Sec. 3.2: Logarithmic Functions and Their Graphs

Definition of a Logarithmic Function

For $x > 0$, $b > 0$, and $b \neq 1$, the logarithmic function with base b is denoted $f(x) = \log_b x$, where

$$y = \log_b x \text{ if and only if } x = b^y$$

Log With Special Base

1) $\log_{10}$: Common log (log)

2) $\log_e$: Natural Log (ln)

The function defined by $f(x) = \log_e x = \ln x,\ x > 0$ is called the **natural logarithmic function**.

Ex. 1: Write each equation in its equivalent exponential form.

a. $2 = \log_5 \boldsymbol{x}$

b. $x = \ln 64$

Ex. 2: Write each equation in its equivalent logarithmic form.

a. $y = e^4$

b. $9 = \sqrt{81}$

Ex. 3: Evaluate

a) $\log_2 16$

b) $\log_{169} 13$

c) $\log_6 \left(\frac{1}{36}\right)$

d) $\log_{1/3} 27$

e) $\log_9 \sqrt{9}$

f) $\log_{100} 10$

g) $log_{\frac{16}{9}} \frac{27}{64}$

Properties of Logarithms

1. $\log_b 1 = 0$

2. $\log_b b = 1$

3. $\log_b b^x = x \text{ and } b^{\log_b x} = x$ Inverse Properties

4. $\text{If } \log_b x = \log_b y, \text{ then } x = y.$ One-to-One Property

Ex.4: Use the properties of logarithms to simplify each expression.

