Notes

Pre-Calculus
Sec. 3.4
Exponential and Logarithmic Equations
(day 2)

Solving More Complex Log. Equations

$$\log A + \log B = C$$

1. **Combine** and **Isolate** the log.

2. Rewrite the equation in **exponential form**.

3. **Solve** and **check** for extraneous solutions.

Ex. 1: Solve for x.

a) $\log_5(x-7) = 2$

$x-7>0$

$x>7$

domain

$$\cancel{5}^{\cancel{\log_5}(x-7)} = 5^2$$

$$x-7 = 25$$

$$\boxed{x = 32}$$

✓ check to make sure the log's argument remains > 0

b) $\frac{6\ln(2x)}{6} = \frac{30}{6}$

$$\ln(2x) = 5$$

$$\cancel{e}^{\cancel{\ln}(2x)} = e^5$$

$$2x = e^5$$

$$\boxed{x = \frac{e^5}{2}}$$

✓

c) $\log_2 x + \log_2 (x-7) = 3$

Condense first

$$\log_2[x^2-7x] = 3$$

$$\cancel{2^{\log_2}}[x^2-7x] = 2^3$$

$$x^2 - 7x = 2^3$$

$$x^2 - 7x = 8$$

$$x^2 - 7x - 8 = 0$$

Factor
SR
QF

$$(x-8)(x+1) = 0$$

$$\boxed{x = 8} \quad \cancel{x = -1}$$

check in the original arg > 0

$d)\quad (\log x)^2 - \log x^3 + 2 = 0$

$(\log x)^2 - 3\log x + 2 = 0$ Let $d = \log x$

Substitute here. $d^2 - 3d + 2 = 0$ "Quadratic in form"

$(d-2)(d-1) = 0$

$d = 2$	$d = 1$
$\log x = 2$	$\log x = 1$
$\cancel{10^{\log}}x = 10^2$	$\cancel{10^{\log}}x = 10^1$
$x = 10^2$	$x = 10^1$
✓ $\boxed{x = 100}$	$\boxed{x = 10}$ ✓

sub. back

$e)\ \log_2(x-4)=3$

$\cancel{2}^{\cancel{\log_2}(x-4)} = 2^3$

$x-4 = 2^3$

$x-4 = 8 \quad \boxed{x=12}$ ✓

$f)\ \log_{\underline{\underline{2}}}(x-1)+\log_{\underline{\underline{2}}}(x+1)=3$

condense first

$\log_2[(x-1)(x+1)] = 3$

$\cancel{2}^{\cancel{\log_2}(x^2-1)} = 2^3$

w/ square roots

$x^2-1 = 8$

$\sqrt{x^2} = \sqrt{9}$

$|x| = 3 \quad x = \pm 3$

or

w/factoring

$x^2-9=0$

$(x+3)(x-3)=0$

$\cancel{x=-3} \quad \boxed{x=3}$

arg>0

All different bases

g) $\log_2 x + \log_4 x + \log_{16} x = 7$ $\boxed{D: x > 0}$

• use change of base.

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 16} = 7$$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 2^2} + \frac{\log x}{\log 2^4} = 7$$

$$\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} + \frac{\log x}{4\log 2} = 7$$

Get a common denom.

$$\frac{4\log x + 2\log x + \log x}{4\log 2} = 7$$

$$\frac{7\log x}{4\log 2} = \frac{7}{1}$$

$$\frac{7\log x}{7} = \frac{28\log 2}{7}$$

$$\log x = 4\log 2$$

$$\cancel{\log} x = \cancel{\log} 2^4$$

$$\boxed{x = 16}$$

$\boxed{OR}$

condenses with change of base.

