6.6 Vectors in the Plane

Objective: To represent vectors as directed line segments, write the component forms of vectors, perform basic vector operations and represent vectors graphically, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

Vectors consist of change in the x and y direction. These numbers are called the _***Components*** of the vector.

Definitions:

Vector: ***Quantities described by a direction and magnitude (size)***

Vectors are represented by an: ***arrowhead***

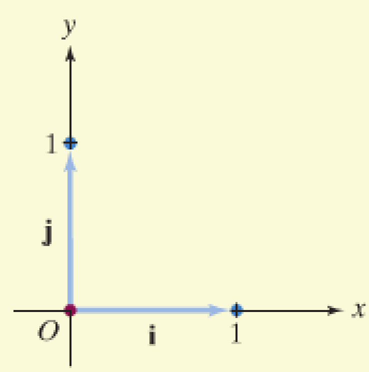
Velocity represents: ***direction of movement and speed***

Force represents: ***direction and strength***

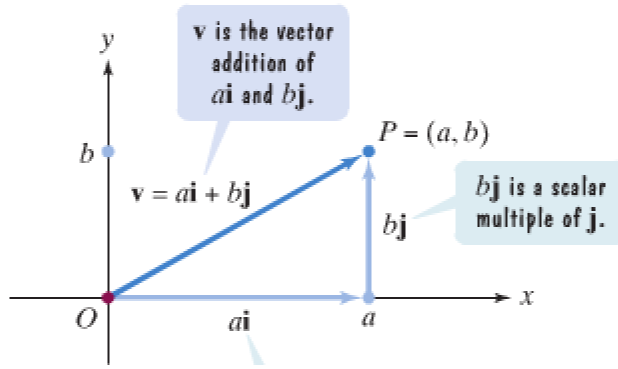
In this PreCalculus book vectors are represented as **ai+bj (linear combination) instead of $\langle a, b\rangle$.

The i and j Unit Vectors

Vector **i** is the unit vector whose direction is along the positive x-axis. Vector **j** is the unit vector whose direction is along the positive y-axis.



Why are the unit vectors **i** and **j** important? Vectors in the rectangular coordinate system can be represented in terms of **i** and **j**. For example, consider vector **v** with initial point at the origin, (0, 0), and terminal point at $P = (a, b)$. The vector **v** is shown in **Figure 6.55**. We can represent **v** using **i** and **j** as $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$.

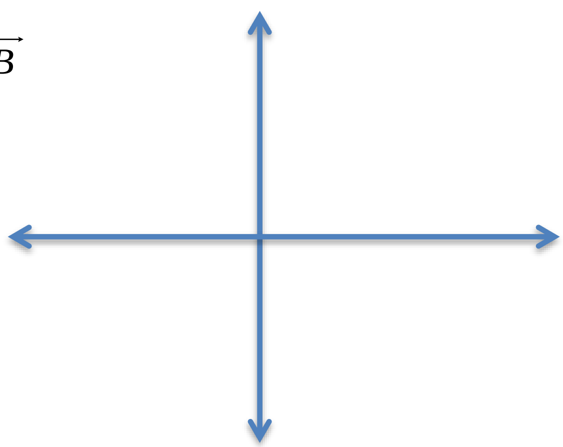


The *__directed line segment__* whose intial point is the origin is often the most convenient representative of a set of equivalent directed line segments. This representative of a vector v is referred to as : *__standard position vector__*

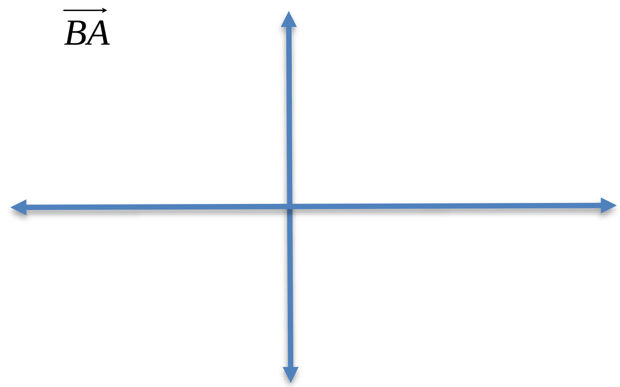
Vector notation: $\langle \boldsymbol{a}, \boldsymbol{b} \rangle$

Example. Given Point A is $(4,5)$ and Point B is $(-3,-5)$. Find $\overrightarrow{AB}$ and $\overrightarrow{BA}$. Find $\left\|\overrightarrow{AB}\right\|$.

