Pre-Calc. Sec. 6.5

Trigonometric Form of Complex Numbers (Polar Form)

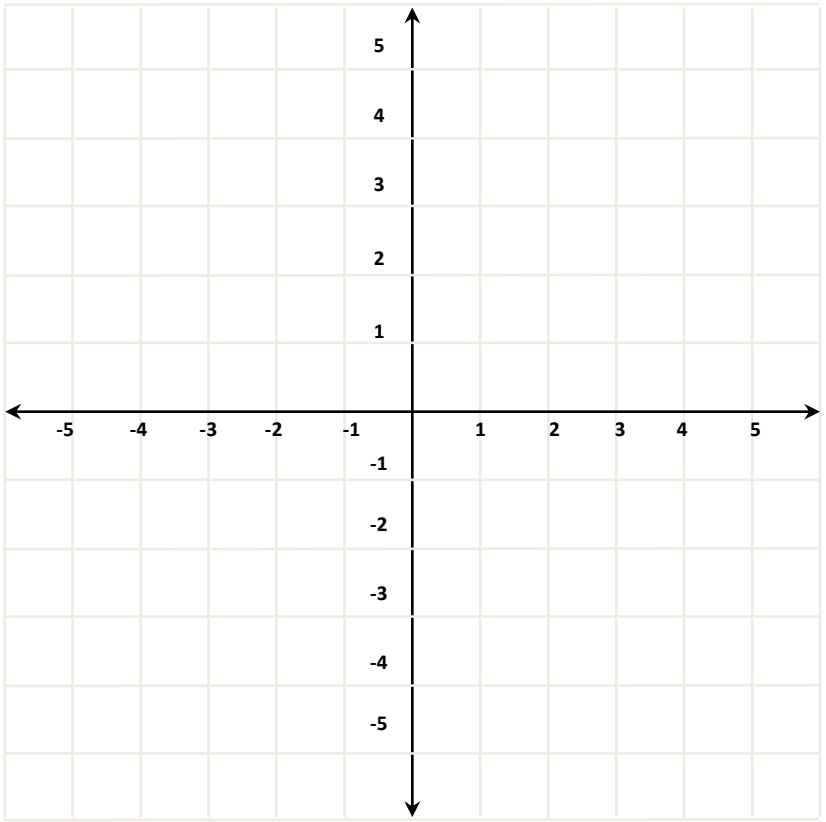
A complex number, $z = a + bi$, can be expressed as a point (a, b) in the complex plane. This is standard (or rectangular) form.

The absolute value of a complex # $a + bi$ is defined as the distance between the origin (0,0) and the point (a,b).

$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

Ex.1) Plot the complex # and find its absolute value.

$$z = -1 + \sqrt{3}i$$

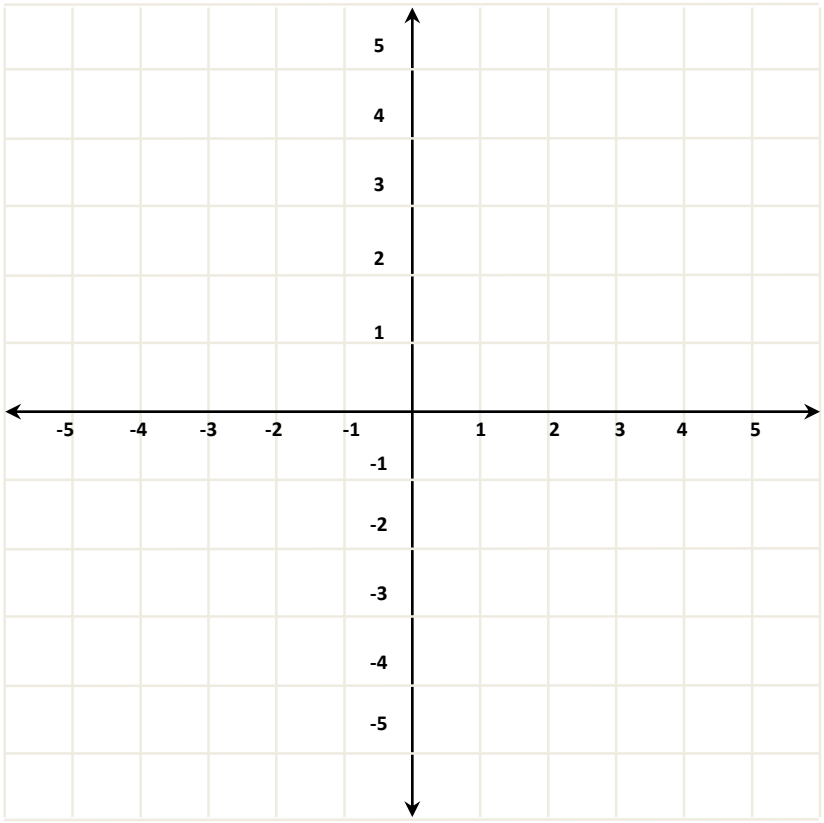


The trigonometric form (also called polar form) of a complex number is given by:

$$z = r(\cos\theta + i\sin\theta) \text{ or } rcis\theta$$

Recall :

$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r} \qquad \tan\theta = \frac{y}{x}$$