$$\log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = 7$$

• like terms

Add $1 + \frac{1}{2} + \frac{1}{4}$

$\frac{4}{4} + \frac{2}{4} + \frac{1}{4}$

$$\frac{4}{\cancel{7}} \cdot \frac{\cancel{7}}{\cancel{4}}\log_2 x = \cancel{7} \cdot \frac{4}{\cancel{7}}$$

$$\log_2 x = 4$$

$$x = 2^4 \quad \boxed{x = 16}$$

Round to Hundredths

Ex. 2: How long will it take \$25,000 to grow to \$500,000 at 9% annual interest compounded monthly? $n = 12$

$$A = P\left(1+\frac{r}{n}\right)^{nt} \quad \text{or} \quad A = Pe^{rt}$$

$P = 25{,}000$

$A = 500{,}000$

$r = 9\% = .09$

$$\frac{500{,}000}{25{,}000} = \frac{25{,}000\left(1+\frac{.09}{12}\right)^{12t}}{25{,}000}$$

$$\log 20 = \log(1.0075)^{12t}$$

$$\frac{\log 20}{12\log(1.0075)} = \frac{12t\log(1.0075)}{12\log(1.0075)}$$

$$\boxed{t = \frac{\log 20}{12\log(1.0075)}} \qquad \boxed{t \approx 33.41 \text{ yrs}}$$

Round to Hundredths

Ex.3 How long will an investment of \$30,000 take to grow to \$450,000 at 3% interest rate that is compounded quarterly?

$n=4$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$P = 30{,}000$

$A = 450{,}000$

$r = 3\% = .03$

$$\frac{\cancel{450{,}000}}{\cancel{30{,}000}} = \frac{\cancel{30{,}000}\left(1+\frac{.03}{4}\right)^{4t}}{\cancel{30{,}000}}$$

$$\log 15 = \log(1.0075)^{4t}$$

$$\frac{\log 15}{4\log(1.0075)} = \frac{4t\log(1.0075)}{4\log(1.0075)}$$

$$\boxed{t = \frac{\log 15}{4\log(1.0075)}} \qquad \boxed{t \approx 90.61 \text{ yrs}}$$

don't actually need

Ex. 4: Suppose you invested ~~$6500~~ into a savings account with a 5.4% annual interest rate that is compounded continuously. $A = Pe^{rt}$

a) How long will the investment take to triple the value?

$A = 3P$

$r = 5.4\% = .054$

$$A = Pe^{rt}$$

Triple

$$\frac{3\cancel{P}}{\cancel{1P}} = \frac{\cancel{1P}e^{(.054)t}}{\cancel{1P}}$$

$$\ln 3 = \ln e^{(.054)t}$$

$$\frac{\ln 3}{.054} = \frac{.054t}{.054}$$

$$\boxed{t = \frac{\ln 3}{.054}} \quad \boxed{t \approx 20.34 \text{ yrs}}$$

$$\frac{19500}{6500} = \frac{6500e^{rt}}{6500}$$

$$3 = 1e^{rt}$$

b) How much longer (in years) would the investment take for the money to triple if the interest is only compounded monthly?

$n = 12$

$r = .054$

$A = 3P$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$3P = $

$3 = \left(1 + \frac{.054}{12}\right)^{12t}$

$\log 3 = \log(1.0045)^{12t}$

$\frac{\log 3}{12\log(1.0045)} = \frac{12\cancel{t}\log(1.0045)}{12\log(1.0045)}$

$\boxed{t = \frac{\log 3}{12\log(1.0045)}}$

$\boxed{t \approx 20.39 \text{ yrs}}$

$20.39 - 20.34 \simeq \boxed{0.05 \text{ yrs longer}}$

Ex. 5: What should the annual interest rate be in order for \$1200 to grow to \$5200 in 8 years with continuous compounding?

$t = 8$

$P = 1200$

$A = 5200$

$A = Pe^{rt}$

$$\frac{5200}{1200} = \frac{\cancel{1200}\, e^{r(8)}}{\cancel{1200}}$$

$$\ln\left(\frac{52}{12}\right) = \ln e^{8r}$$

$$\ln\left(\frac{13}{3}\right) = 8r$$

$$\boxed{\frac{\ln\left(\frac{13}{3}\right)}{8} = r}$$

* change to a percentage

$r \approx 0.1832921$

$\times 100$

$r \approx 18.32921$

$\boxed{r \approx 18.33\%}$