Pre-Calc. Sec. 6.5 continued…

multiply Divide

Product and Quotient Rules for the trigonometric form of

r(cosθ+isinθ)

complex numbers:

Given : $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

Product : $z_1 z_2 = r_1 r_2\left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$

multiply multiply Add θ's

Quotient : $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$

Divide Divide Subtract θ's

Ex.5) Perform the operation, leave in trigonometric form.

Given :

$$z_1 = \underbrace{\frac{1}{2}}_{r_1}(\cos \underbrace{115^\circ}_{\theta_1} + i\sin 115^\circ) \quad z_2 = \underbrace{\frac{4}{5}}_{r_2}(\cos \underbrace{300^\circ}_{\theta_2} + i\sin 300^\circ)$$

Find : $z_1 z_2$

Product (multiply)

$$z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{5}\left[\cos(115^\circ + 300^\circ) + i\sin(115^\circ + 300^\circ)\right]$$

$$= \frac{2}{5}\left[\cos 415^\circ + i\sin 415^\circ\right]$$

Use coterminal

$415^\circ - 360^\circ = 55^\circ$

$$\boxed{z_1 \cdot z_2 = \frac{2}{5}(\cos 55^\circ + i\sin 55^\circ)}$$

or $\boxed{\frac{2}{5}\operatorname{cis} 55^\circ}$

Ex.6) Perform the operation, leave in trigonometric form.

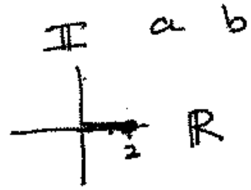
Given : $z_1 = 2 \quad z_2 = \sqrt{3} - i$ Given in standard form $(a+bi)$

a) Find : $\dfrac{z_1}{z_2}$ Using the Quotient Rule (trigonometric form).

$Z_1 = 2$

$= 2 + 0i$

$(2, 0)$
a b



$r = 2$

$\theta = 0°$

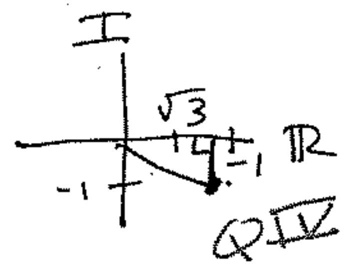
If you only have a real or only have an imaginary part... then it's a QA

$Z_1 = r(\cos\theta + i\sin\theta)$

$Z_1 = 2(\cos 0° + i\sin 0°)$

$Z_2 = \sqrt{3} - i$

$(\sqrt{3}, -1)$
a b



$r = \pm\sqrt{a^2+b^2}$

$r = \oplus\sqrt{(\sqrt{3})^2+(-1)^2}$

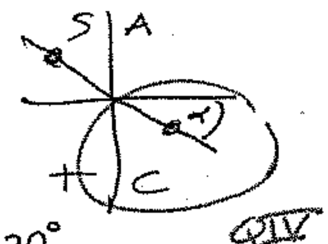
$r = 2$

$\tan\theta = \dfrac{b}{a}$

$\tan\theta = \dfrac{-1}{\sqrt{3}}$

$\tan\theta = -\dfrac{\sqrt{3}}{3}$
neg

$\alpha = \dfrac{\pi}{6} = 30°$



$\theta = 360° - \alpha$

$= 360° - 30°$

$\theta = 330°$

$Z = 2(\cos 330° + i\sin 330°) \rightarrow$

Now Find:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$$

$$= \frac{2}{2}\left[\cos(0°-330°) + i\sin(0°-330°)\right]$$

$$= 1\,(\cos(\underline{\underline{-330°}}) + i\sin(-330°)]$$

use coterminal

$$-330° + 360° = 30°$$

$$\frac{z_1}{z_2} = \boxed{1\,(\cos 30° + i\sin 30°)}$$ polar or trigonometric form

don't need the one

or $\boxed{1\,\mathrm{cis}\,30°}$

b) Using standard form (Alg2), we would have multiplied by the conjugate. The Quotient Rule works best when the # is *already* in trigonometric form.

$$\frac{z_1}{z_2} = \frac{2}{\left(\sqrt{3}-i\right)} \cdot \frac{\left(\sqrt{3}+i\right)}{\left(\sqrt{3}+i\right)} \text{ FOIL}$$

