Sec. 3.1: Exponential Functions and Their Graphs

The **exponential function *f* with base *b*** is denoted by

$$\boldsymbol{f(x)=b^{x}} \overset{\swarrow \text{input}}{}$$

$f(x) = a \cdot \underline{\underline{b}}^{x}$

where $b > 0$, $b \neq 1$ and x is any real number.

Ex. 1: Evaluate Exponential Functions. Round your result to three decimal places if needed.

Given:

$$f(x)=3^{x},\ g(x)=\left(\frac{1}{4}\right)^{x},\ h(x)=10^{x-2}$$

★ Do not leave answers in scientific notation.

Find: $a)\ f(-\sqrt{2})$ $b)\ g(\pi)$ $c)\ h(-6.4)$

$\boxed{\wedge}$

$= 3^{-\sqrt{2}}$

$\approx \boxed{.211}$

$= \left(\frac{1}{4}\right)^{\pi}$

$\approx \boxed{.013}$

$= 10^{(-6.4-2)} = 10^{-8.4}$

$= 3981 \times 10^{-9} = \boxed{0}$

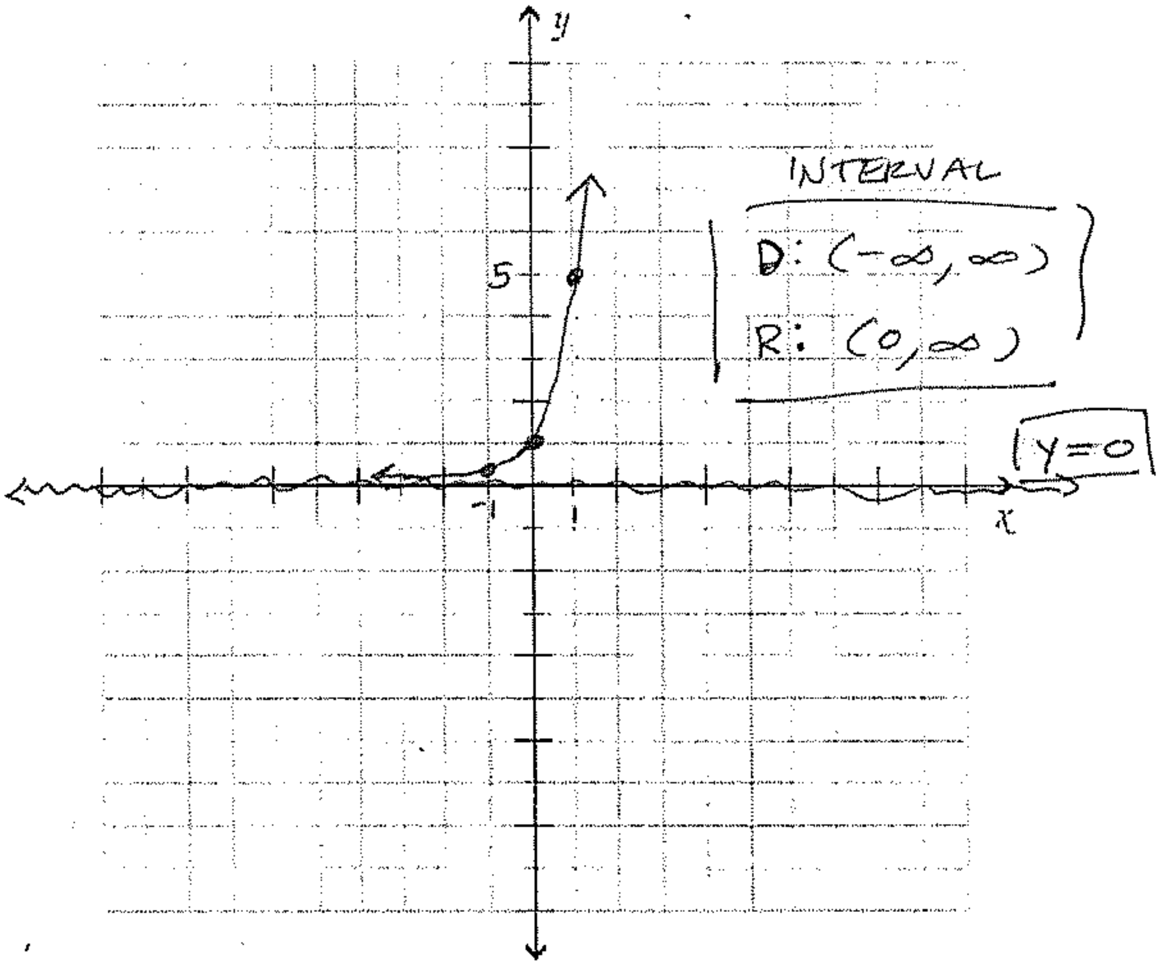
Graphs of Exponential Functions

Ex. 2: Graph each function.

a) $f(x) = \underline{5}^{x \leftarrow \text{positive}}$

5>1 growth (increase)

