Notes

Pre-Calculus
Sec. 6.4
Graphs of Polar Equations

1) CIRCLES:

$\frac{x}{r}$

A) r = n cos θ

"center on the x-axis with radius = n/2 or diameter = n"

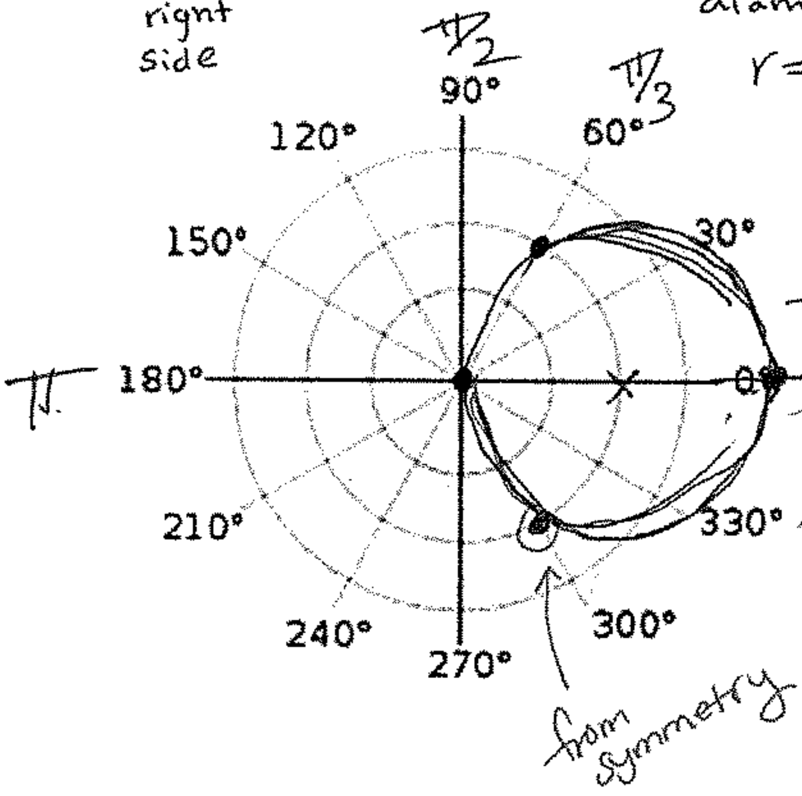
Ex. r = 4 cos θ

x-axis symmetry

Positive right side

diam = 4

r = 2



r	θ
4.	0
0 pole	π/2
~~-4~~	~~π~~
2	π/3

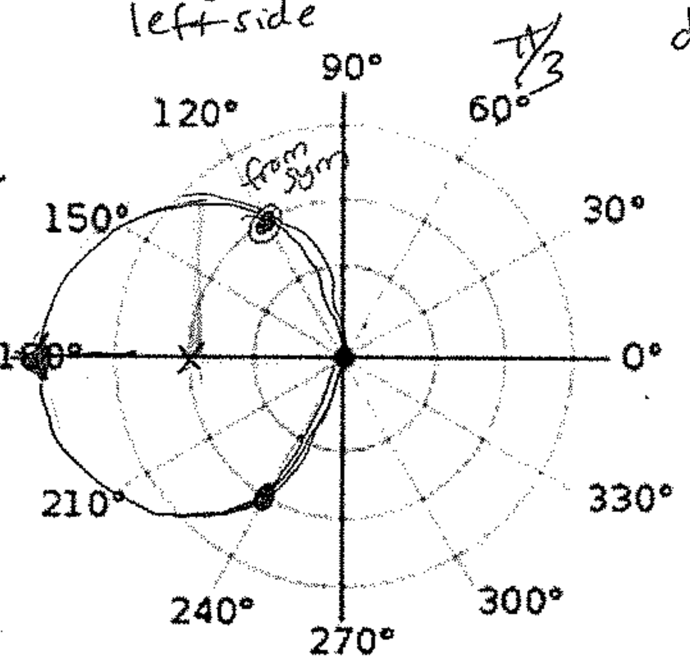
Ex. r = - 4 cos θ

x-axis sym.

