Sec. 3.2: Logarithmic Functions and Their Graphs

Definition of a Logarithmic Function

For $x > 0$, $b > 0$, and $b \neq 1$, the logarithmic function with base b is denoted $f(x) = \log_b x$, where

Exponent from Exponential form. → $y = \log_{\underline{b}} x$ if and only if $x = \underline{b}^y$

LOGARITHMIC FORM

EXPONENTIAL FORM

Log With Special Base

1) $\log_{10}$: Common log (log) $\log 5$

NO BASE WRITTEN = 10

2) $\log_e$: Natural Log (ln) $\ln 2 \longrightarrow \log_e 2$

$e \approx 2.72$

The function defined by $f(x) = \log_e x = \ln x,\ x > 0$ is called the **natural logarithmic function**.

Ex. 1: Write each equation in its equivalent exponential form.

a. $2 = \log_{\underline{\underline{5}}} x$

$\boxed{5^2 = x}$

b. $x = \ln 64$

$x = \log_{\underline{\underline{e}}} 64$

$\boxed{e^x = 64}$

Ex. 2: Write each equation in its equivalent logarithmic form.

a. $y = \underline{\underline{e}}^4$

Base

$\log_e y = 4$

$\boxed{\ln y = 4}$

b. $9 = \sqrt{81}$

$9 = 81^{\frac{1}{2}}$

$\boxed{\log_{81} 9 = \frac{1}{2}}$

change forms: $6^x = \frac{1}{36}$ $6^{x} = 6^{-2}$

Ex. 3: Evaluate

a) $\log_2 16 = \boxed{4}$

$\log_{2^1} 2^4 = \frac{4}{1}$

b) $\log_{169} 13 = \boxed{\frac{1}{2}}$

$\log_{13^2} 13^1 = \frac{1}{2}$

c) $\log_6 (\frac{1}{36}) = \boxed{-2}$

$\frac{1}{6^2}$

$\log_{6^1} (6^{-2}) = \frac{-2}{1}$

d) $\log_{1/3} 27 = \boxed{-3}$

$\log_{3^{-1}} 3^3 = \frac{3}{-1}$

e) $\log_9 \sqrt{9} = \boxed{\frac{1}{2}}$

$\log_{9^1} 9^{\frac{1}{2}} = \frac{\frac{1}{2}}{1}$

f) $\log_{100} 10 = \boxed{\frac{1}{2}}$

$\log_{10^2} 10^1 = \frac{1}{2}$

g) $log_{\frac{16}{9}} \frac{27}{64} \rightarrow \log_{(\frac{4^2}{3^2})} (\frac{3^3}{4^3}) \rightarrow \log_{(\frac{4}{3})^2} (\frac{3}{4})^3$

$(\frac{4}{3})^2 \rightarrow (\frac{3}{4})^{-2}$

$\rightarrow \log_{(\frac{3}{4})^{-2}} (\frac{3}{4})^3 = \boxed{-\frac{3}{2}}$

Properties of Logarithms

1. $\log_b 1 = 0$ $b^0 = 1$ $\log_2 32 = \boxed{5}$ $2^{\log_2 32} = \boxed{32}$

$b > 0$ $2^5 = 32$ ✓

2. $\log_b b = 1$ $b^1 = b$

Exponent is written as ONE log w/ the same base as the base.

3. $\log_b b^x = x$ *and* $\boxed{b^{\log_b x} = x}$ Inverse Properties

$\log_3 9 = 2$

$\log_3 3^2 = 2$

$b^x = b^x$

4. *If* $\log_b x = \log_b y$, *then* $x = y$. One-to-One Property

One log w/ same base on each side of the equation.

- Then set the arguments equal.

Ex.4: Use the properties of logarithms to simplify each expression.

a) $\log_{1.5} 1 = \boxed{0}$

$\log_{\pi} 1 = 0$

$\ln 1 = 0$

b) $9^{\log_9 15} = 15$

$\log_9 15 = \boxed{}$

would need a calc.