L → R



INTERVAL

D: $(-\infty, \infty)$

R: $(0, \infty)$

X	f(x)
-1	1/5
0	1
1	5
2	25

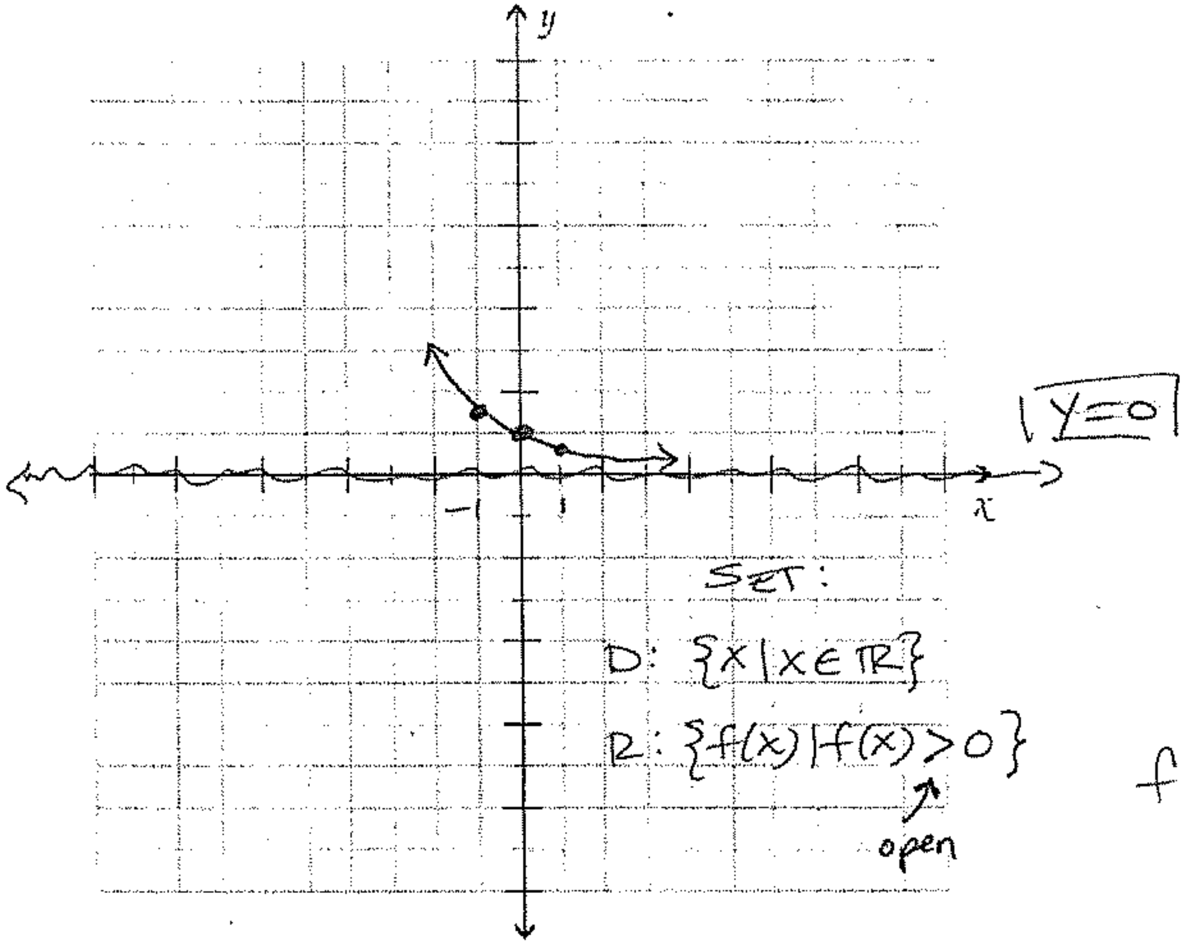
increases

Graphs of Exponential Functions

b) $f(x)=\left(\underline{\frac{2}{3}}\right)^{x} \leftarrow \text{positive}$

$0 < \frac{2}{3} < 1$ decay (decreasing)

L → R



SET:

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{f(x) \mid f(x) > 0\}$ ← open

X	f(x)
−1	$\frac{3}{2}$
0	1
1	$\frac{2}{3}$
2	$\frac{4}{9}$

↓ decreases

$f(-1) = \left(\frac{2}{3}\right)^{-1} = \left(\frac{3}{2}\right)^{1}$

Characteristics of Exponential Functions

$$f(x) = b^x, \quad b > 0, \quad b \neq 1$$ Parent graph

Domain: $(-\infty, \infty)$ Range: $(0, \infty)$ open

Y- Intercept: $(0, 1)$ Horizontal Asymptote: $\boxed{y=0}$

f(x) is increasing if *b > 1*. growth

f(x) is decreasing if *0 < b < 1*. decay

(*) No tricky negative. (No reflection)

One-to-One Function: [✓ Yes] or No Passes both VLT and HLT

Transformations Involving Exponential Functions

$$f(x) = a \cdot b^{(x-h)} + k$$

Vertical Stretch or

Left or right Horizontal shift

Up or down Vertical shift creates a new horizontal asymptote. $\boxed{y=k}$

Ex.3: Graph each function. Then state the domain and range.

