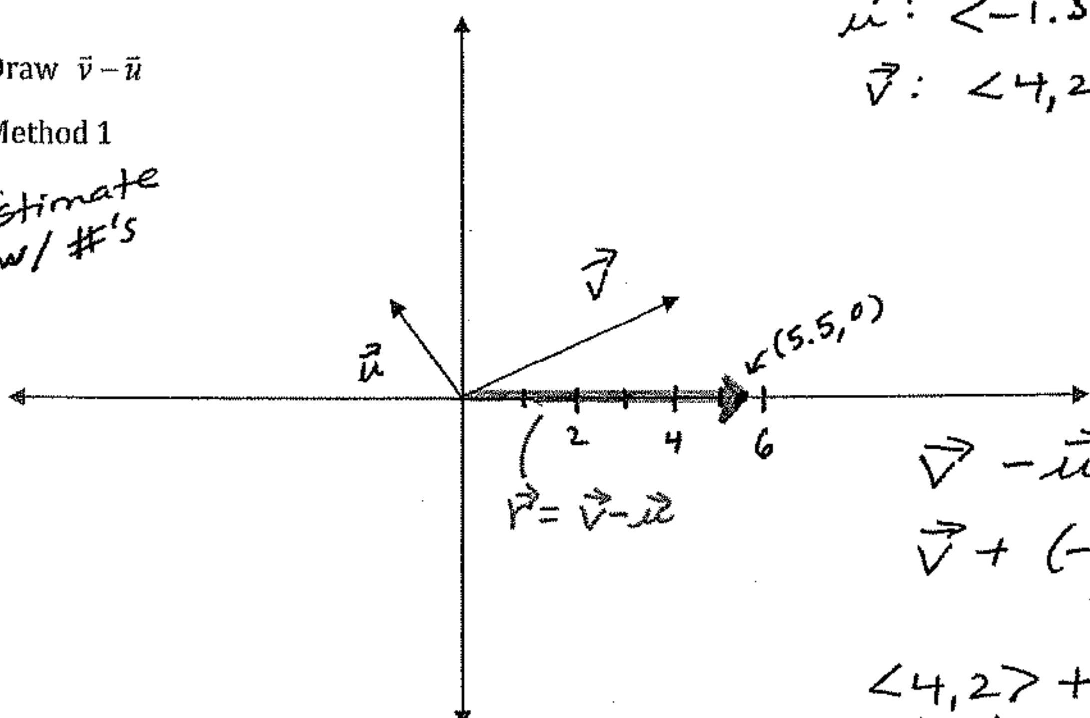
$\vec{u}: \langle -1.5, 2 \rangle$

$\vec{v}: \langle 4, 2 \rangle$

Draw $\vec{v}-\vec{u}$

Method 1

Estimate w/ #'s



$\vec{v} - \vec{u}$

$\vec{v} + (-\vec{u})$ change signs

$\langle 4, 2 \rangle + \langle 1.5, -2 \rangle$

$= \langle 5.5, 0 \rangle$ resultant

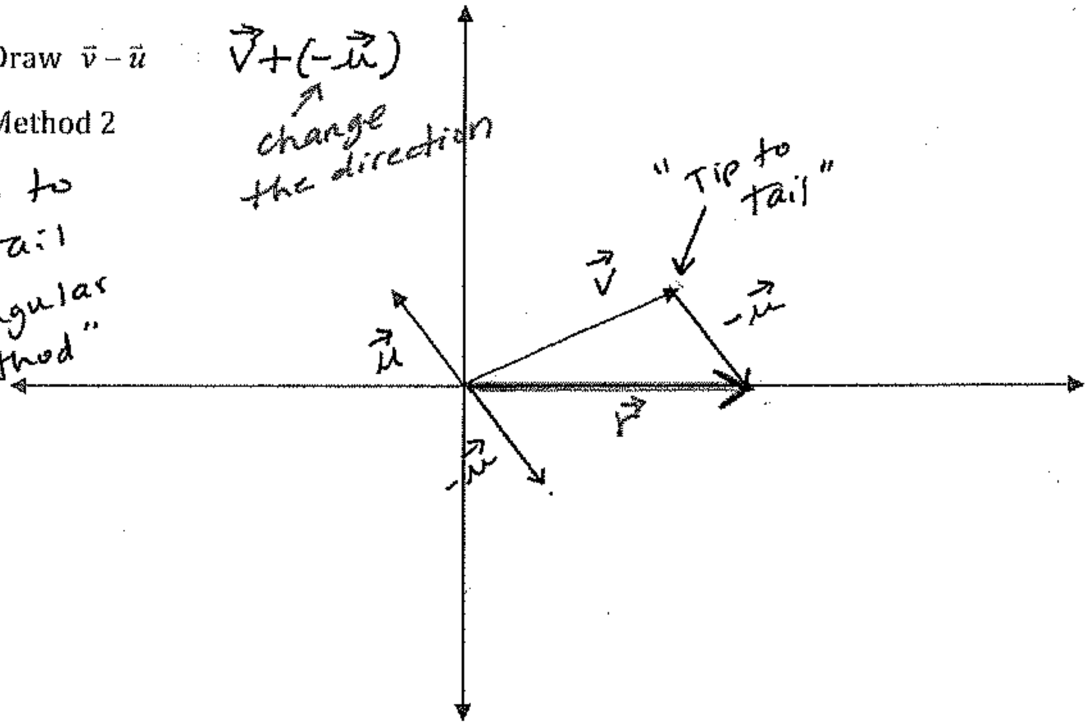
Draw $\vec{v} - \vec{u}$: $\vec{v} + (-\vec{u})$

change the direction

Method 2

Tip to tail

"Triangular method"



If **v** = a**i** + b**j**, and has a direction angle θ, then $\boxed{\mathbf{v} = \|\mathbf{v}\|\langle\cos\theta, \sin\theta\rangle}$ and the direction angle can be determined from $\tan\theta = \frac{b}{a}$.

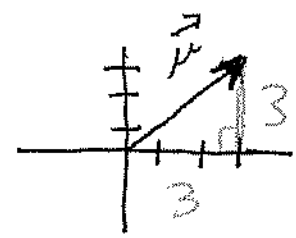
Component form: $\langle a, b\rangle$

Example #5: Find the direction angle of the vector **u**= 3**i** + 3**j**.