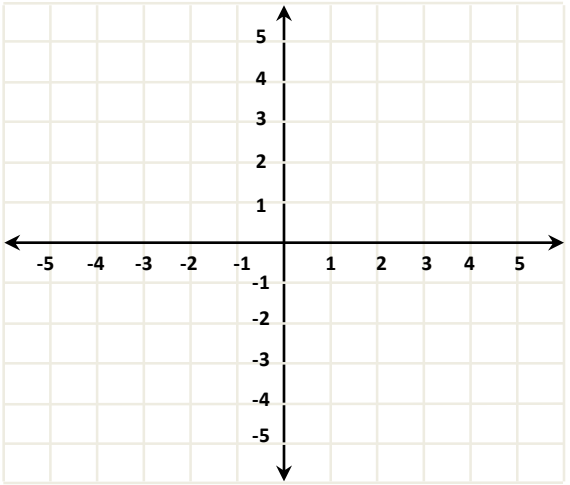
Now :

$$\cos\theta = \frac{a}{r} \qquad \sin\theta = \frac{b}{r} \qquad \tan\theta = \frac{b}{a}$$

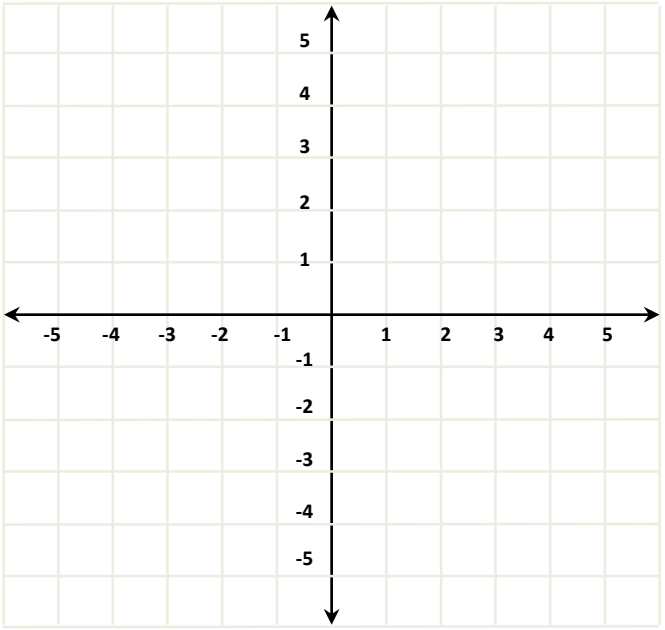
So.....

$$a = r\cos\theta \qquad b = r\sin\theta$$

Ex.2)Write the trigonometric form (polar form) for the complex number: $z = -1 + \sqrt{3}i$



Ex.3) Graph Z, then find the trigonometric form (polar form) for the complex number: $z = 2\sqrt{2} - i$

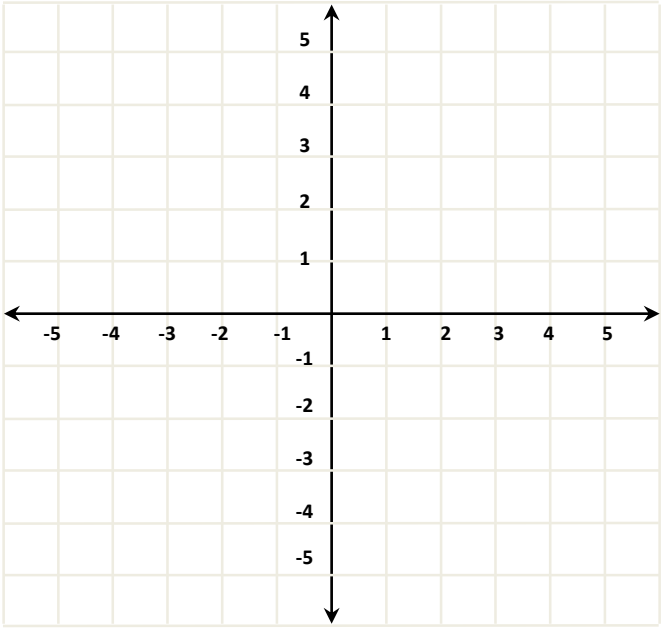


Ex.4) Represent the complex # graphically, then find the number in standard form.

$$z = 5(\cos 135° + i \sin 135°) \text{ or } 5cis135°$$

$r =$

$\theta =$



Pre-Calc. Sec. 6.5 continued…

Product and Quotient Rules for the trigonometric form of complex numbers:

$$\text{Given}: \; z_1 = r_1(\cos\theta_1 + i\sin\theta_1) \quad z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$$

$$\text{Product}: \; z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right]$$

$$\text{Quotient}: \; \frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right]$$

Ex.5) Perform the operation, leave in trigonometric form.

Given :

$$z_1 = \frac{1}{2}(\cos 115° + i \sin 115°) \quad z_2 = \frac{4}{5}(\cos 300° + i \sin 300°)$$

Find : $z_1 z_2$

Ex.6) Perform the operation, leave in trigonometric form.

Given : $z_1 = 2 \quad z_2 = \sqrt{3} - i$

a) Find : $\frac{z_1}{z_2}$ Using the Quotient Rule (trigonometric form).

DeMoivre's Theorem, for the trigonometric form of complex numbers, is used to find *powers* of complex #'s.

$$z^n = \left[r\left(\cos\theta + i\sin\theta\right)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

7) Using DeMoivre's Theorem find:

$$\left[2\left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)\right]^5$$

8) Using DeMoivre's Theorem find : $4\left(1-\sqrt{3}i\right)^3$