Pre-Calc. Sec. 6.5

Trigonometric Form of Complex Numbers (Polar Form)

standard form or rectangular form

A complex number, $z = a + bi$, can be expressed as a point (a, b) in the complex plane. This is standard (or rectangular) form.

The absolute value of a complex # $a + bi$ is defined as the distance between the origin (0,0) and the point (a,b).

$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

Ex.1) Plot the complex # and find its absolute value.

$z = -1 + \sqrt{3}i$

$a + bi$ form

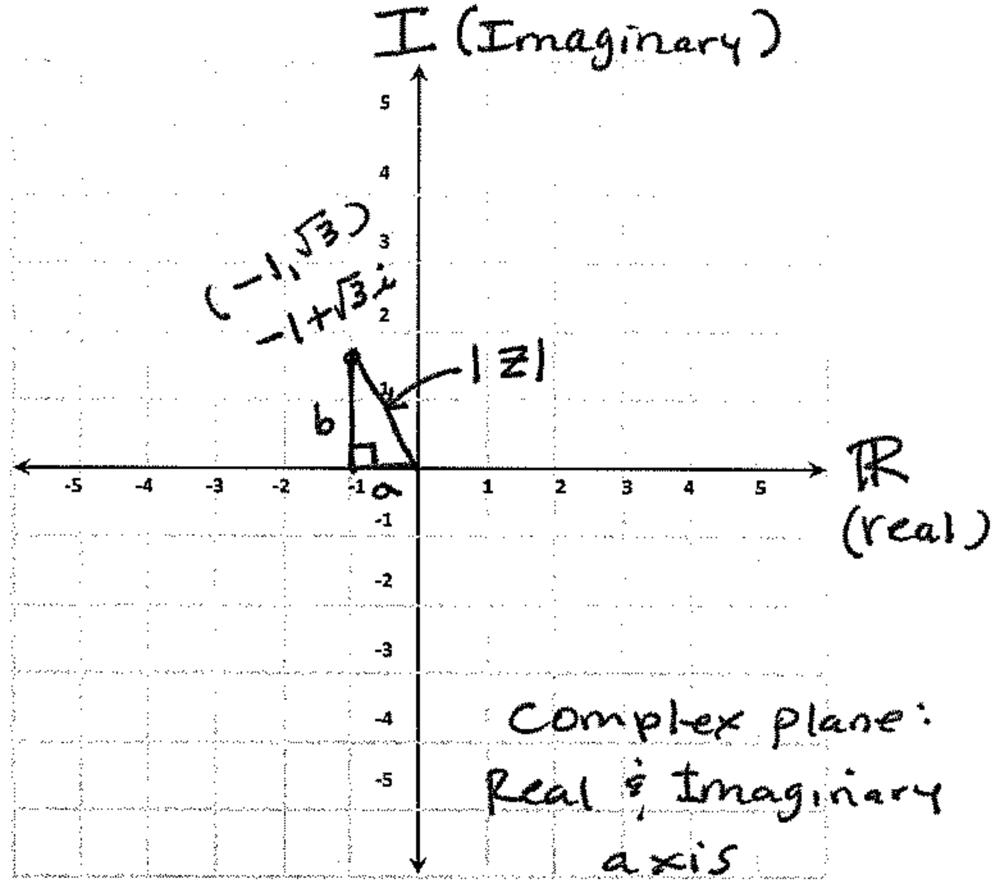
$a = -1$

$b = \sqrt{3}$ — $\sqrt{1} = 1$, $\sqrt{4} = 2$

(a, b)

$(-1, \sqrt{3})$

Real part   imag. part.



$$|z| = |-1+\sqrt{3}i| = \sqrt{a^2+b^2}$$

$$= \sqrt{(-1)^2 + (\sqrt{3})^2}$$

$$= \sqrt{1+3}$$

$$|z| = \boxed{2}$$ absolute value

The trigonometric form (also called polar form) of a complex number is given by:

$$z = r(\cos\theta + i\sin\theta) \text{ or } r\,cis\,\theta$$

radius

radius

Expanded

condensed

Recall :