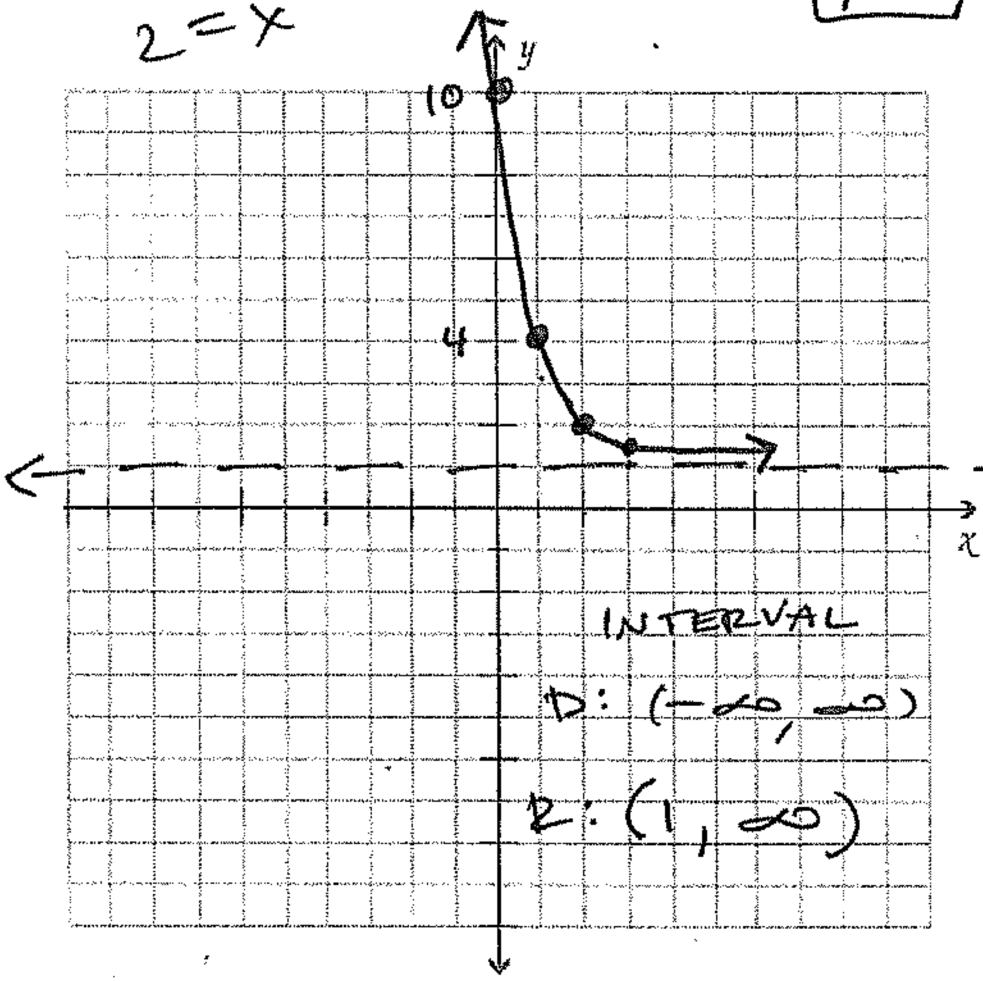
$a) f(x) = 3^{2-x} + 1$ up one

$f(x) = 3^{-x+2} + 1$ ← Tricky negative Y-axis reflection

$2-x=0$
$2=x$

HA: $\boxed{y=1}$

$3>1$ ~~growth~~ ~~increase~~
"decay" decrease L→R
growth w/reflection



INTERVAL

D: $(-\infty, \infty)$

R: $(1, \infty)$