- measured from the positive x-axis: $[0, 360°)$ or $[0, 2\pi)$

$\langle 3, 3\rangle$

$\langle a, b\rangle$



$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{3}{3}$$

$$\tan\theta = 1$$

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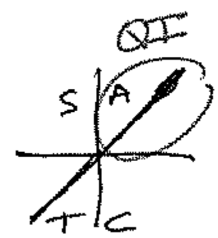

$$\boxed{\theta = 45° \text{ or } \frac{\pi}{4}}$$

direction angle

$$\|\vec{u}\| = \sqrt{a^2 + b^2}$$

$$= \sqrt{3^2 + 3^2}$$

$$= \sqrt{18} <^{2}_{9}$$

$$\|\vec{u}\| = 3\sqrt{2}$$

Let's check:

$$\vec{u} = \|\vec{u}\| \langle\cos\theta, \sin\theta\rangle$$

$$= \underbrace{3\sqrt{2}}_{\text{scalar}} \langle\cos 45°, \sin 45°\rangle$$

$$= \langle 3\sqrt{2}\cos 45, 3\sqrt{2}\sin 45°\rangle$$

$$= \langle 3\sqrt{2}\left(\tfrac{\sqrt{2}}{2}\right), 3\sqrt{2}\left(\tfrac{\sqrt{2}}{2}\right)\rangle = \langle 3, 3\rangle$$ The Given Vector ✓