CAH $\cos\theta = \frac{x}{r}$ SOH $\sin\theta = \frac{y}{r}$ TOA $\tan\theta = \frac{y}{x}$

Now :

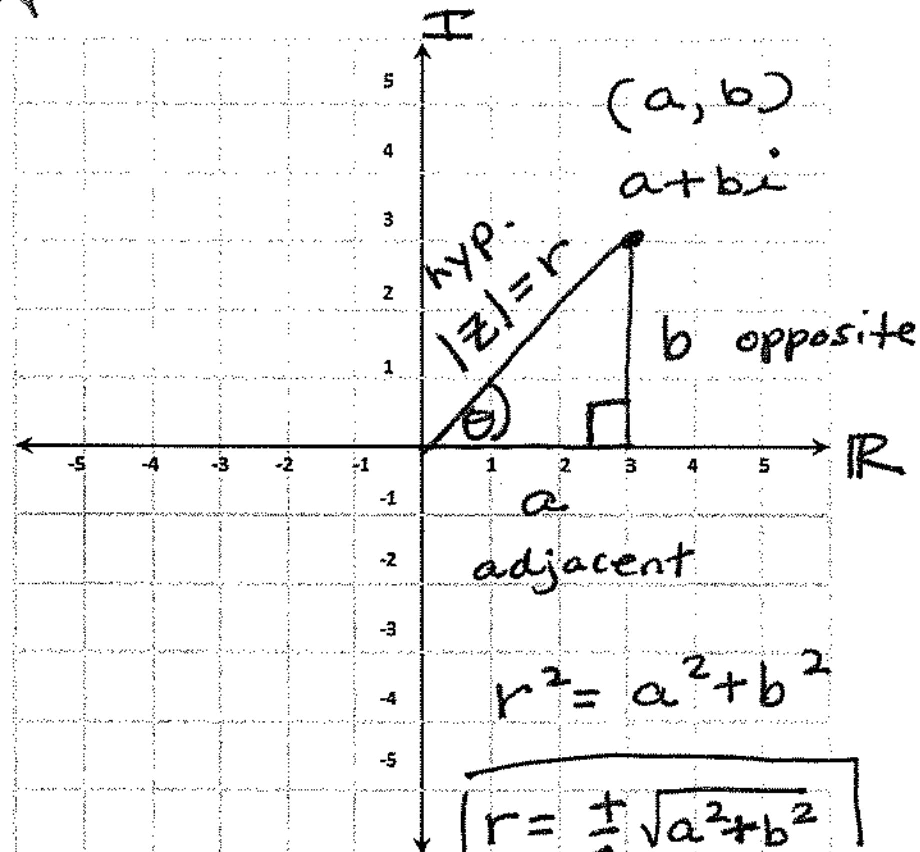
$\cos\theta = \frac{a}{r}$ $\sin\theta = \frac{b}{r}$ $\tan\theta = \frac{b}{a}$

$\frac{\cos\theta}{1} = \frac{a}{r}$ $\frac{\sin\theta}{1} = \frac{b}{r}$

So.....

$\boxed{a = r\cos\theta}$ $\boxed{b = r\sin\theta}$

$\underline{\underline{a}} + \underline{\underline{b}}i$ (standard form)



$r^2 = a^2 + b^2$

$\boxed{r = \pm\sqrt{a^2+b^2}}$

can be either in polar form

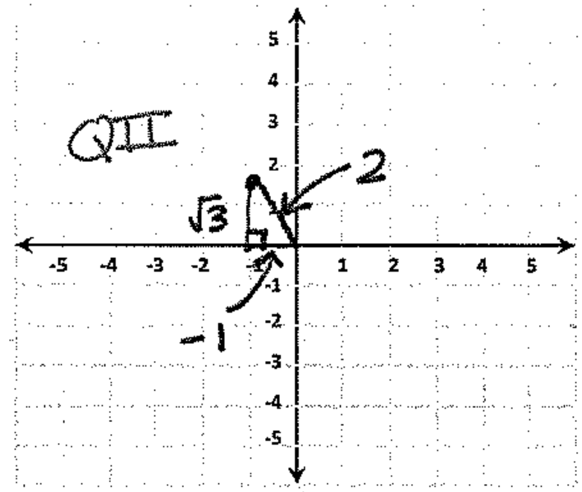
Ex.2) Write the trigonometric form (polar form) for the complex number: $z = -1 + \sqrt{3}\, i$

plot the # !!!