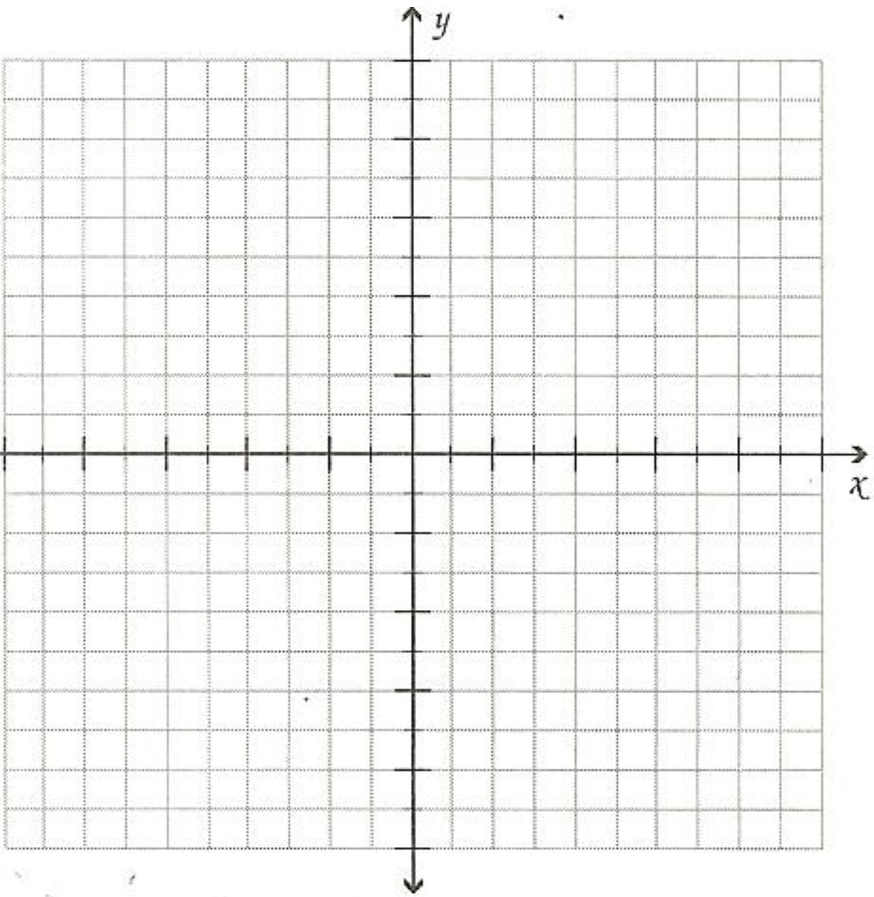
a) $\log_{1.5} 1 =$

b) $9^{\log_9 15} =$

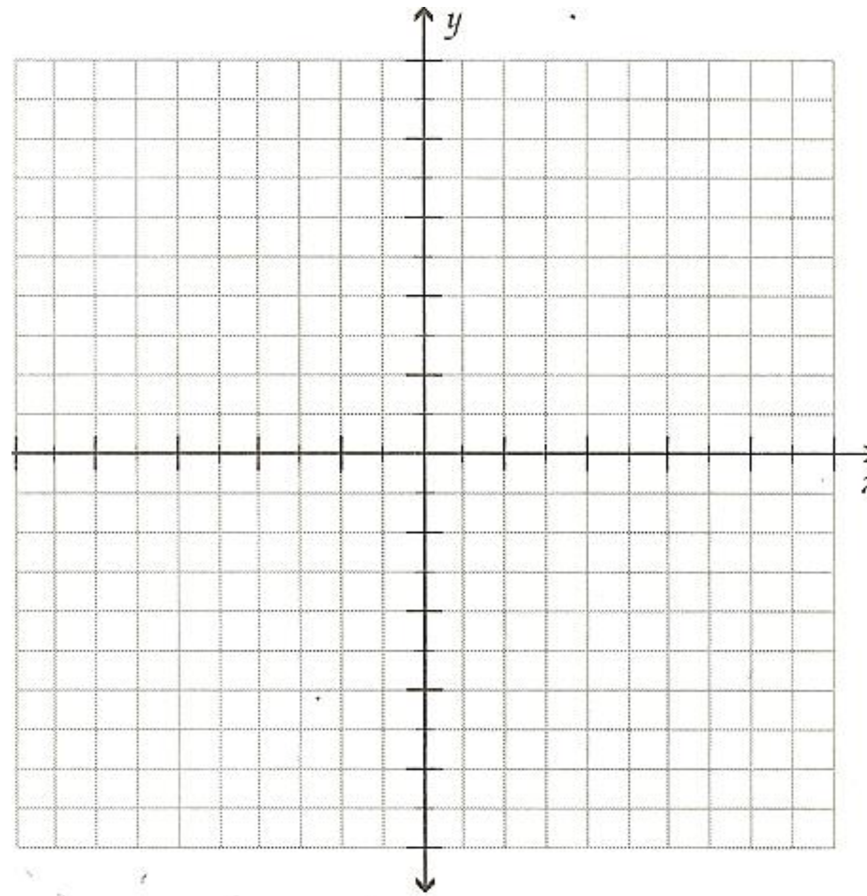
Graphs of Logarithmic Functions

$$f(x) = \log_b x$$

$b > 1$



$0 < b < 1$



Comparison of Inverse Functions
$f(x) = b^x$ and $f(x) = \log_b x$

Exponential: $y = b^x$

Logarithmic: $y = \log_b x$

y-int:

x-int:

Domain:

Domain:

Range:

Range:

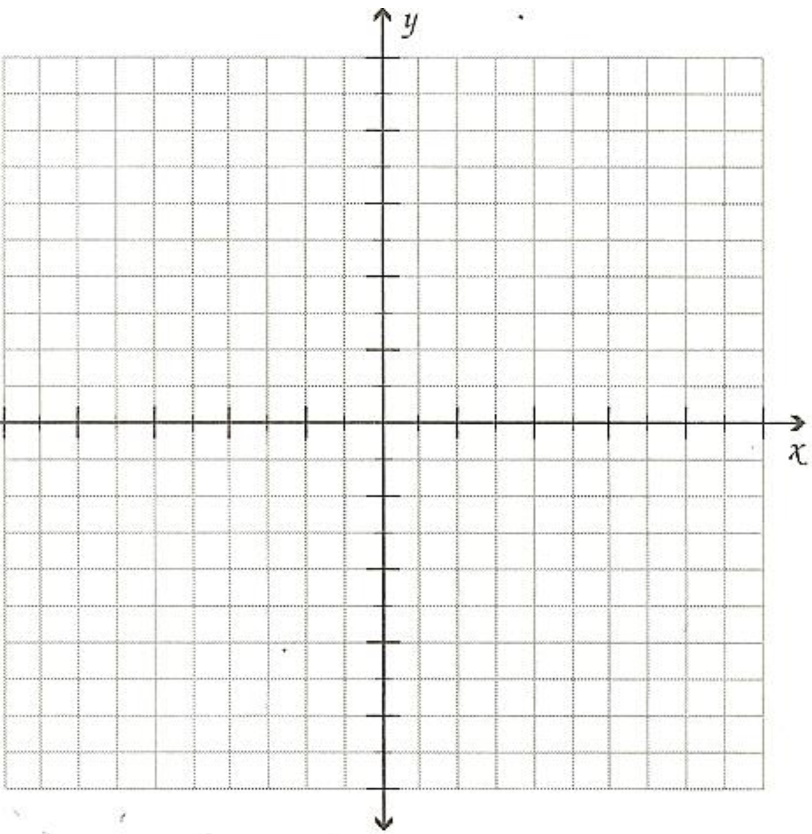
Horizontal Asymp.:

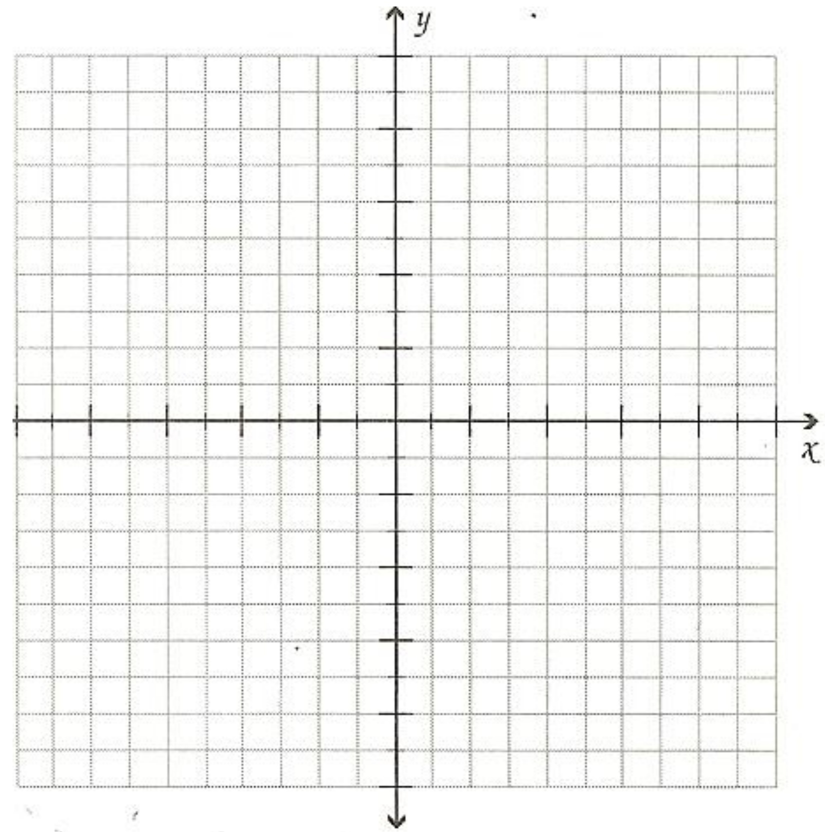
Vertical Asymp.:

Ex. 5: Find the domain, x-intercept, and vertical asymptote of the logarithmic function. Then graph.

