Notes

Pre-Calculus
Sec. 3.4
Exponential and Logarithmic Equations
(day 3)

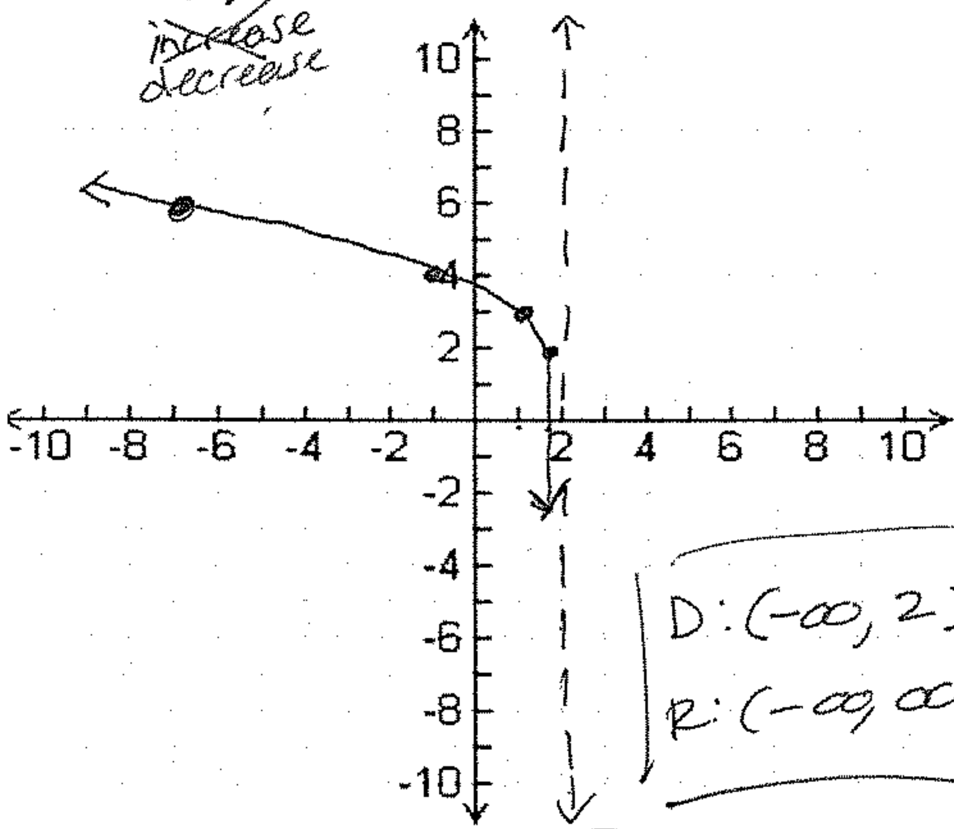
Review :

Graph. State domain and range in interval notation.

$f(x) = \log_3(2 - x) + 3$

$3 > 1$

~~increase~~ decrease



$\boxed{x=2}$

D: $(-\infty, 2)$

R: $(-\infty, \infty)$

$2 - x > 0$

$2 > x$

$x < 2$ domain

$x = 2$ VA

X	Y
$1\frac{2}{3}$	2
1	3
-1	4
-7	5

$0 = \log_3(2-x) + 3$

$-3 = \log_3(2-x)$

$3^{-3} = \cancel{3}^{\cancel{\log_3}(2-x)}$

$3^{-3} = 2 - x$

$\frac{1}{27} = 2 - x$

$\frac{1}{27} - 2 = -x$

$2 - \frac{1}{27} = x$

$1\frac{26}{27}$ or $\frac{53}{27} = x$

1) Given $\log_a 5 = x$ and $\log_a 3 = y$

Evaluate the following in terms of x and/or y

$\log_a(.12)$

$$\log_a\left(\frac{12}{100}\right) = \log_a\left(\frac{3}{25}\right) = \log_a 3 - \log_a 25$$

$$\log_a 3 - \log_a 5^2$$

$$\log_a 3 - 2\log_a 5$$

$$\boxed{y - 2x}$$

$$\log_a 3 - \log_a(5\cdot 5)$$

$$\log_a 3 - [\log_a 5 + \log_a 5]$$

$$\log_a 3 - \log_a 5 - \log_a 5$$

$$y - x - x$$

$$\boxed{y - 2x}$$

2) Solve for x : $\left(\sqrt[3]{2}\right)^{2-x} = 2^{x^2}$

$$\left[\left(2\right)^{1/3}\right]^{2-x} = 2^{x^2}$$

* set exponents equal when same base.