$a + bi$

$(-1, \sqrt{3})$

(a, b)



① Find r:

$r = \pm\sqrt{a^2+b^2}$

$r = 2$

② Find θ:

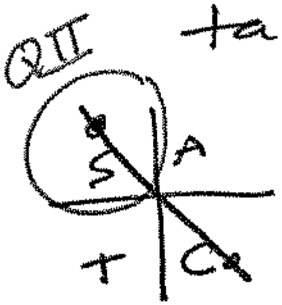
TOA $\quad \tan\theta = \frac{b}{a}$

$\tan\theta = \frac{\sqrt{3}}{-1}$

QII $\quad \tan\theta = -\sqrt{3}$

neg

$\alpha = \pi/3$



$\theta = \frac{2\pi}{3}$

③ Trig. Form:

$z = r(\cos\theta + i\sin\theta)$

need r and θ ← θ should be btwn $[0, 2\pi)$ or $[0, 360)$.

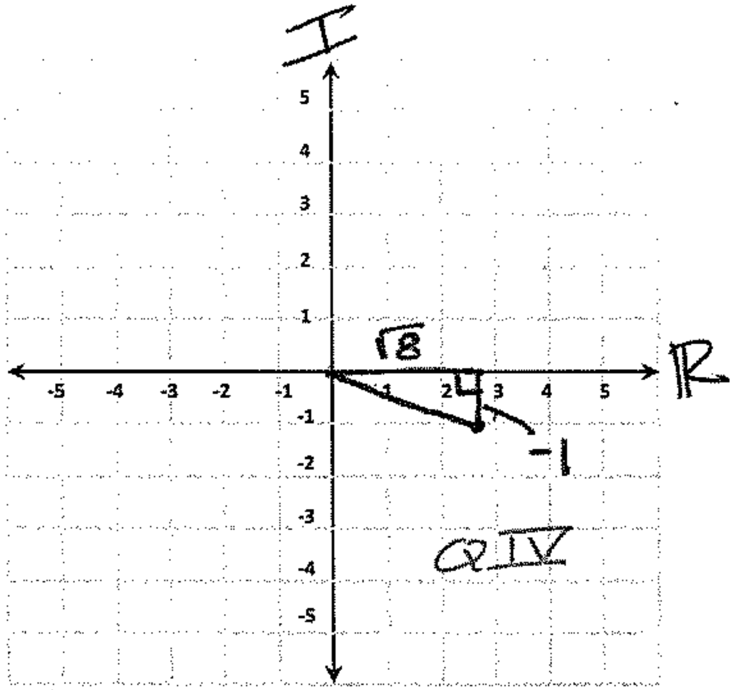
$$\boxed{z = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)}$$

or

$z = r\,\text{cis}\,\theta$

$$\boxed{z = 2\,\text{cis}\,\frac{2\pi}{3}}$$

Ex.3) Graph Z, then find the trigonometric form (polar form) (in degrees) for the complex number: $z = 2\sqrt{2} - i$ Plot the # !!!



Standard form

$a + bi$

$(2\sqrt{2}, -1)$

(a, b)

$(\sqrt{8}, -1)$

$2\sqrt{2}$

$\sqrt{4 \cdot 2}$

$\sqrt{8}$ — $\sqrt{4}$ 2, $\sqrt{9}$ 3

③ Trig form:

$z = r(\cos\theta + i\sin\theta)$

$\boxed{z = 3(\cos 340.53° + i\sin 340.53°)}$

or

$z = r\operatorname{cis}\theta$

$\boxed{z = 3\operatorname{cis} 340.53°}$

Btwn $[0, 360°)$

① need r

$r = \pm\sqrt{a^2+b^2}$

$r = \pm\sqrt{(\sqrt{8})^2+(-1)^2}$

$r = \pm\sqrt{8+1}$

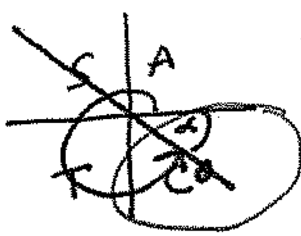
$r = 3$

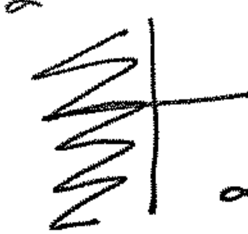
② need θ

$\tan\theta = \frac{b}{a}$

$\tan\theta = \frac{-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\tan\theta = \frac{-\sqrt{2}}{4}$ ← neg





$\alpha = \tan^{-1}\left(+\frac{\sqrt{2}}{4}\right)$

$\alpha \approx 19.47°$

$\theta = 360° - \alpha$

$\theta = 360° - 19.47°$ $\theta \approx 340.53°$

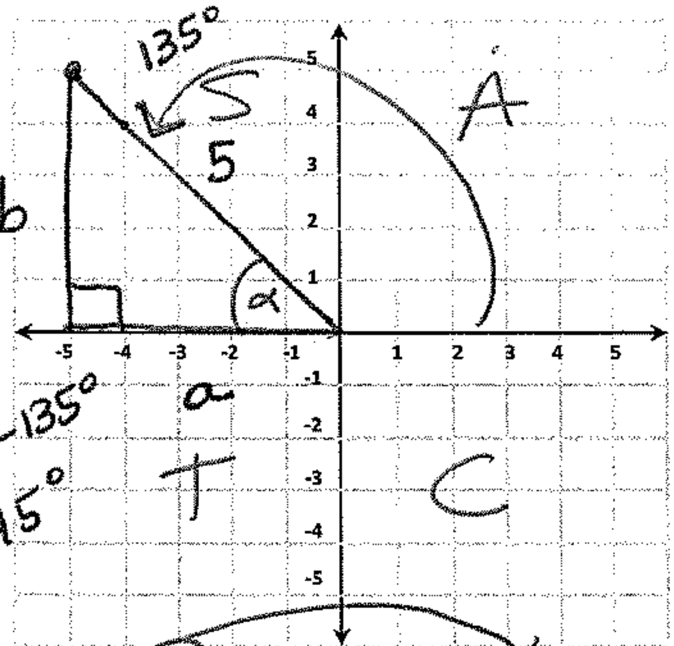
Ex.4) Represent the complex # graphically, then find the number in standard form (rectangular form)
$a+bi$

Plot it!!!

$$z = \underset{r}{5}(\cos \underset{\theta}{135°} + i \sin 135°) \text{ or } 5cis135°$$

$r = 5$

$\theta = 135°$



$\alpha = 180° - 135°$

$\alpha = 45°$

① $z = 5(\cos 135° + i \sin 135°)$

$= 5\cos 135° + 5\sin 135° i$

$= 5(\pm\cos 45°) + 5(\pm\sin 45°)i$

$= 5\left(-\frac{\sqrt{2}}{2}\right) + 5\left(\frac{\sqrt{2}}{2}\right)i$

or ② CAH

$\frac{\cos\theta}{1} = \frac{a}{r}$

$a = r\cos\theta$

$a = 5\cos 135°$

$a = 5(\pm\cos 45°)$

$a = 5\left(-\frac{\sqrt{2}}{2}\right)$

$a = -\frac{5\sqrt{2}}{2}$

SOH

$\frac{\sin\theta}{1} = \frac{b}{r}$

$b = r\sin\theta$

$b = 5\sin 135°$

$b = 5(\pm\sin 45°)$

$b = 5\left(\frac{\sqrt{2}}{2}\right)$

$b = \frac{5\sqrt{2}}{2}$

$z = \boxed{\frac{-5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2} i}$

Standard form

$a + bi$