Recall

$$\boxed{i^2 = -1}$$

when given in standard form $(a+bi)$, use the conjugate

$$= \frac{2\left(\sqrt{3}+i\right)}{3 + \cancel{i\sqrt{3}} - \cancel{i\sqrt{3}} - i^2}$$

$$-(-1)$$

$$+1$$

$$\frac{z_1}{z_2} = 1\left(\cos 30^\circ + i \sin 30^\circ\right)$$

$$= \cos 30^\circ + \sin 30^\circ\, i$$

$$= \boxed{\frac{\sqrt{3}}{2} + \frac{1}{2} i}$$

$$= \frac{\cancel{2}\left(\sqrt{3}+i\right)}{\cancel{4}_2}$$

$$= \frac{\sqrt{3}+i}{2}$$

$$= \boxed{\frac{\sqrt{3}}{2} + \frac{1}{2} i}$$

Rectangular or standard form $a + bi$

r(cosθ+isinθ)

DeMoivre's Theorem, for the trigonometric form of complex numbers, is used to find *powers* of complex #'s.

power power power

$$z^{n} = \left[r\left(\cos\theta + i\sin\theta\right)\right]^{n} = r^{n}\left(\cos \underline{n\theta} + i\sin n\theta\right)$$

multiply θ by the power

7) Using DeMoivre's Theorem find :

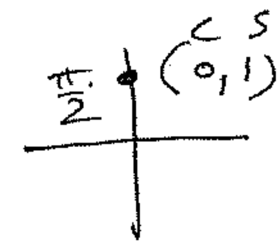
$$\left[2\left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)\right]^5$$

r θ power

$$= 2^5\left[\cos\left(\cancel{5}\cdot\frac{\pi}{\cancel{10}_2}\right) + i\sin\left(\cancel{5}\cdot\frac{\pi}{\cancel{10}_2}\right)\right]$$

$$= \boxed{32\left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right]}$$ Trig form.

or $\boxed{32\,\text{cis}\,\frac{\pi}{2}}$



Standard form:

$$32\cos\frac{\pi}{2} + 32\sin\frac{\pi}{2}\,i$$

$$\cancel{32(0)} + 32(1)\,i$$

Standard/ rectangular form $\boxed{32i}$ QA

8) Using DeMoivre's Theorem find: $4\left(1-\sqrt{3}i\right)^3$

need trigonometric form. $= 4(z)^3$

$z = 1 - \sqrt{3}i$

QIV

$(1, -\sqrt{3})$
$\ \ a \quad b$

$r = \pm\sqrt{a^2+b^2}$

$r = \oplus\sqrt{(1)^2+(-\sqrt{3})^2}$

$r = \oplus\sqrt{4}$

$r = 2$

$\tan\theta = \frac{b}{a}$

$\tan\theta = -\frac{\sqrt{3}}{1}$

$\alpha = \frac{\pi}{3}$

$\theta = 2\pi - \alpha$

$\theta = \frac{5\pi}{3}$

$z = r(\cos\theta + i\sin\theta)$

$z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

Now find $z^3 = \left[2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\right]^3$

$= 2^3\left(\cos\left(\cancel{3}\cdot\frac{5\pi}{\cancel{3}}\right) + i\sin\left(\cancel{3}\cdot\frac{5\pi}{\cancel{3}}\right)\right]$

$= 8\left(\cos 5\pi + i\sin 5\pi\right)$

use coterminal $5\pi - 4\pi = \pi$

$z^3 = 8(\cos\pi + i\sin\pi)$

→

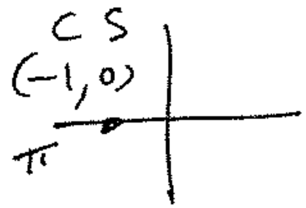
Finally find: $4z^3 = 4\left[8(\cos\pi + i\sin\pi)\right]$

$$= \boxed{32(\cos\pi + i\sin\pi)}$$

Trig. form / polar form

change to standard form

$$32(\cos\pi + i\sin\pi)$$



$$32\cos\pi + 32\sin\pi\, i$$

$$32(-1) + \cancel{32(0)\, i}$$

$$= \boxed{-32}$$

QA → Standard form / rectangular form