Notes

Pre-Calculus
Sec. 3.3
Properties of Logarithms

The Change-of-Base Formula

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then $\log_b x$ can be converted to a different base as:

Base a	Base 10	Base e
$\log_b x = \dfrac{\log_a x}{\log_a b}$	$\log_b x = \dfrac{\log x}{\log b}$	$\log_b x = \dfrac{\ln x}{\ln b}$

Ex.1: Rewrite the logarithm as a ratio of (a) common logarithms and (b) natural logarithms. Then evaluate. Round your result to three decimal places.

a) $\log_{\sqrt{2}} 3$

a $\dfrac{\log 3}{\log \sqrt{2}}$ b $\dfrac{\ln 3}{\ln \sqrt{2}}$

$\approx \boxed{3.170}$

b) $\log_{\pi} 53.2$

a $\dfrac{\log 53.2}{\log \pi}$ b $\dfrac{\ln 53.2}{\ln \pi}$

$\approx \boxed{3.472}$

Properties of Logarithms

1. Product Rule: $\log_b(mn) = \log_b m + \log_b n$

Condense $\longleftrightarrow$ Expand

ex. a) $ln\,(4x) =$

Expands to . . .

$\boxed{\ln 4 + \ln x}$

b) $\log_5 10 + \log_5 y =$

Condenses to

$\boxed{\log_5 (10y)}$

2. Quotient Rule: $\log_b \left[\frac{m}{n}\right] = \log_b m - \log_b n$

Condense $\longleftrightarrow$ Expand

ex. a) $\log_3 \left(\frac{9}{y}\right) =$

Expands to . . .

$\log_3 9 - \log_3 y =$

$\boxed{2 - \log_3 y}$

always simplify if possible

b) $\log_{10} 25 - \log_{10} 5 =$

Condenses to . . .

$\log \frac{25}{5} =$

$\boxed{\log 5}$

3. Power Rule: $\log_b x^c = c \log_b x$

Condensed inside the log

Expanded when in front

ex. a) $\log_5 x^3 = 3 \log_5 x$

→ Expands

b) $x \log 3 = \log 3^x$

→ Condense

Ex.2: Expand the logarithmic expression.

$\log_2 (-)^2$

a) $\log_2 \frac{5x^2}{3y}$

$= \log_2 5 + \log_2 x^2 - \log_2 3 - \log_2 y$

$= \boxed{\log_2 5 + 2\log_2 x - \log_2 3 - \log_2 y}$

or

$\log_2 5x^2 - \log_2 3y$

$\log_2 5 + \log_2 x^2 - (\log_2 3 + \log_2 y)$

$\boxed{\log_2 5 + 2\log_2 x - \log_2 3 - \log_2 y}$

b) $\ln\sqrt{x^2(x+2)}$

$\ln\left[x^2(x+2)\right]^{1/2}$

$\frac{1}{2}\ln\left[x^2(x+2)\right]$

$\frac{1}{2}\left[\ln x^2 + \ln(x+2)\right]$

$\frac{1}{2}\left[2\ln x + \ln(x+2)\right]$

$\boxed{\ln x + \frac{1}{2}\ln(x+2)}$

or

$\ln\left[x^2 \cdot (x+2)\right]^{1/2}$

mult

$\ln\left[(x^2)^{1/2}(x+2)^{1/2}\right]$

$\ln\left[x \cdot (x+2)^{1/2}\right]$

$\ln x + \ln(x+2)^{1/2}$

$\boxed{\ln x + \frac{1}{2}\ln(x+2)}$

$$c)\ \log_5\left(\frac{25}{x\sqrt{x-1}}\right) = \underbrace{\log_5 25} - \log_5 x - \log_5 (x-1)^{1/2}$$

mult.