negative left side

diam = 4

r = 2



r	θ
-4	0.
0 pole	π/2
-2	π/3

$\frac{y}{r}$

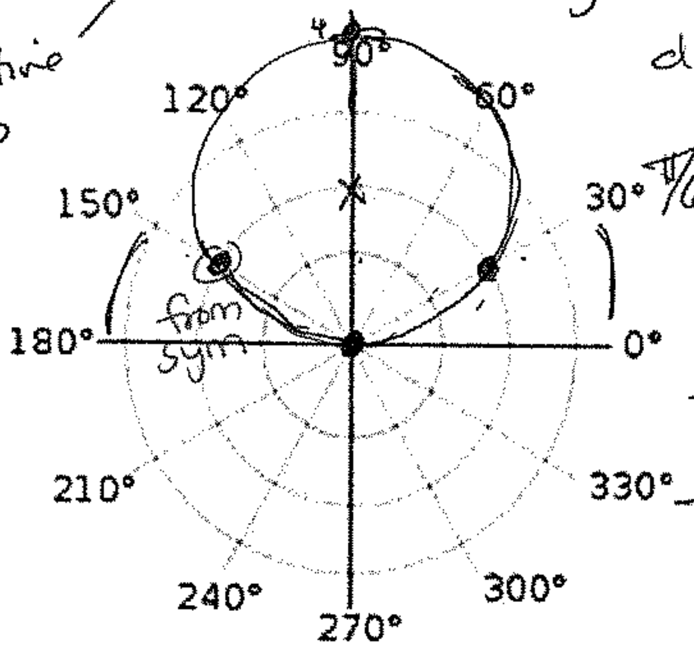
B) $r = n \sin \theta$

"center on y-axis with radius = n/2 or diameter = n"

Ex. $r = 4 \underline{\sin \theta}$

y-axis Symmetry

Positive up top



diam = 4

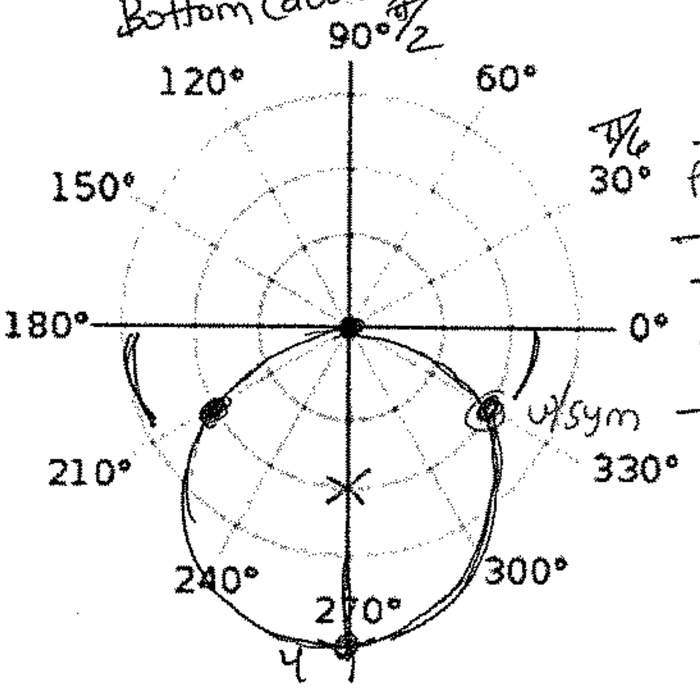
r = 2

r	θ
pole 0	0
4	π/2
2	π/6

Ex. $r = - 4 \underline{\sin \theta}$

negative Bottom (down)

y-axis sym



r	θ
pole 0	0
−4	π/2
−2	π/6

Sounds like

2) Limaçons "Limasons"