Graphs of Logarithmic Functions

Parent Functions (LOF)

$\boxed{f(x) = \log_b x}$

domain $x > 0$

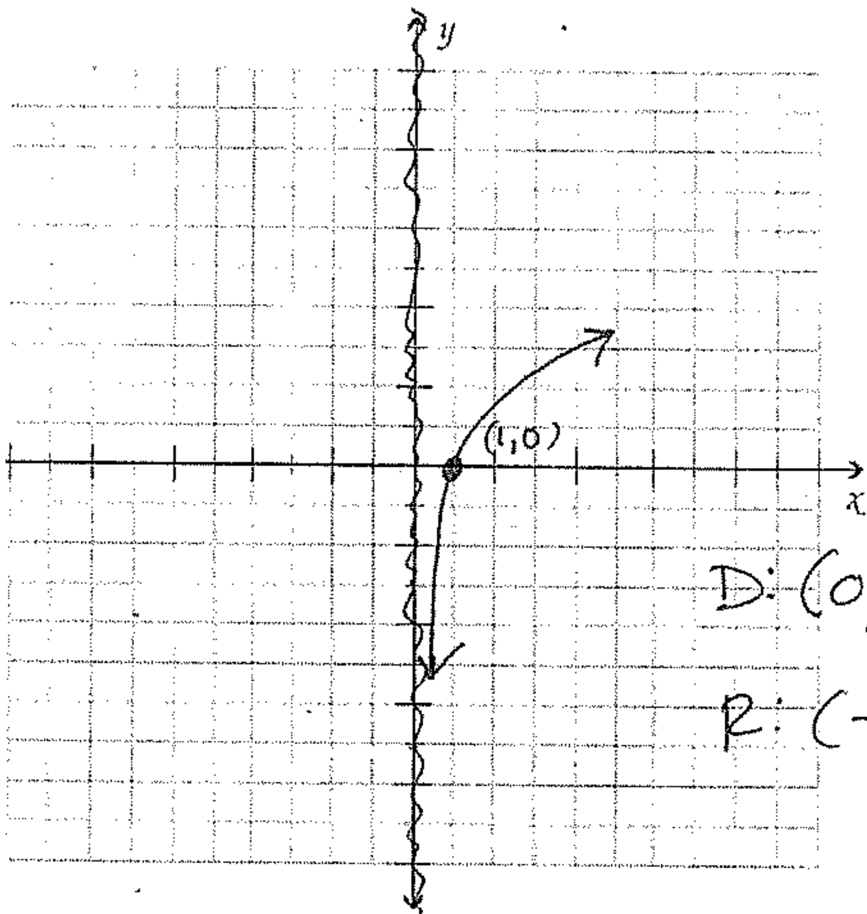
VA: $x = 0$

vertical asymptote

$b > 1$

Logarithmic Growth

- Increases from L → R



VA: $\boxed{x = 0}$

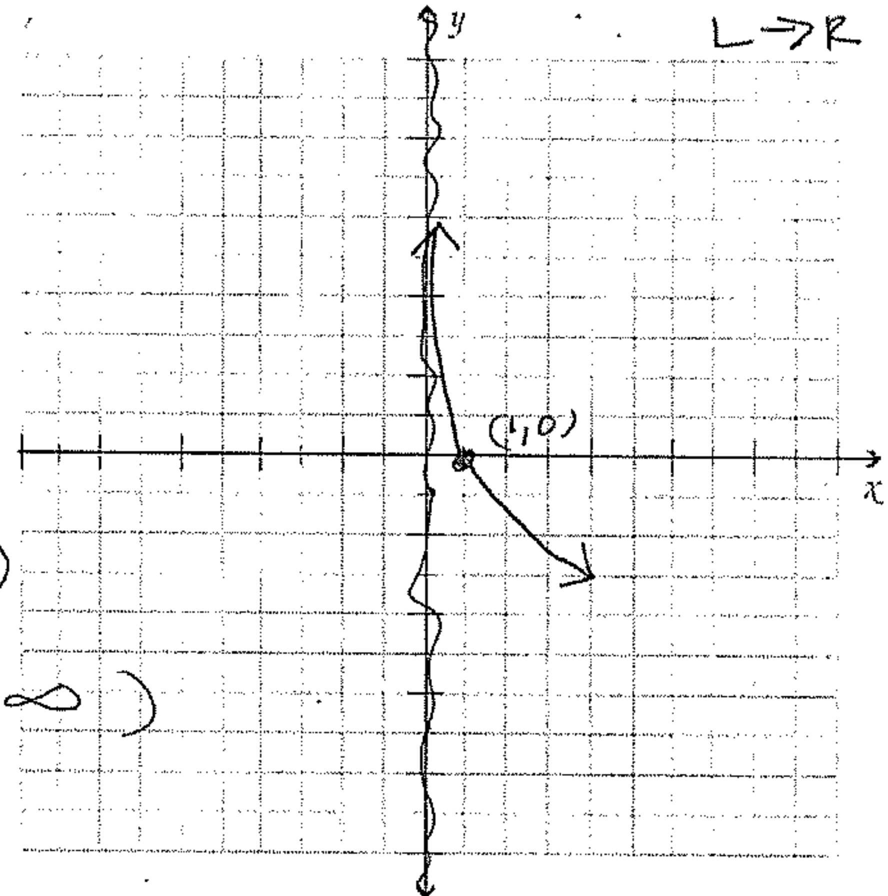
D: $(0, \infty)$

R: $(-\infty, \infty)$

$0 < b < 1$

Logarithmic Decay

- Decreases from L → R



VA $\boxed{x = 0}$

switch x and y

Comparison of Inverse Functions

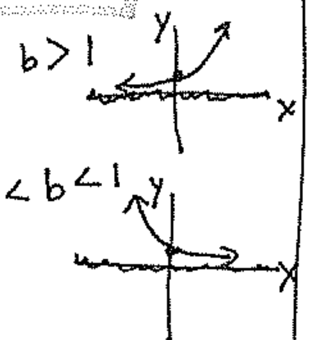
$f(x) = b^x$ and $f(x) = log_b\, x$

$\boxed{\text{VS.}}$

Exponential: $\boxed{y = b^x}$	Logarithmic: $\boxed{y = \log_b x}$
y-int: $(\underset{x}{0}, \underset{y}{1})$	x-int: $(\underset{x}{1}, \underset{y}{0})$
Domain: $(-\infty, \infty)$	Domain: $(0, \infty)$
Range: $(0, \infty)$	Range: $(-\infty, \infty)$
Horizontal Asymp.: ~~HA~~ $\boxed{y = 0}$	Vertical Asymp.: VA $\boxed{x = 0}$ log

b > 1

0 < b < 1



Ex. 5: Find the domain, x-intercept, and vertical asymptote of the logarithmic function. Then graph.