$$= \boxed{2 - \log_5 x - \frac{1}{2}\log_5 (x-1)}$$

$$d)\ \ln\left(\frac{e^{-2}}{4}\right) = \ln e^{-2} - \ln 4$$

$$= -2\underbrace{\ln e}_{(1)} - \ln 4$$

$$= \boxed{-2 - \ln 4}$$

$\ln e = 1$

$\log_e e = 1$

$\ln e^{-2}$

$\log_e e^{-2} = \frac{-2}{1}$

e) $\log\left(\frac{10x}{(x^2-1)}\right) \rightarrow \log\left[\frac{10x}{(x+1)(x-1)}\right]$

factor the rational expression

$$\underbrace{\log_{10} 10} + \log_{10} x - \log_{10}(x+1) - \log_{10}(x-1)$$

$$\boxed{1 + \log x - \log(x+1) - \log(x-1)}$$

e) $\log\left(\dfrac{10x}{(x^2-1)}\right) \longrightarrow \log\left[\dfrac{10\cdot x}{(x+1)\cdot(x-1)}\right]$

$$\underbrace{\log_{10} 10} + \log_{10} x - \log_{10} (x^2-1)$$

factors

Dos

$$1 \quad + \log x - \log_{10}[(x+1)(x-1)]$$

$$1 \quad + \log x - [\log_{10}(x+1) + \log_{10}(x-1)]$$

$$\boxed{1 \quad + \log x - \log(x+1) - \log(x-1)}$$

Ex. 3: Condense the expression as a single logarithm.

a) $\log_4 8 + \log_4 2 = \log_4 (8 \cdot 2)$

$= \log_4 16$

$= \boxed{2}$

b) $2[3\ln(x+1) - \ln x] - \ln(x^2 - 2)$

$6\ln(x+1) - 2\ln x - \ln(x^2-2)$

$+ \ln(x+1)^6 - \ln x^2 - \ln(x^2-2)$

$\boxed{\ln\left[\frac{(x+1)^6}{x^2(x^2-2)}\right]}$ or $\boxed{\ln\left[\frac{(x+1)^6}{x^4-2x^2}\right]}$

c) $\frac{2}{3}\ln 8 - \frac{1}{3}[2\ln y - \frac{1}{2}\ln(y-1)]$

$8^{2/3} \rightarrow (\sqrt[3]{8})^2$

$(2^{\cancel{3}})^{2/\cancel{3}}$ $(2)^2$

4 4

$\frac{2}{3}\ln 8 - \frac{2}{3}\ln y + \frac{1}{6}\ln(y-1)$

$\underbrace{\ln 8^{2/3}} - \ln y^{2/3} + \ln(y-1)^{1/6}$

$+\ln 4 - \ln y^{2/3} + \ln(y-1)^{1/6}$

$\boxed{\ln\left[\frac{4(y-1)^{1/6}}{y^{2/3}}\right] \quad \underline{\underline{\text{or}}} \quad \ln\left[\frac{4\sqrt[6]{y-1}}{\sqrt[3]{y^2}}\right]}$

* not rationalized here.

d) $\log_2(x-1)+\log_2(x+1)-\frac{1}{2}\log_2 25-2\log_2(x+1)$

$$+\log_2(x-1)+\log_2(x+1)-\log_2 25^{1/2}-\log_2(x+1)^2$$

$$-\log_2 5$$

$$\log_2\left[\frac{(x-1)\cancel{(x+1)}}{5(x+1)^{\cancel{2}}}\right]$$

$$\boxed{\log_2\left[\frac{x-1}{5(x+1)}\right]}$$

Recall: $\frac{X^5}{X^3} = X^{5-3} = X^2$

Ex.4: Use the properties of logarithms to rewrite and simplify the expression.

a) $\log_2(4^2 \cdot 3^4)$

$\log_2 4^2 + \log_2 3^4$

$2\underbrace{\log_2 4}_{} + 4\log_2 3$

$2(2)$

$\boxed{4 + 4\log_2 3}$

b) $2\underbrace{\ln e^6}_{} - \underbrace{\ln e^5}_{}$

$\ln e^6$

$\log_e e^6 = \frac{6}{1}$

$2(6) - 5$

$12 - 5$

$= \boxed{7}$

or $2\ln e^6 - \ln e^5$

$(2\cdot 6)\ln e - 5\ln e$

$12\ln e - 5\ln e$

$12(1) - 5(1)$

$12 - 5 = \boxed{7}$

* **c)** $8^{\log_2 6 - \log_8 6}$

use exponent rules to work backwards

mult

$\frac{8^{\log_2 6}}{8^{\log_8 6}} = \frac{(2^3)^{\log_2 6}}{6}$

w/ Rule

$= \frac{2^{3\log_2 6}}{6} = \frac{2^{\log_2 6^3}}{6}$ w/rule $= \frac{6^3}{6^1} = 6^2 = \boxed{36}$