• always use the quadrantals and extra points

r = a ± b sin θ or r = a ± b cos θ

one of these

A) If |a| = |b|: "Heart Shape" called a Cardioid

$$\boxed{\frac{|a|}{|b|} = 1}$$

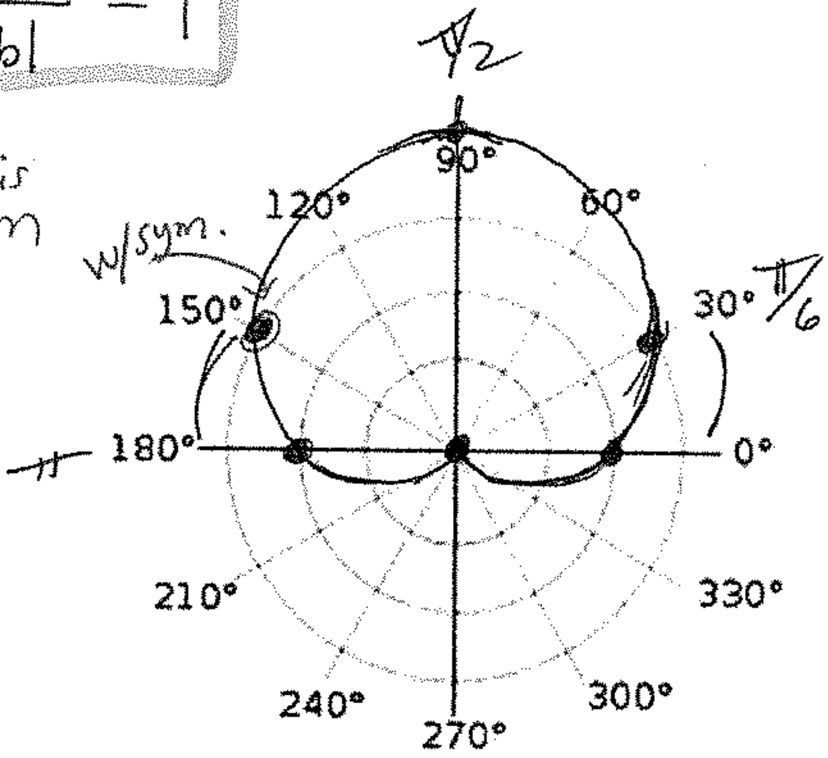
Ex. $r = \overset{a}{2} + \overset{b}{2}\ \underline{\sin\theta}$

y-axis sym

Positive (up top)

|a| = 2

|b| = 2



r	θ
2	0
4	π/2
2	π
0 pole	3π/2
3	π/6
1	7π/6

$r = \overset{a}{2} - \overset{b}{2}\underline{\cos\theta}$ **Limaçon**: Cardioid

negative (leftside)