a) $y = \log_2 x$ ← $X > 0$ domain

VA: $X = 0$

$2 > 1$ increases

X	y
$\frac{1}{2}$	-1
1	0
2	1
4	2

X-int

$Y = \log_2 2$ $y = \log_2 \frac{1}{2}$

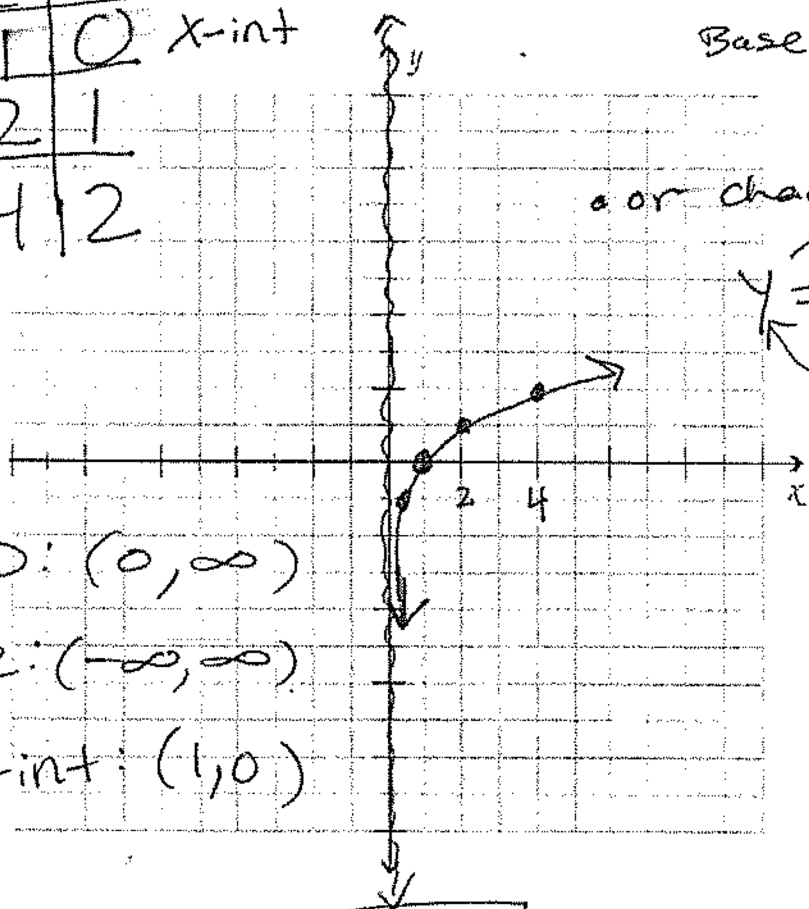
Base 2: $\underbrace{\frac{1}{2} \quad 2 \quad 4 \quad 8}$

or change forms

$Y = \log_2 X$

$2^Y = X$

plug in y values.



