Notes

Pre-Calculus: Sec. 3.4
Exponential and Logarithmic Equations(day 1)

Solving More Complex Exponential Equations:

1. Isolate the exponential expression.

2. Change to logarithmic form or take ***log*** or ***ln*** on both sides.

3. Solve for x.

Ex. 1: Solve for x. Round your answer to three decimal places.

a) $4^{2x-3} - 3 = 137$

$\quad +3 \qquad +3$

$\log 4^{(2x-3)} = \log 140$

can we get the same base?

$(2x-3)\log 4 = \log 140$

$2x\log 4 - 3\log 4 = \log 140$

$\quad +3\log 4 \qquad +3\log 4$

$$\frac{2x\log 4}{2\log 4} = \frac{\log 140 + 3\log 4}{2\log 4}$$

$$\boxed{x = \frac{\log 140 + 3\log 4}{2\log 4}}$$

prefer as one fraction

$$\boxed{x \approx 3.282}$$

equivalent to:

$$x = \frac{\log 140}{2\log 4} + \frac{3\cancel{\log 4}}{2\cancel{\log 4}}$$

$$\boxed{x = \frac{\log 140}{2\log 4} + \frac{3}{2}}$$

or

$$\boxed{x = \frac{1}{2}\left[\frac{\log 140}{\log 4} + 3\right]}$$

b) $4e^{7x} + 10 = 22$

$\quad -10 \quad -10$

NOT: $(4e)^{7x}$

$$\frac{\cancel{4}e^{7x}}{\cancel{4}} = \frac{12}{4}$$

$$e^{7x} = 3$$

can we get the same base?

use log or ln

$$\ln e^{7x} = \ln 3$$

$7x \cdot \ln e$

$7x(1)$

$$7x = \ln 3$$

NOT: $\ln \frac{3}{7}$

$$\boxed{x = \frac{\ln 3}{7}} \quad \boxed{x \approx 0.157}$$

$$x = \frac{\log 5 + 3\log 3}{2\log 3}$$

$$\frac{\log 5}{2\log 3} + \frac{3\cancel{\log 3}}{2\cancel{\log 3}}$$

$$\boxed{x = \frac{\log 5}{2\log 3} + \frac{3}{2}}$$

equivalent answer

or

$$\boxed{x = \frac{1}{2}\left[\frac{\log 5}{\log 3} + 3\right]}$$

start @ step 3 from previous page

$$3^{2x-3} = 5$$

or change to logarithmic form

$$\log_3 5 = 2x - 3$$

use change of base

$$\frac{\log 5}{\log 3} = 2x - 3$$

$$+3 \qquad\qquad +3$$

$$\frac{1}{2}\cdot\frac{\log 5}{\log 3} + 3\cdot\frac{1}{2} = 2x\cdot\frac{1}{2}$$

$$\boxed{\frac{\log 5}{2\log 3} + \frac{3}{2} = x}$$

Combine with like denominators (LCD) to get one fraction (on previous page)

d) $\dfrac{119}{e^{6x}-14} = \dfrac{7}{1}$

$7\overline{)119}$

$\dfrac{7(e^{6x}-14)}{7} = \dfrac{119}{7}$

$$\begin{aligned} e^{6x} - 14 &= 17 \\ +14 &\quad +14 \end{aligned}$$

$e^{6x} = 31$

$\ln e^{6x} = \ln 31$

$6x = \ln 31$

$\boxed{x = \dfrac{\ln 31}{6}}$

$\boxed{x \approx 0.572}$

e) $\left(\underbrace{4-\frac{2.471}{40}}\right)^{9t}=21$

Let $A \approx 3.938\ldots$

(Store it)

$$A^{9t} = 21$$

$$\log A^{9t} = \log 21$$

$$9t \log A = \log 21$$

$$\boxed{t = \frac{\log 21}{9 \log A}}$$

"Recall A"

$$\boxed{t \approx 0.247}$$

$f)\ e^{4x-5}+10=12$

$\quad\quad -10 \quad -10$

$$e^{4x-5} = 2$$

$$\ln e^{4x-5} = \ln 2$$

$$4x-5 = \ln 2$$

$$\quad +5 \quad\quad +5$$

$$4x = 5 + \ln 2$$

Better to write in front to avoid mistakes later.

$$\boxed{x = \frac{5+\ln 2}{4}}$$

$$\boxed{x \approx 1.423}$$

vs.

$$4x = \ln 2 + 5$$

easy to make a mistake

$\neq \ln 7$

Solving Quadratic Form Equations

Ex. 2: Solve for x.

$a)\ e^{2x} - 5e^{x} + 6 = 0$

$(e^x - 2)(e^x - 3) = 0$

or use a substitution:

Let $d = e^x$

$(d)^2 = (e^x)^2$

$d^2 = e^{2x}$

or sub } $d^2 - 5d + 6 = 0$

$(d-2)(d-3) = 0$

$d = 2$	$d = 3$
sub back } $e^x = 2$	$e^x = 3$
$\ln e^{x} = \ln 2$	$\ln e^{x} = \ln 3$
$\boxed{x = \ln 2}$	$\boxed{x = \ln 3}$

b) $3^{x} + 4(3^{-x}) = 5$

$$3^{x} + 4\left(\frac{1}{3^{x}}\right) = 5$$

$$3^{x} \cdot 3^{x} + \frac{4}{\cancel{3^{x}}} \cdot \cancel{3^{x}} = 5 \cdot 3^{x} \neq 15x$$

$\neq 9^{2x}$

$9x^{2}$

same base... add exponents

$$3^{2x} + 4 = 5 \cdot 3^{x}$$

$$3^{2x} - 5 \cdot 3^{x} + 4 = 0$$

$$(3^{x} - 4)(3^{x} - 1) = 0$$

or substitute:

Let $d = 3^{x}$

$d^{2} = 3^{2x}$

continues $\longrightarrow$

or sub $\}$ $d^2 - 5d + 4 = 0$

$(d-4)(d-1) = 0$

$d = 4$

sub back $\}$ $3^x = 4$

$\log 3^x = \log 4$

$x \log 3 = \log 4$

$\boxed{x = \frac{\log 4}{\log 3}}$

or

$\boxed{x = \log_3 4}$

w/ change of base

$d = 1$

$3^x = 1$

$\boxed{x = 0}$

$\log 3^x = \log 1$

$x \log 3 = \log 1$

$x = \frac{\log 1}{\log 3}$

$x = \frac{0}{\log 3}$

$x = 0$

c) $e^x - 36e^{-x} = -9$

$$e^x \cdot e^x - \frac{36}{\cancel{e^x}} \cdot \cancel{e^x} = -9 \cdot e^x$$

$$e^{2x} - 36 = \underbrace{-9 \cdot e^x}$$

$$e^{2x} + 9 \cdot e^x - 36 = 0$$

$$(e^x + 12)(e^x - 3) = 0$$

$$\cancel{e^x = -12}$$

$$\ln e^x = \ln(-12)$$

$$x = \ln(\cancel{-12})$$

Rule: log's arg > 0

$$e^x = 3$$

$$\ln e^{x} = \ln 3$$

$$\boxed{x = \ln 3}$$

or

Let

$d = e^x$

$d^2 = e^{2x}$

Ex. 3: Solve for x. Round your answer to three decimal places. *a)* $2^{x+1} = 3^{2x-1}$

can't get the same base.

$$\log 2^{(x+1)} = \log 3^{(2x-1)}$$

$$(x+1)\log 2 = (2x-1)\log 3$$

Get all x's on same side.

$$x\log 2 + \log 2 = 2x\log 3 - \log 3$$

factor

$$x\log 2 - 2x\log 3 = -\log 3 - \log 2$$

$$x(\log 2 - 2\log 3) = -\log 3 - \log 2$$

$$\boxed{x = \frac{-1(-\log 3 - \log 2)}{-1(\log 2 - 2\log 3)}}$$

Too many negatives here.

$$\boxed{x = \frac{\log 3 + \log 2}{2\log 3 - \log 2}}$$

* this answer

$$\boxed{x \approx 1.191}$$

b) $5^{4x-2} = 2^{x+1}$

$$\log 5^{(4x-2)} = \log 2^{(x+1)}$$

$$(4x-2)\log 5 = (x+1)\log 2$$

$$4x\log 5 - 2\log 5 = x\log 2 + \log 2$$

move right

move left

$$4x\log 5 - x\log 2 = \log 2 + 2\log 5$$

factor

$$x(4\log 5 - \log 2) = \log 2 + 2\log 5$$

$$\boxed{x = \frac{\log 2 + 2\log 5}{4\log 5 - \log 2}}$$

$$\boxed{x \approx 0.681}$$

Ex.4 : Factor each. Simplify completely.

$x^5 - x^2$

Least

$x^2(x^3 - 1)$

a) $7x^{3/7} - 14x^{6/7} + 21x^{10/7}$

3 terms

GCF:

$$7x^{3/7}\left(1x^{\overset{0}{(3/7-3/7)}} - 2x^{\overset{3/7}{(6/7-3/7)}} + 3x^{\overset{7/7}{(10/7-3/7)}}\right)$$

$x^0 = 1$

$$7x^{3/7}\left(1 - 2x^{3/7} + 3x^1\right)$$

$$\boxed{7x^{3/7}\left(3x - 2x^{3/7} + 1\right)}$$

$x^{3/7}$ doubled

$= x^{6/7}$

$\therefore$ does not factor further, not a quadratic "in form" question

Factor

b) $x^{7/3} - x^{4/3} - 2x^{1/3}$ 3 terms

$x^0 = 1$

GCF:

$$x^{1/3}\left(x^{\overset{4/3}{(7/3 - 1/3)}} - x^{\overset{3/3}{(4/3 - 1/3)}} - 2x^{\overset{0}{(1/3 - 1/3)}}\right)$$

$$x^{1/3}\left(x^2 - x - 2\right)$$

$$\boxed{x^{1/3}(x-2)(x+1)}$$

Factor

c) $2(x+5)^{-1/2} - (x+5)^{-3/2}$ 2 terms

$$(x+5)^{-3/2}\left[2(x+5)^{\overset{2/2}{(-1/2+(+3/2))}} - (x+5)^{\overset{0}{(-3/2+(+3/2))}}\right]$$

$$(x+5)^{-3/2}\left[2(x+5) - 1\right]$$ $(x+5)^0 = 1$

$$(x+5)^{-3/2}\left[2x+10-1\right]$$

$$(x+5)^{-3/2}(2x+9)$$

$$= \boxed{\frac{2x+9}{(x+5)^{3/2}}}$$

d) $18(x-1)^{-2} - 9x(x-1)^{-4}$ 2 terms

$9(x-1)^{-4}\left[2(x-1)^{(-2+(+4))} - 1x(x-1)^{(-4+(+4))}\right]$

$(x-1)^{0} = 1$

$9(x-1)^{-4}\left[2(x-1)^{2} - x\right]$

$\left[2(x-1)(x-1) - x\right]$

FOIL

$\left[2(x^2-2x+1) - x\right]$

$\left[2x^2 - 4x + 2 - x\right]$

$9(x-1)^{-4}\left[2x^2-5x+2\right]$

stays in the numerator

$$= \boxed{\frac{9(2x-1)(x-2)}{(x-1)^4}}$$