→ $\boxed{y=1}$ HA

y-int →

x	f(x)
0	10
1	4
2	2
3	1 1/3

decrease

$3^{2-3} + 1$

$3^{-1} + 1$

$\frac{1}{3} + 1$

$b)\ g(x) = -\left(\frac{1}{4}\right)^{x-3} + 1$ ← up one, new HA: $\boxed{y=1}$

exponent
$x-3=0$
$x=3$

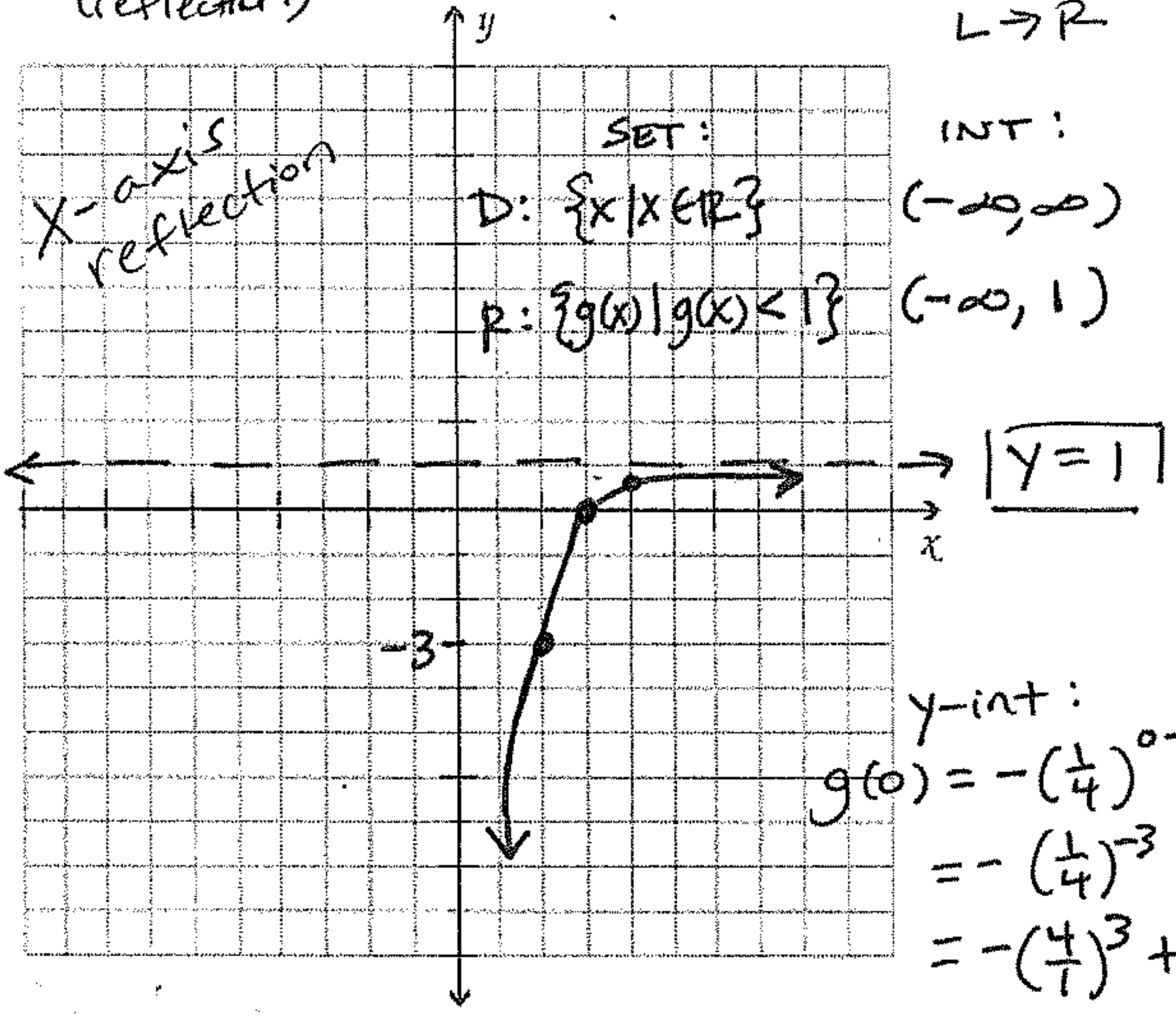
Tricky negative (reflection)