D: $(0, \infty)$

R: $(-\infty, \infty)$

X-int: $(1, 0)$

VA $\boxed{X = 0}$

b) $f(x) = \log_{\frac{1}{2}} x$

$\log_{2^{-1}} 2^2$

domain $X > 0$

VA: $X = 0$

$0 < \frac{1}{2} < 1$ decrease

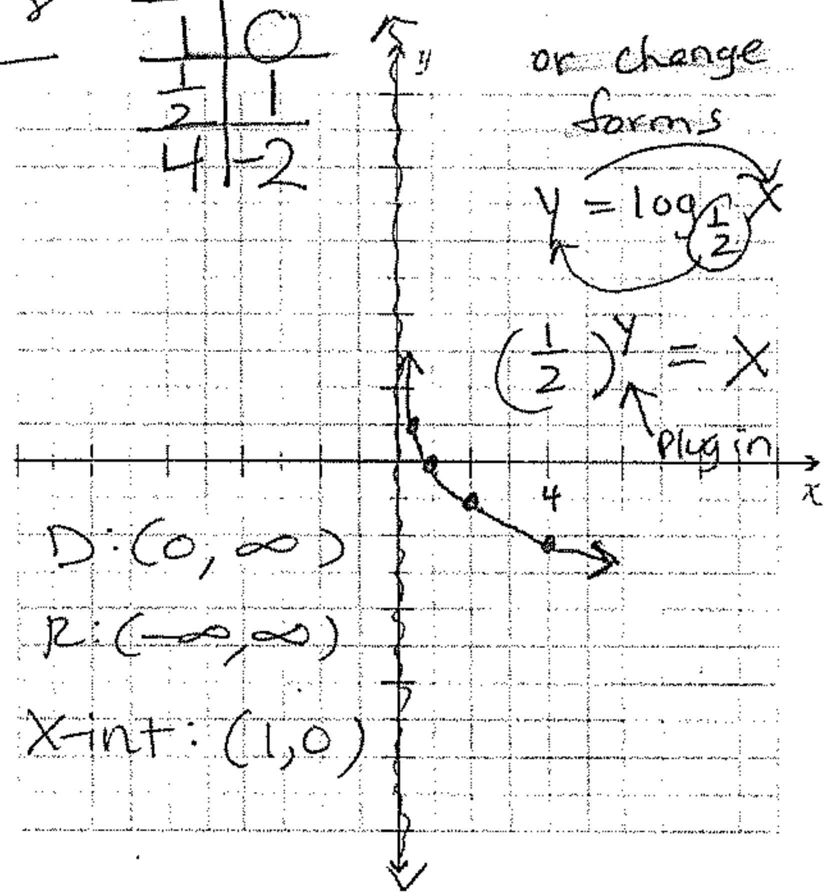
X	Y
2	-1
1	0
$\frac{1}{2}$	1
4	-2

or change forms

$Y = \log_{\frac{1}{2}} X$

$\left(\frac{1}{2}\right)^Y = X$

Plug in



D: $(0, \infty)$

R: $(-\infty, \infty)$

X-int: $(1, 0)$

VA $\boxed{X = 0}$

$\log_3 \frac{1}{3}$ $\log_3 3^{-1}$ $\log_3 9$ $\log_3 3^2$

Ex. 6: Find the domain, x-intercept, and vertical asymptote of the logarithmic function. Then graph.

a) $f(x) = \log_3(x-1)+3$

$3>1$ increase

$x-1>0$

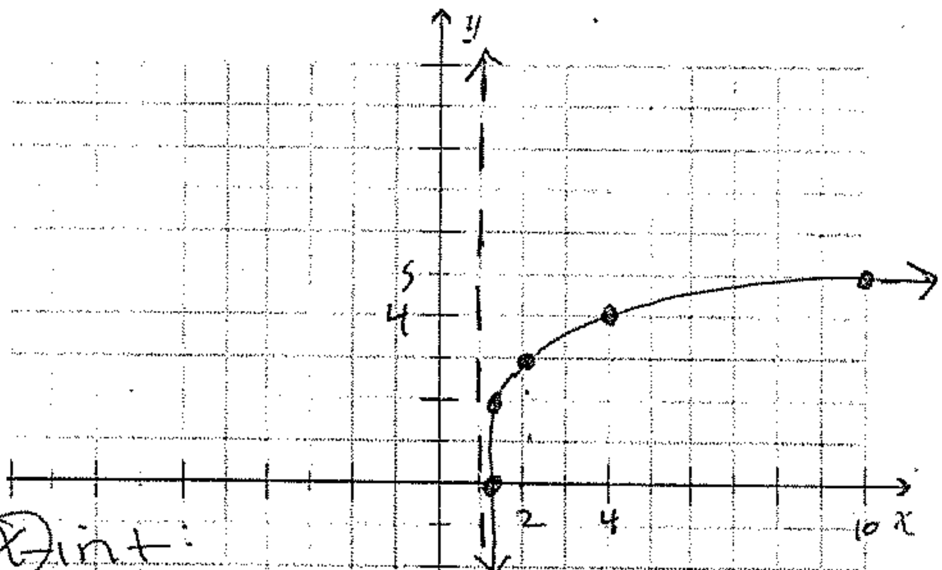
$x>1$ domain

VA: $x=1$

Base 3:

$\frac{1}{3}$ 3 9 27

x	y $f(x)$
$1\frac{1}{3}$	2
2	3
4	4
10	5



x-int:

$y = \log_3(x-1)+3$

$0 = \log_3(x-1)+3$

$-3 = \log_3(x-1)$

$3^{-3} = x-1$

$\frac{1}{27} = x-1$

$x = 1\frac{1}{27}$

D: $(1, \infty)$

R: $(-\infty, \infty)$

x-int: $(1\frac{1}{27}, 0)$ or $(\frac{28}{27}, 0)$

y-int: none

VA $\boxed{x=1}$

• or change forms

$y = \log_3(x-1)+3$

$y-3 = \log_3(x-1)$

$3^{(y-3)} = x-1$

plug in y's $3^{(y-3)}+1 = x$

b) $y = \log_{\frac{1}{2}}(4 - x)$

Y-axis reflection

$0 < \frac{1}{2} < 1$

~~decrease~~

increase

$4 > x$

$4 - x > 0$

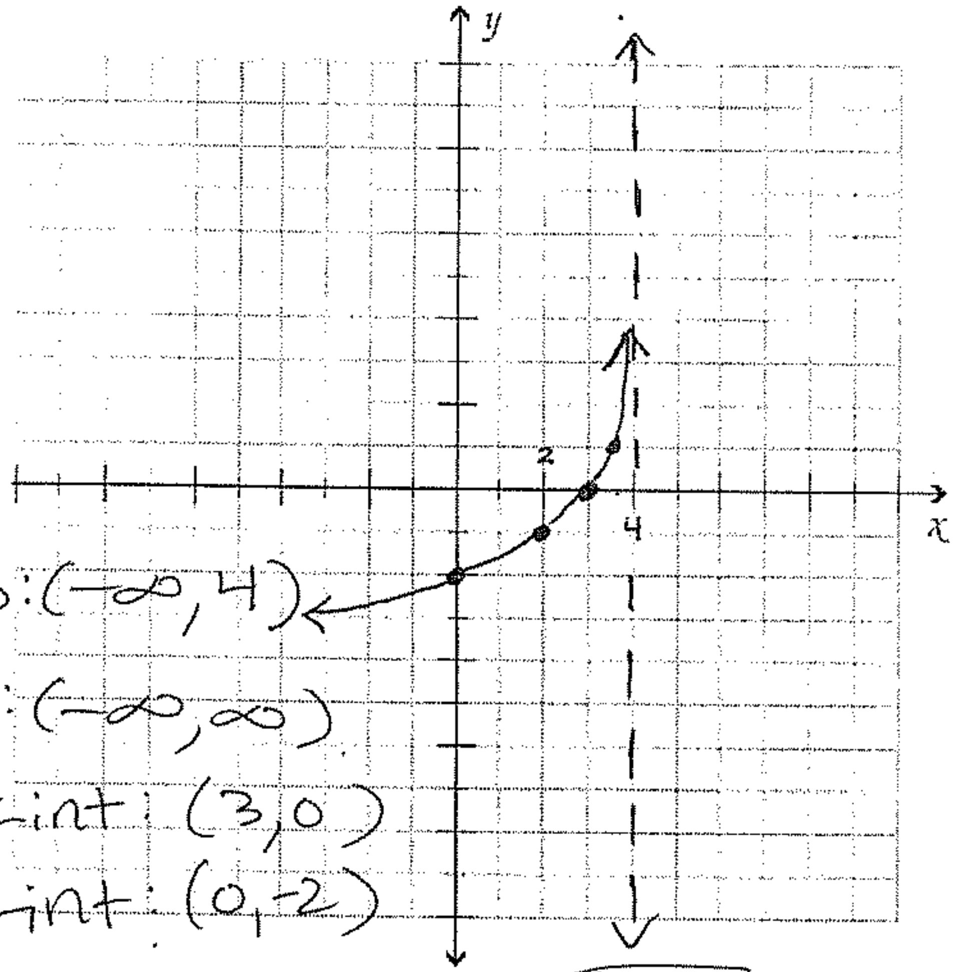
$-x > -4$

flip

$x < 4$ domain

VA: $\boxed{x = 4}$

x	y
2	-1
3	0
$3\frac{1}{2}$	1
0	-2



D: $(-\infty, 4)$

R: $(-\infty, \infty)$

x-int: $(3, 0)$

y-int: $(0, -2)$

VA $\boxed{x=4}$

$\log_{\frac{1}{2}}(4-x)$

$\log_{2^{-1}}(2^{2}) = -2$

or change forms...