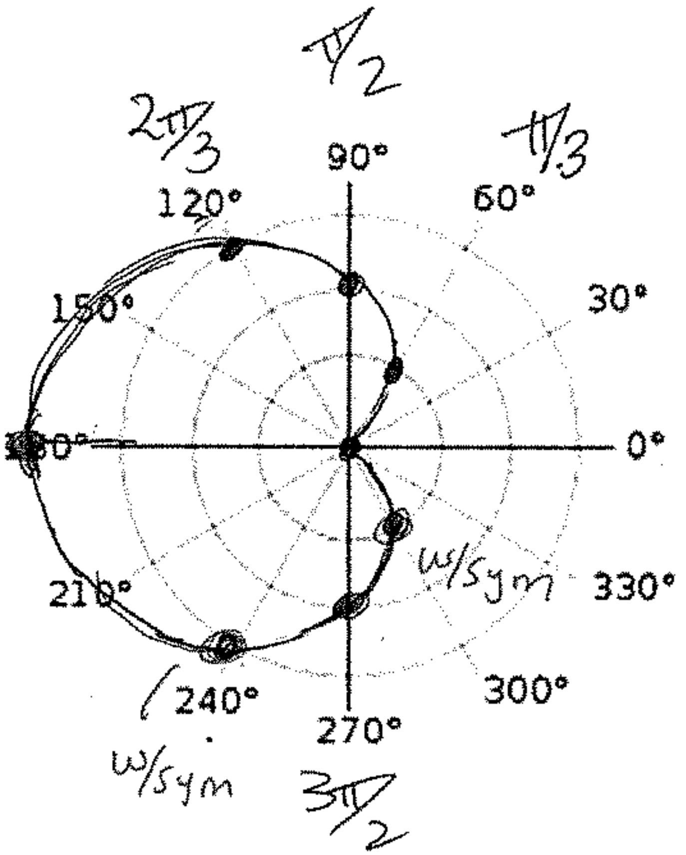
X-axis symmetry

$\frac{|a|}{|b|} = 1$

$\frac{|2|}{|-2|} = 1$

r	θ
0 pole	0
2	π/2
4	π
2	3π/2
1	π/3
3	2π/3

#



Less than

B) If |a| < |b| Less **Limaçon:** $\boxed{\frac{|a|}{|b|} < 1}$ Less than

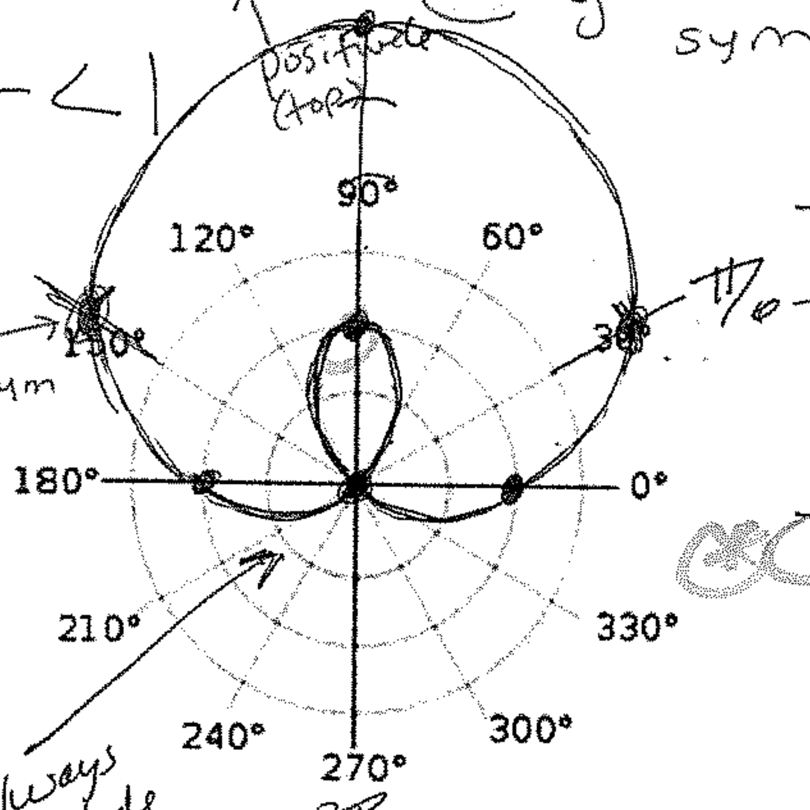
"Inner Loop - crosses or loops at the pole"

- All loops include a point at the pole, although it doesn't come from a QA

Ex. $r = \overset{a}{2} + \overset{b}{4}\ \underline{\sin\theta}$

$\frac{2}{4} < 1$

y-axis sym



Always include the pole for inner loops

r	θ
2	0
6	π/2
2	π
-2	3π/2
4	π/6

Study!

$r = 2 + 4\sin\theta$

pole

$0 = 2 + 4\sin\theta$

$-2 = 4\sin\theta$

$-\frac{1}{2} = \sin\theta$

$\alpha = \frac{\pi}{6}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

C) If |a| > |b| **Limaçon:** "Curve surrounding the pole"

$$1 < \frac{|a|}{|b|} < 2$$

(NO pole, NO loop, With DIMPLE)

$$\frac{|a|}{|b|} \geq 2$$

(NO pole, NO loop, NO dimple)

Ex. $r = \overset{a}{3} + \overset{b}{2}\cos\theta$ (Right) x-axis sym

(NO pole, NO loop, With DIMPLE)