$0 < \frac{1}{4} < 1$ de~~cr~~ease

increase L→R

	X	g(x)
y-int:	0	-63
	2	-3
*	3	0
	4	3/4

↓ increase



X-axis reflection

SET:	INT:
D: $\{x \mid x \in \mathbb{R}\}$	$(-\infty, \infty)$
R: $\{g(x) \mid g(x) < 1\}$	$(-\infty, 1)$

y-int:

$g(0) = -\left(\frac{1}{4}\right)^{0-3} + 1$
$= -\left(\frac{1}{4}\right)^{-3} + 1$
$= -\left(\frac{4}{1}\right)^{3} + 1$
$= -64 + 1 = -63$

$g(2) = -\left(\frac{1}{4}\right)^{2-3} + 1$
$= -\left(\frac{1}{4}\right)^{-1} + 1$
$= -\left(\frac{4}{1}\right)^{1} + 1$
$= -4 + 1$
$= -3$

The Natural Base *e*

In many applications, the most convenient choice for a base is the irrational number: $e \approx 2.718281828\ldots$

$\pi \approx 3.14$

$\boxed{e \approx 2.72}$

The function, $f(x) = e^x$ is called natural exponential function.

↖ base e

To evaluate: $e^{-3} = \boxed{0.050}$

$e \boxed{\wedge} (-3)$

$e^{2.5} = \boxed{12.182}$

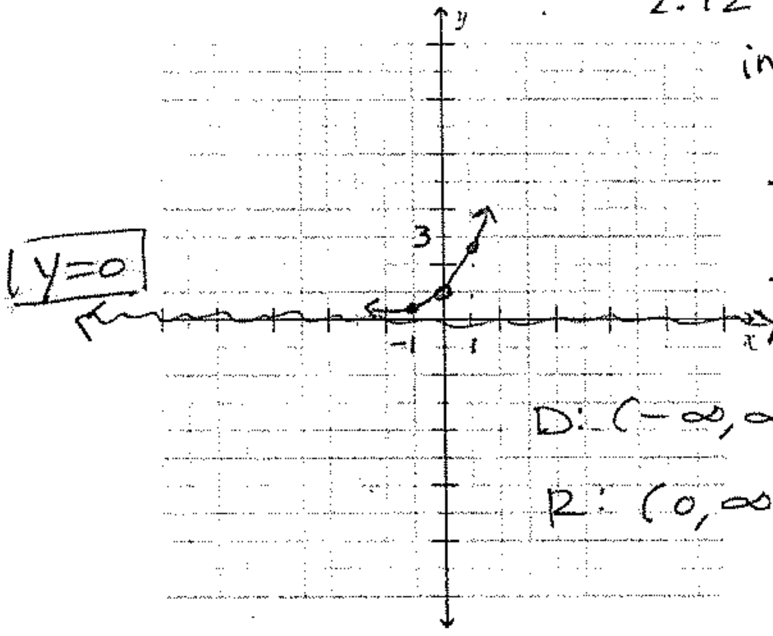
$e \boxed{\wedge} (2.5)$

To graph: $f(x) = \underline{e}^x$

$e > 1$

$2.72 > 1$

increase



$\boxed{y=0}$

D: $(-\infty, \infty)$

R: $(0, \infty)$

X	f(x)
-1	$1/e \approx .37$
0	1
1	$e \approx 2.72$

$g(x) = e^{-x}$ ← Tricky negative (Y-axis reflection)

$e > 1$

in~~crease~~

decrease

$g(x) = e^{-1 \cdot x}$

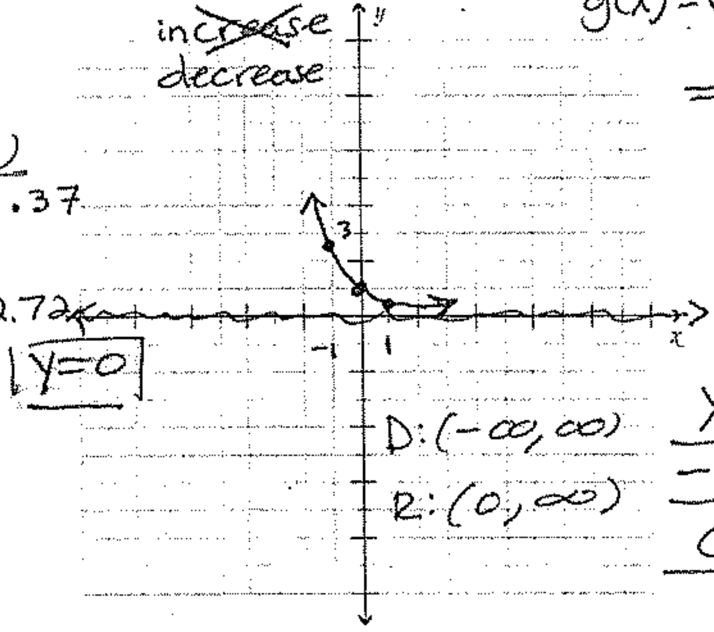
$g(x) = (e^{-1})^x$

$= \left(\frac{1}{e}\right)^x$

↑

$0 < \frac{1}{e} < 1$