$$\cancel{3} \cdot \frac{1}{\cancel{3}}(2-x) = x^2 \cdot 3$$

$$2 - x = 3x^2$$

$$0 = 3x^2 + x - 2$$

$$0 = (3x-2)(x+1)$$

$$\boxed{x = \frac{2}{3} \quad x = -1}$$

3) State the domain in interval notation :

$$\log_3\left(\frac{x^2-4}{x+3}\right)$$

$>$ $<$

$\geq$ $\leq$

arg > 0

$$\frac{x^2-4}{x+3} > 0$$

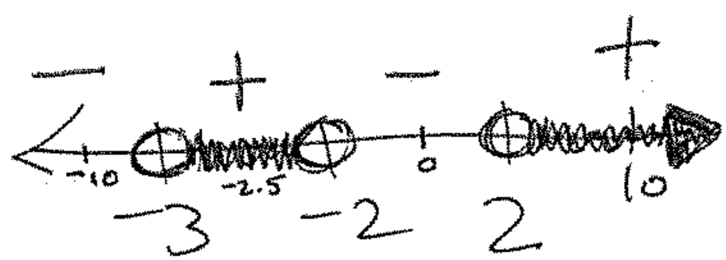
Alg 2

arg > 0

$$\frac{(x+2)(x-2)}{x+3} > 0$$

$$\frac{(-)(-)}{+} \qquad \frac{+}{+}$$

Num	denom
$(x+2)(x-2)=0$	$x+3=0$
$x=\pm 2$	$x=-3$



D : $\boxed{(-3,-2)\cup(2,\infty)}$

4) Solve for x :

$$\log_6 \sqrt[3]{-2x-8} - \log_6 \sqrt[3]{x-5} = \log_6 \sqrt[3]{x+4}$$

$$\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}}$$

$$\cancel{6}^{\cancel{\log_6}} \sqrt[3]{\frac{-2x-8}{x-5}} = \cancel{6}^{\cancel{\log_6}} \sqrt[3]{x+4}$$

$$\left(\cancel{\sqrt[3]{}}\frac{-2x-8}{x-5}\right)^{\cancel{3}} = \left(\cancel{\sqrt[3]{}}x+4\right)^{\cancel{3}}$$

$$\frac{-2x-8}{x-5} \cancel{=} \frac{x+4}{1}$$

$$-2x-8 = (x+4)(x-5)$$

$$-\cancel{2x}-\cancel{8} = x^2 - x - 20$$

$$+\cancel{2x}+\cancel{8} \qquad +2x \quad +8$$

$$0 = x^2 + x - 12$$

$$0 = (x+4)(x-3)$$

$$\cancel{x=-4} \qquad \cancel{x=3}$$

$$\boxed{\text{NO Solution}}$$

* 5) Solve for x : $3^{2x} + 3^{2+x} - 10 = 0$

$$3^{2x} + \overbrace{3^2 \cdot 3^x} - 10 = 0$$

$$3^{2x} + 9 \cdot 3^x - 10 = 0$$

Quadratic in form.

Let $d = 3^x$

$d^2 = 3^{2x}$

w/ sub

$$d^2 + 9d - 10 = 0$$

$$(d+10)(d-1) = 0$$

$d = -10$	$d = 1$
$\cancel{3^x = -10}$	$3^x = 1$
$\log 3^x = \cancel{\log(-10)}$	$\boxed{x = 0}$
arg>0	

sub back.

Don't need to do...