$\overrightarrow{AB}$



$\overrightarrow{BA}$



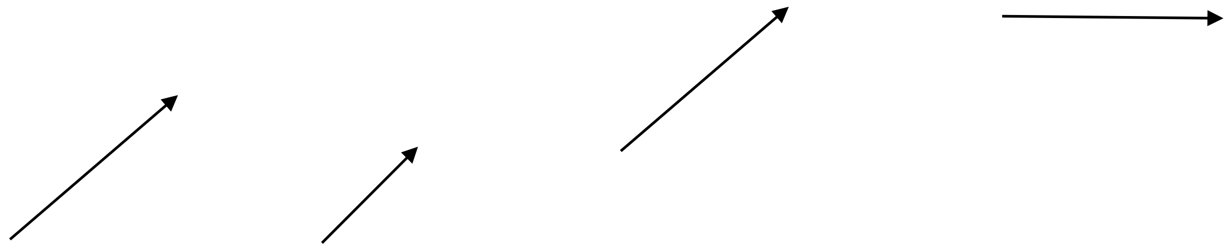
To find the component form of a vector you do the following operation:

Terminal point subtract initial point: $\left[(x_2 - x_1),(y_2 - y_1) \right]$

To find the magnitude (aka. Speed/size/length/weight) of a vector (distance formula)

$$\left\|\vec{v}\right\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Two vectors are ________________________ if they have the same ________________ and
____



$|\vec{u}| = 7$ $\qquad$ $|\vec{w}| = 5$ $\qquad$ $|\vec{z}| = 7$ $\qquad$ $|\vec{m}| = 7$

Which vectors are equal?____

Example #2: Given A = $(5,1)$ and B = $(1,-5)$. Give the component form of $\overrightarrow{AB}$ and $\left\|\overrightarrow{AB}\right\|$.

Example #2b :Given A = $(-9,1)$ and B = $(-2,8)$. Give the component form of $\overrightarrow{BA}$ and $\left\|\overrightarrow{BA}\right\|$.

Example #3: If $\mathbf{v} = \langle 1,3 \rangle$ and $\mathbf{u} = \langle -2,-5 \rangle$ find:

a. $3\mathbf{v}$ $\qquad$ b. $\mathbf{v} + \mathbf{u}$ $\qquad$ c. $\mathbf{v} - \mathbf{u}$ $\qquad$ d. $2\mathbf{v} + 3\mathbf{u}$

DEFINITIONS

Fill in the blanks.

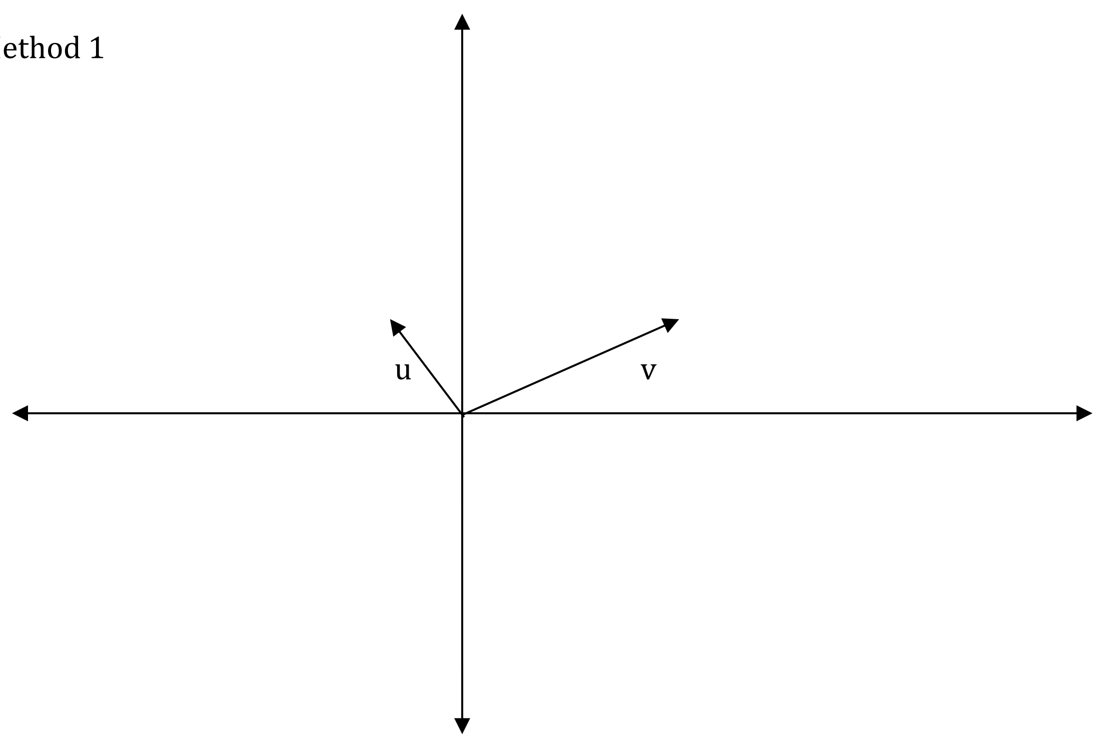
1) A ________________________________ can be used to represent a quantity that involves both magnitude and direction.
2) The directed line segment $\overrightarrow{PQ}$ has _____________ point P and _______________ Q.
3) The _________________ of the directed line segment $\overrightarrow{PQ}$ is denoted by $\|\overrightarrow{PQ}\|$
4) The directed line segment whose initial point is the origin is said to be in ____________________________
5) The two basic vector operations are scalar______________ and vector________________.
6) The vector **u + v is called the ________________ of vector addition**