$a) \quad y = \log_2 x$

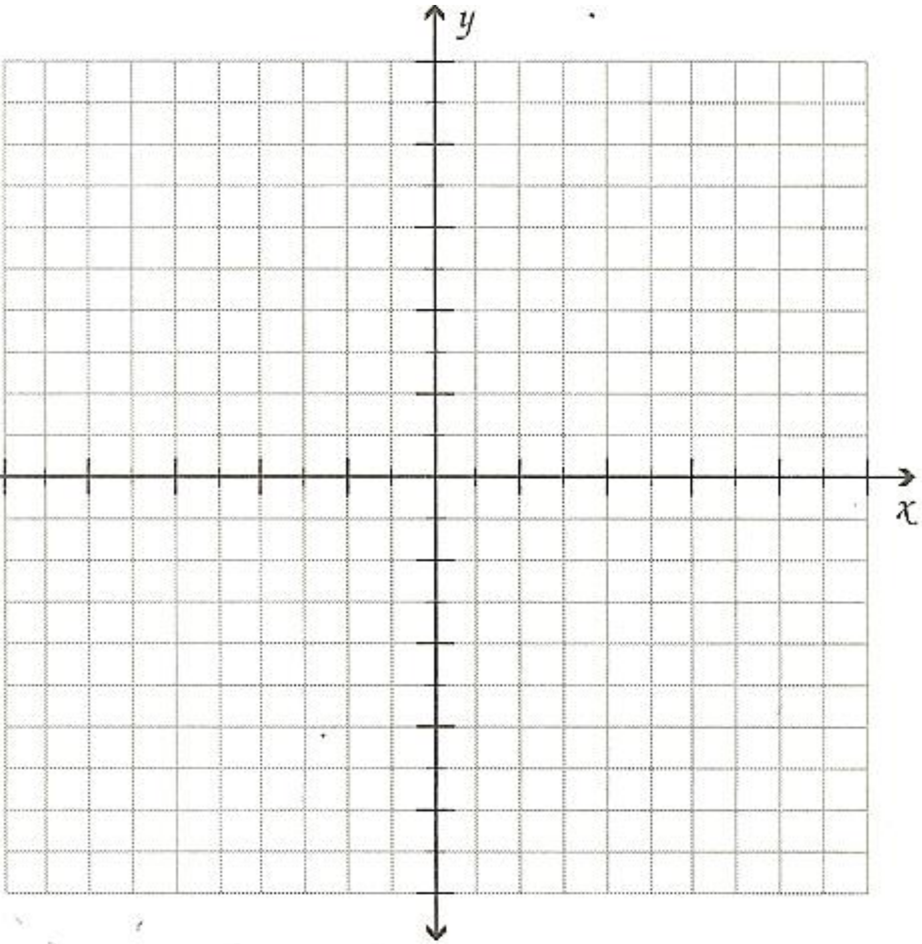
$b) \quad f(x) = \log_{\frac{1}{2}} x$



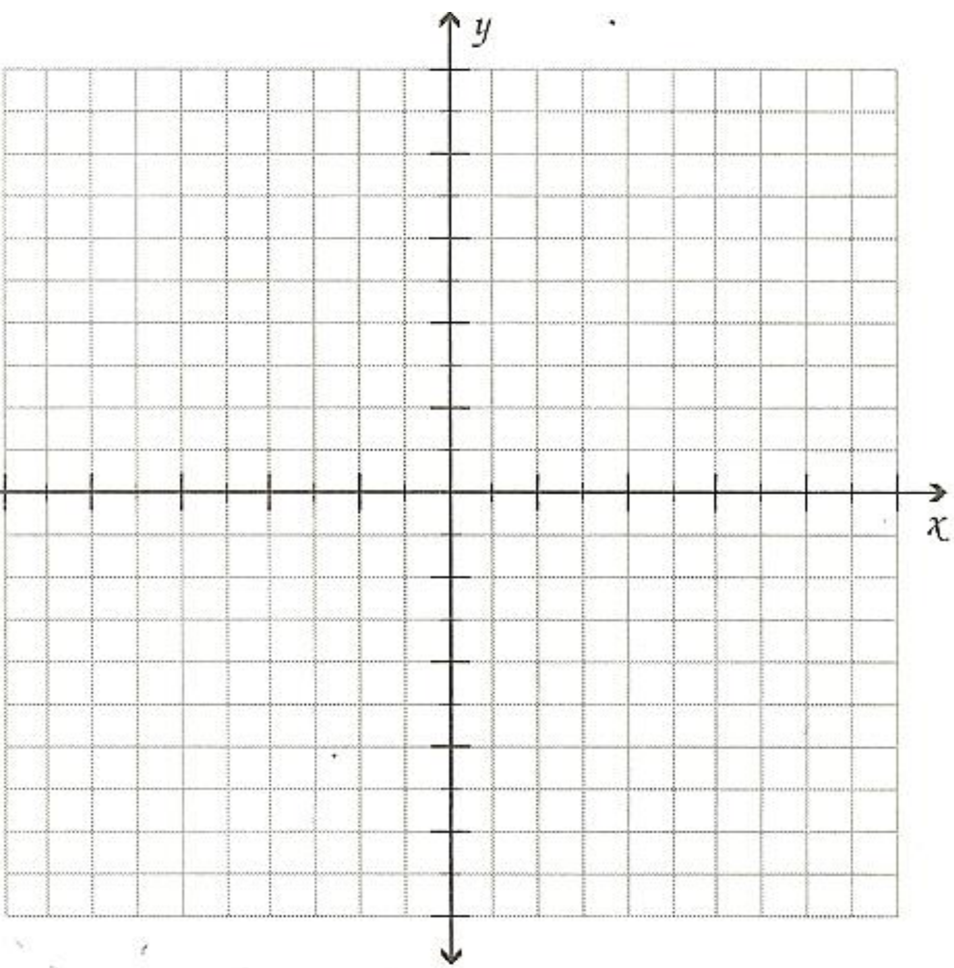


Ex. 6: Find the domain, x-intercept, and vertical asymptote of the logarithmic function. Then graph.

$a) \quad f(x) = \log_3(x-1) + 3$



b) $y = \log_{\frac{1}{2}}(4 - x)$



$c)\quad y = -\ln(x-2)$