decrease



$\boxed{y=0}$

D: $(-\infty, \infty)$

R: $(0, \infty)$

X	g(x)
-1	e
0	1
1	$\frac{1}{e}$

Applications: Formulas for Compound Interest

After *t* years, the balance *A* in an account with principal *P* and annual interest rate *r* (in decimal form) is given by:

(not as a %)

1) For *n* compoundings per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$

n	called
1	annually
2	Semi-annually
4	Quarterly
12	monthly

n	called
52	weekly
365	daily (non-leap yr)
24 6	bi-monthly < twice a month every 2 months

2) For $\boxed{\text{continuous}}$ compounding: $A = P\underline{e}^{rt}$

Base "e"

$$A = P\underline{\underline{e}}^{(rt)}$$

Ex.4: If you deposit $12,000 into a savings account that pays 6.75% interest, compounded continuously, while another account pays 7% interest, compounded quarterly. Which one of these accounts yields a better return over **6** years?

6.75% acct

Continuous

$A = Pe^{(rt)}$

$r = 6.75\% \rightarrow .0675$

$P = 12,000$

$t = 6$

$A = 12000\,\underbrace{(e)^{(.0675)(6)}}$

$A \approx \$17,991.63$

include

nearest penny

7% acct

Quarterly $n = 4$

$A = P(1 + \frac{r}{n})^{nt}$

$r = 7\% \rightarrow .07$

$P = 12,000$

$t = 6$

$A = 12000\,(\underline{1 + \frac{.07}{4}})^{(4)(6)}$

$A = 12000\,\underbrace{(1.0175)^{24}}$

$\boxed{A \approx \$18,197.31}$

The 7% acct. yields the better return.