There are 2 methods to choose from. Method #1 : Estimate the components of each vector and perform the indicated operation to find the resultant vector

Method#2: Use a ruler and measure each vector and draw "Tip to Tail" to find the resultant vector

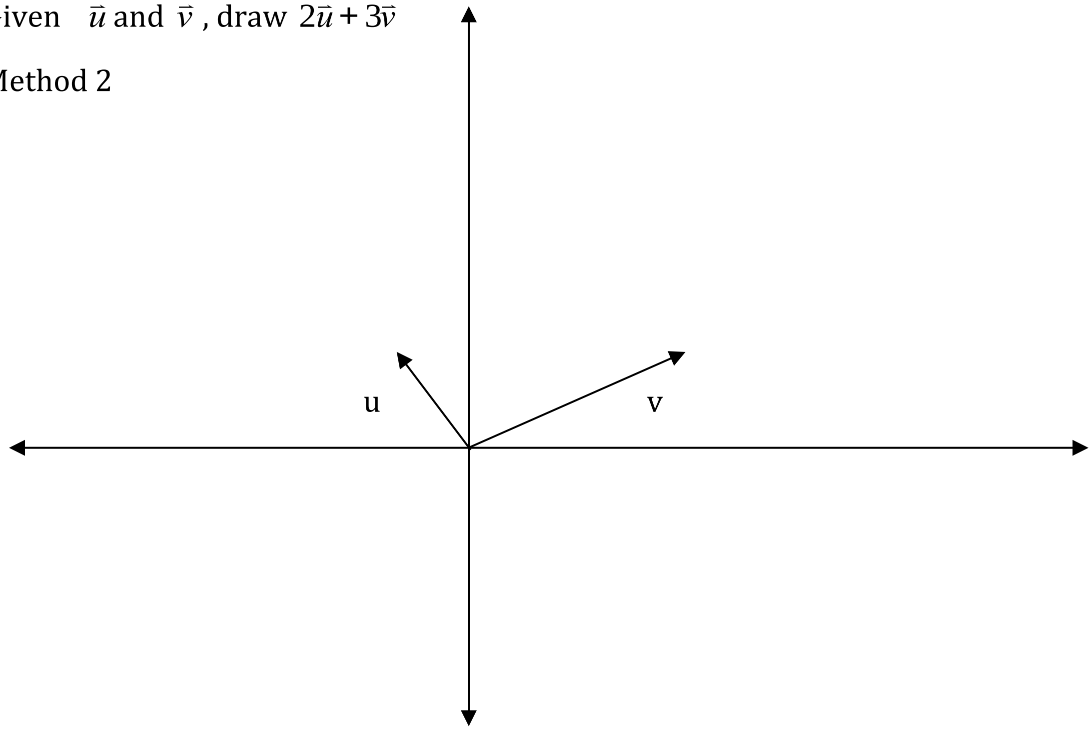
Given $\vec{u}$ and $\vec{v}$, draw $2\vec{u}+3\vec{v}$