$$\log 3^x = \log 1$$

$$x \log 3 = \log 1$$

$$x = \frac{\log 1}{\log 3} = \frac{0}{\log 3}$$

$$x = 0$$

6) Solve for x: $2e^{x} - 4e^{-x} = 7$

$$e^x \cdot 2e^x - \frac{4}{\cancel{e^x}} \cdot \cancel{e^x} = 7 \cdot e^x$$

$$2e^{2x} - 4 = 7 \cdot e^x$$

$$2e^{2x} - 7e^x - 4 = 0$$

Let $d = e^x$

$d^2 = e^{2x}$

w/ sub $2d^2 - 7d - 4 = 0$

$$(2d+1)(d-4) = 0$$

$d = -\frac{1}{2}$ | $d = 4$

sub back $\cancel{e^x = -\frac{1}{2}}$ | $e^x = 4$

$\ln e^x = \cancel{\ln\left(-\frac{1}{2}\right)}$ | $\ln e^{x} = \ln 4$

arg>0

$$\boxed{x = \ln 4}$$

7) Solve for x : $\log_6(x-2)+\log_6 x=\log_6 8$

$$\cancel{6}^{\cancel{\log_6}[x^2-2x]} = \cancel{6}^{\cancel{\log_6} 8}$$

$$x^2-2x=8$$

$$x^2-2x-8=0$$

$$(x-4)(x+2)=0$$

$$\boxed{x=4} \quad \Big| \quad \cancel{x=-2}$$

8) Solve for x : $\log_9(4+x) = \log_9(2x-1)$

$\cancel{9} \qquad \cancel{9}$

$4+\cancel{x} = 2x-\cancel{1}$

$+1 \ \cancel{-x} \qquad -x \ \cancel{+1}$

$\boxed{5 = x}$ ✓

10) *Solve for x*: $\log_x 32 = \frac{5}{2}$

base > 0

$$\left[x^{5/2}\right]^{2/5} = \left[32\right]^{2/5}$$

← odd root will not generate ± on answer

$$x = \left(2^5\right)^{2/5}$$

$$\boxed{x = 4}$$ ✓

11) *Solve for x*: $3^{6x} = 27\sqrt{81}$

$$3^{6x} = 3^3(3^4)^{1/2}$$

$$3^{6x} = 3^3 \cdot 3^2$$

$$3^{6x} = 3^5$$

$$\cancel{3}^{6x} = \cancel{3}^5$$

$$6x = 5$$

$$\boxed{x = \frac{5}{6}}$$

<u>What if...</u>

$$3^{6x} = 27\sqrt{3}$$ ← changed

$$= 3^3 \cdot (3)^{1/2}$$

$$= 3^3 \cdot 3^{1/2}$$

$$= 3^{3\frac{1}{2}}$$

$$\cancel{3}^{6x} = \cancel{3}^{7/2}$$

$$\frac{1}{\cancel{6}} \cdot \cancel{6}x = \frac{7}{2} \cdot \frac{1}{6}$$

$$\boxed{x = \frac{7}{12}}$$

12) Solve for x: $3^x - 3^{-x} = 4$

Let $d = 3^x$

$3^x - \frac{1}{3^x} = 4$

$d \cdot d - \frac{1}{\cancel{d}} \cdot \cancel{d} = 4 \cdot d$

$d^2 - 1 = 4d$

$d^2 - 4d - 1 = 0$

$\cancel{(d \quad)(d \quad)} = 0$ does not factor.

w/out substitution

$3^x \cdot 3^x - \frac{1}{\cancel{3^x}} \cdot \cancel{3^x} = 4 \cdot 3^x$ $\neq 12^x$

$3^{2x} - 1 = 4 \cdot 3^x$

$3^{2x} - 4 \cdot 3^x - 1 = 0$

$\cancel{(3^x \quad)(3^x \quad)} = 0$

QF

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$
$b = -4$
$c = -1$

$d = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$

$3^x =$

sub back

$3^x = \frac{4 \pm \sqrt{16 + 4}}{2}$

$3^x = \frac{4 \pm \sqrt{20}}{2}$

2 | 20
2 | 10
5

$3^x = \frac{4 \pm 2\sqrt{5}}{2} = \frac{\cancel{2}(2 \pm \sqrt{5})}{\cancel{2}}$

continues on next page

$$3^x = 2 \pm \sqrt{5}$$

$$3^x = 2 + \sqrt{5}$$

$$\log 3^x = \log(2+\sqrt{5})$$

$$x \log 3 = \log(2+\sqrt{5})$$

$$\boxed{x = \frac{\log(2+\sqrt{5})}{\log 3}}$$

$$\boxed{x \approx 1.31}$$

$$3^x = \cancel{2 - \sqrt{5}}$$

$$\log 3^x = \log\cancel{(2-\sqrt{5})}$$

arg > 0

$\sqrt{5}$ — $\sqrt{4}$ 2, $\sqrt{9}$ 3

* you could end up with 0, 1, or 2 solutions
always check each solution