Pre-Calculus
Sec. 3.3
Properties of Logarithms

The Change-of-Base Formula

Let *a, b*, and *x* be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then $\log_b x$ can be converted to a different base as:

Base *a*

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Base 10

$$\log_b x = \frac{\log x}{\log b}$$

Base *e*

$$\log_b x = \frac{\ln x}{\ln b}$$

Ex.1: Rewrite the logarithm as a ratio of (a) common logarithms and (b) natural logarithms. Then evaluate. Round your result to three decimal places.

$a)\quad \log_{\sqrt{2}} 3$

$b)\quad \log_{\pi} 53.2$

Properties of Logarithms

1. Product Rule: $$\log_b(mn) = \log_b m + \log_b n$$

ex. a) ln (4x) =

ln 4 + ln x

b) $\log_5 10 + \log_5 y =$

$\log_5 (10y)$

2. Quotient Rule: $$\log_b \frac{m}{n} = \log_b m - \log_b n$$

ex. a) $\log_3 \left(\frac{9}{y}\right) =$

$\log_3 9 - \log_3 y =$

$2 - \log_3 y$

b) log 25 – log 5 =

$\log \frac{25}{5} =$

log 5

3. Power Rule: $\log_b x^c = c \log_b x$

ex. a) $\log_5 x^3 = 3 \log_5 x$

b) $x \log 3 = \log 3^x$

Ex.2: Expand the logarithmic expression.

$a)\log_2 \frac{5x^2}{3y}$

$\boldsymbol{b)} \quad \mathbf{ln}\sqrt{\boldsymbol{x}^2(\boldsymbol{x}+2)}$

$$c)\ \log_5\left(\frac{25}{x\sqrt{x-1}}\right)$$

$$d)\ \ln\left(\frac{e^{-2}}{4}\right)$$

$e)\ \log\left(\dfrac{10x}{(x^2-1)}\right)$

Ex. 3: Condense the expression as a single logarithm.

$a)\quad \log_4 8 + \log_4 2$

$b)\quad 2[3\ln(x+1) - \ln x] - \ln(x^2 - 2)$

c)

$$\frac{2}{3}\ln 8 - \frac{1}{3}[2\ln y - \frac{1}{2}\ln(y-1)]$$

$$d)\ \log_2(x-1)+\log_2(x+1)-\frac{1}{2}\log_2 25-2\log_2(x+1)$$

Ex.4: Use the properties of logarithms to rewrite and simplify the expression.

$a)\quad \log_2(4^2 \cdot 3^4)$

$b)\quad 2\ln e^6 - \ln e^5$

c) $8^{log_2 6 - log_8 6}$