$$\frac{|3|}{|2|}$$

$$1 < \frac{3}{2} < 2$$

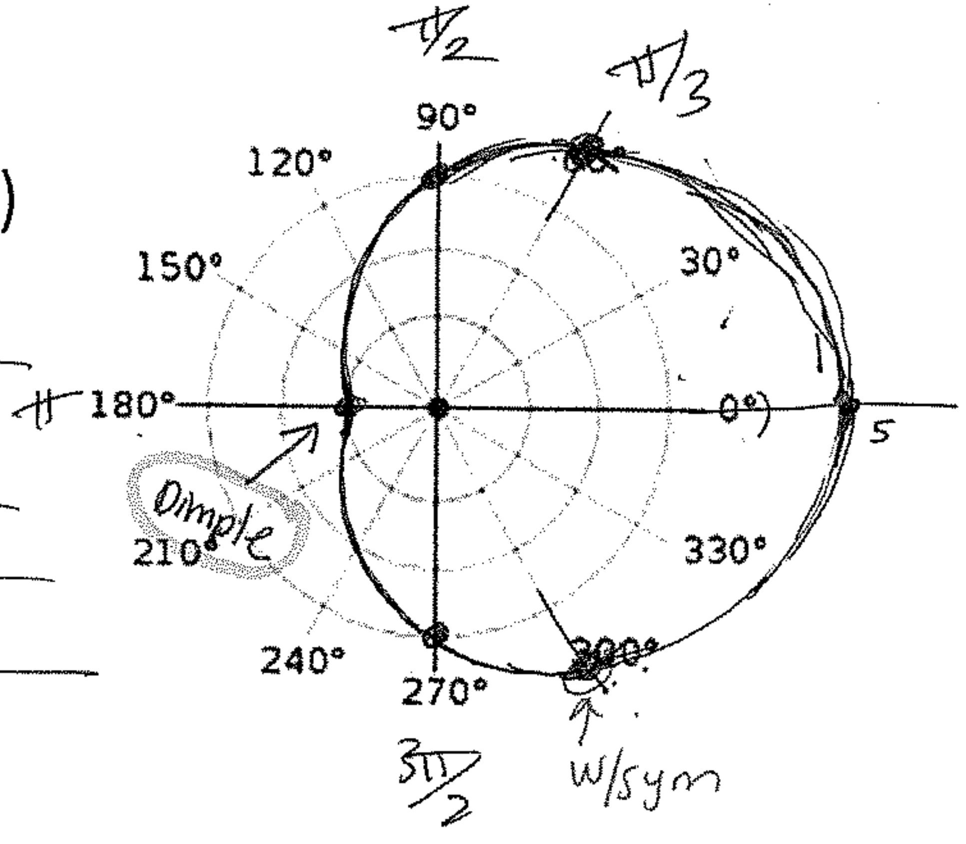
$r = 3 + 2\cos\theta$

$0 = 3 + 2\cos\theta$

$-3 = 2\cos\theta$

$-3/ = \cos\theta$ ∅ B/c range [-1,1]

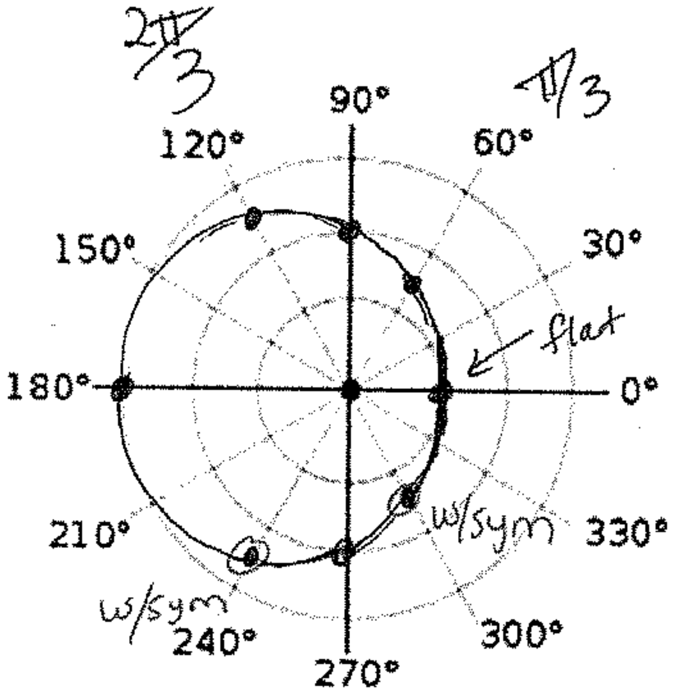
r	θ
5	0
3	π/2
1	π
3	3π/2
4	π/3



negative (left) x-axis is sym $\frac{|2|}{|-1|}$ $2 \geq 2$

Ex. $r = \overset{a}{2} - \overset{b}{1}\cos\theta$ **Limaçon:** Curve surrounding the pole

(NO pole, NO loop, NO dimple)



r	θ
1	0
2	π/2
3	π
2	3π/2
1½	π/3
2½	2π/3

$\frac{S \mid A}{T \mid C}$

3) Roses:

* no plotting by plugging in QA's or other θ

$r = a \sin n\theta$ or $r = a \cos n\theta$

Always include the pole

length of each petal

# of petals

If n is even: # of petals = 2n **(double)**

If n is odd: # of petals = n

n	# of petals
not a rose 1	no Circle ⊙
2	4
3	3 — least possible
4	8
5	5

Ex. Steps for graphing roses: $r = \underset{\text{length of petal} = 3}{3} \sin \underline{2\theta}$ — $n = 2$ (even)

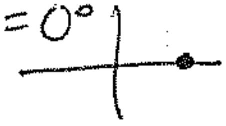
$\#$ of petals $= 4$

Step 1: Find the 1st tip of the petals:

Set sin nθ = 1 or cos nθ = 1 (leave off "a")

$n\theta = 90°$

$n\theta = 0°$



$2\theta = 90°$

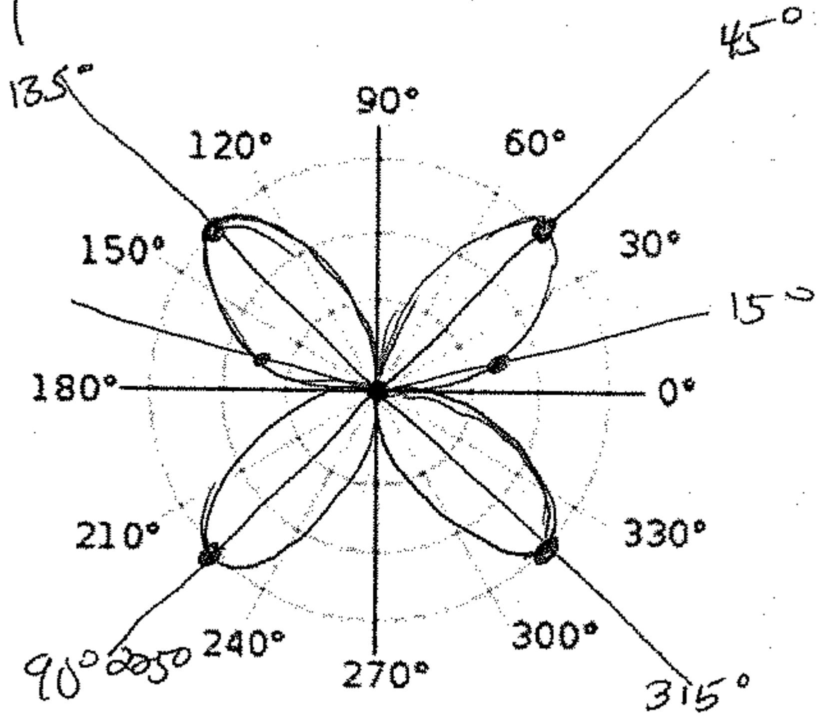
$\boxed{\theta = 45°}$, 1st petal

Step 2: Find the rest of the tips:

360 ÷ # of petals, add to step 1

$$4\overline{)360}^{\;\boxed{90°}}$$

petal every 90°



Petals:

$\boxed{45°}, +90° = \boxed{135°} +90°, = \boxed{225°} +90° = \boxed{315°}$

Rose

Ex. $r = 2 \sin 3\theta$

Petal length = 2

$n = 3$ (odd)

3 petals

Find 1st tip:

Sine

$n\theta = 90^\circ$

$3\theta = 90^\circ$

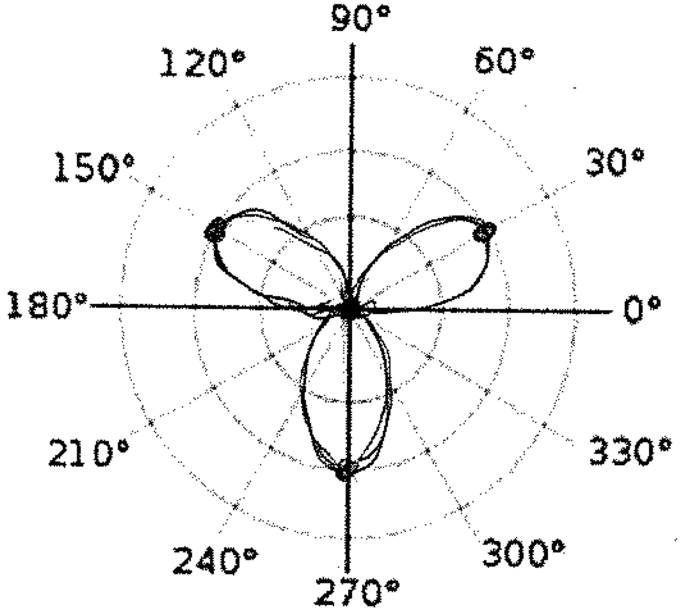
$\theta = 30^\circ$ 1st petal

$3\overline{)360^\circ}$ $\boxed{120^\circ}$ add on to 1st petal

\# of petals

$\boxed{30^\circ} + 120^\circ = \boxed{150^\circ} + 120^\circ = \boxed{270^\circ}$

3 petals



$r = 2\sin 3\theta$

pole $0 = 2\sin 3\theta$

$0 = \sin 3\theta$

Let $u = 3\theta$

$\sin u = 0$

$u = \cancel{0} + \pi n$

$\cancel{\frac{1}{3}}\cancel{3}\theta = \frac{1}{3}\pi n$

$\boxed{\theta = \frac{\pi}{3} n, n \in \mathbb{Z}}$

Ex. $r = 2\cos 3\theta$ Rose

Petal length = 2

$n = 3$ (odd)

(3 petals)

$$3\overline{)360^\circ}^{\;120^\circ}$$

add on

Find 1st tip: Cosine rose

$n\theta = 0^\circ$

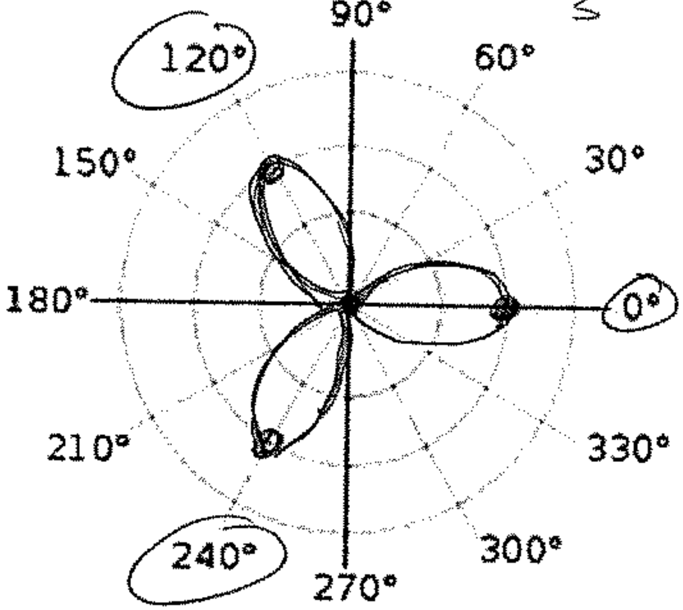
$\frac{3\theta}{3} = \frac{0^\circ}{3}$

$\theta = 0^\circ$

All cosine roses begin at 0° for 1st petal

$\boxed{\boxed{0^\circ} + 120^\circ = \boxed{120^\circ} + 120^\circ = \boxed{240^\circ}}$

3 petals



$r = 2\cos 3\theta$

pole

$0 = 2\cos 3\theta$

$0 = \cos 3\theta$

Let $u = 3\theta$

$\cos u = 0$

$u = \frac{\pi}{2} + \pi n$

$\cancel{\tfrac{1}{3}}\cancel{3}\theta = \frac{\pi}{3\,2} + \frac{\pi}{3}n$

$\boxed{\theta = \frac{\pi}{6} + \frac{\pi}{3}n,\ n \in Z}$

No plug and chug (r, θ)

4) Lemniscates:

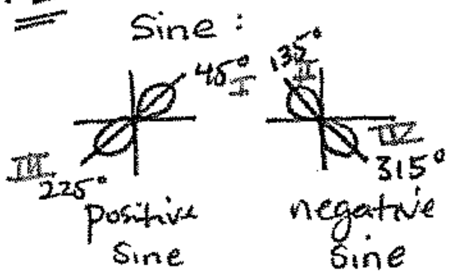
← skate → Figure "8"

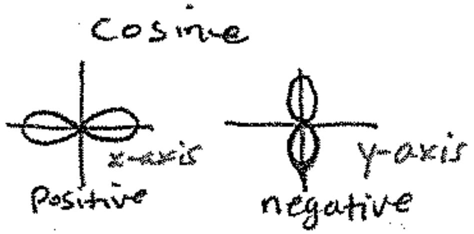
The equation looks like a rose, but with r^2 & a^2.