$y = \log_{\frac{1}{2}}(4-x)$

$\left(\frac{1}{2}\right)^y = 4 - x$

plug in

$x = 4 - \left(\frac{1}{2}\right)^y$

c) $y = -\ln(x-2)$

$x-2>0$

$x>2$ doman

$y = -\log_e(x-2)$

x-axis reflection

$e>1$ ~~increase~~ decrease

VA: $x=2$

$e \approx 2.72$

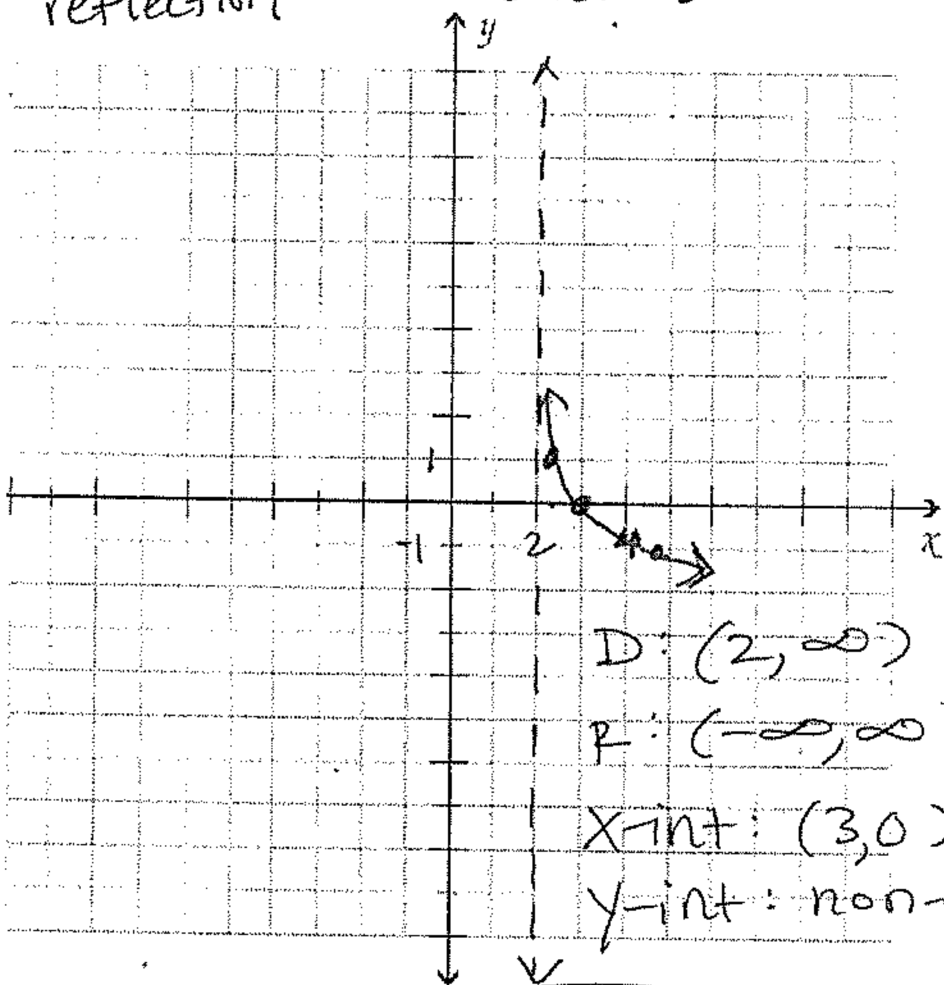
$\frac{1}{e} \approx .37$ (close to $\frac{1}{3}$)

Side note

$y = -\ln x - 2$

$x>0$

VA: $x=0$



D: $(2, \infty)$

R: $(-\infty, \infty)$

X-int: $(3,0)$

Y-int: none

VA $\boxed{x=2}$

$y = -\log_e \frac{1}{e}$

$= -\log_e e^{-1}$

$= -(-1)$

$= 1$

	X	Y
≈ 4.72	$e+2$	-1
Xint	3	0
	$\frac{1}{e}+2$ $\approx 2\frac{1}{3}$	1

or change forms ...

$y = -\log_e(x-2)$

$-y = \log_e(x-2)$

$e^{-y} = x-2$

$x = e^{-y} + 2$ plug in