Method 1



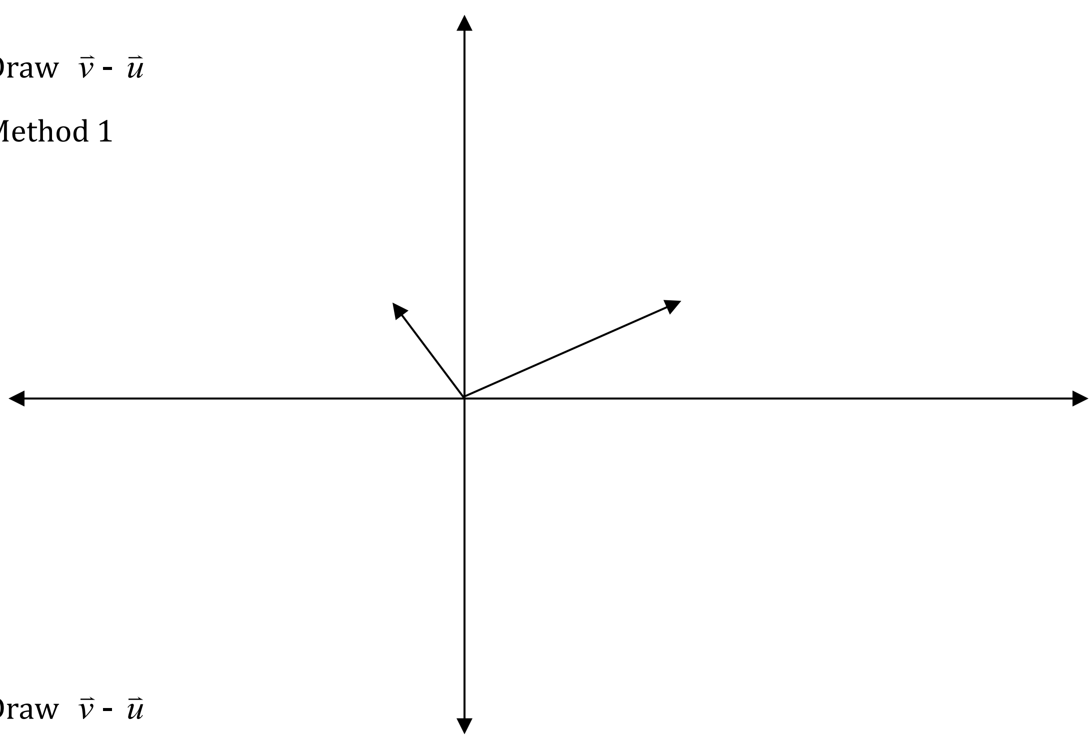
Given $\vec{u}$ and $\vec{v}$, draw $2\vec{u}+3\vec{v}$

Method 2



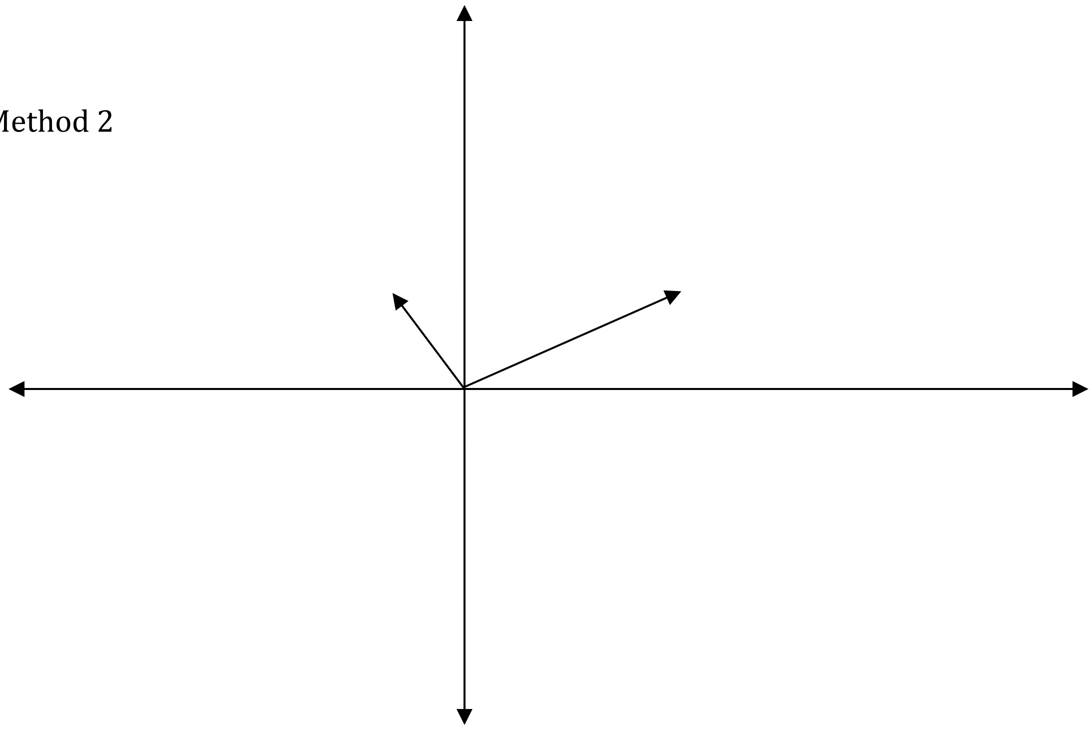
Draw $\vec{v}$ - $\vec{u}$

Method 1



Draw $\vec{v}$ - $\vec{u}$

Method 2



If **v**=**ai** + b**j**, and has a direction angle θ, then **v**= $\|v\|\langle \cos\theta, \sin\theta \rangle$, and the direction angle can be determined from ______________________________________.

Example #5: Find the direction angle of the vector **u=** 3**i** + 3**j.**