$r^2 = a^2 \sin 2\theta$ or $r^2 = a^2 \cos 2\theta$

squared

Always a "2"





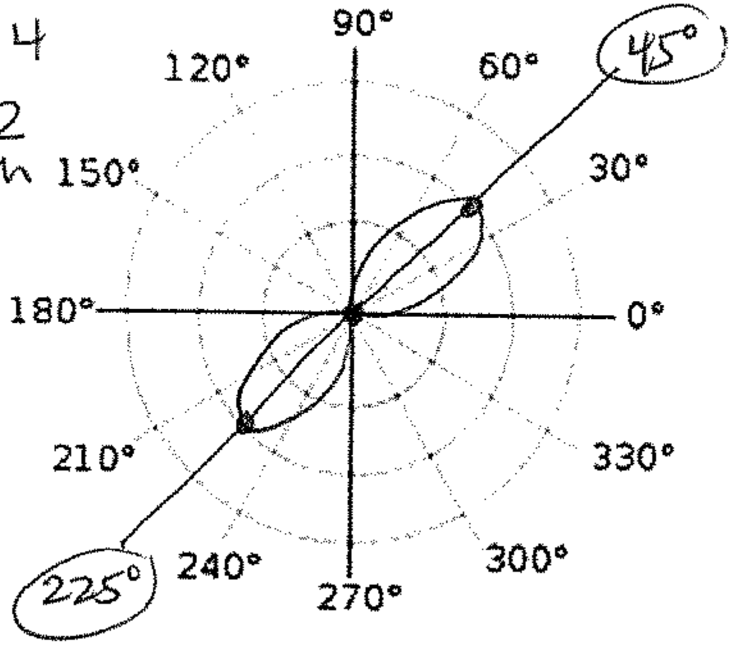
Ex. $r^2 = 4 \sin 2\theta$

positive I/III

$a^2 = 4$

$a = 2$

length



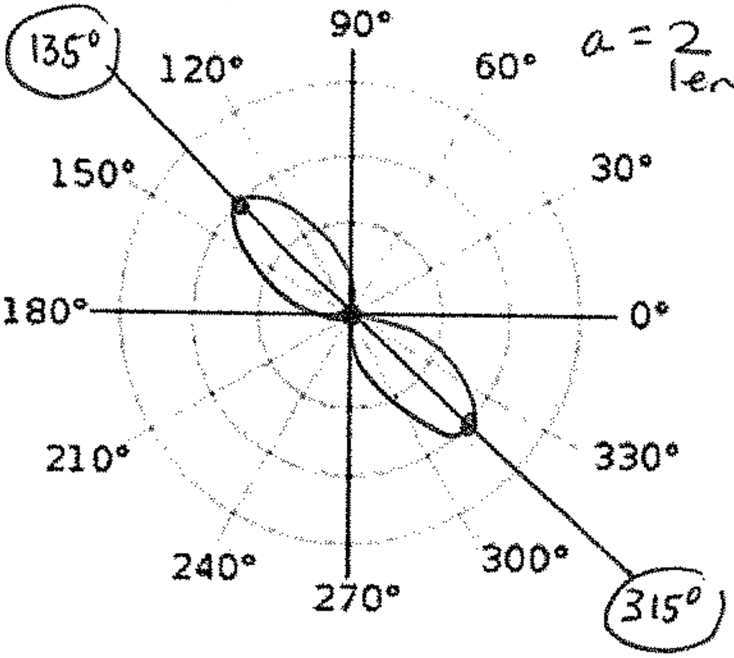
Ex. $r^2 = -4 \sin 2\theta$

negative II/IV

$a^2 = 4$

$a = 2$

length



Lemniscates:

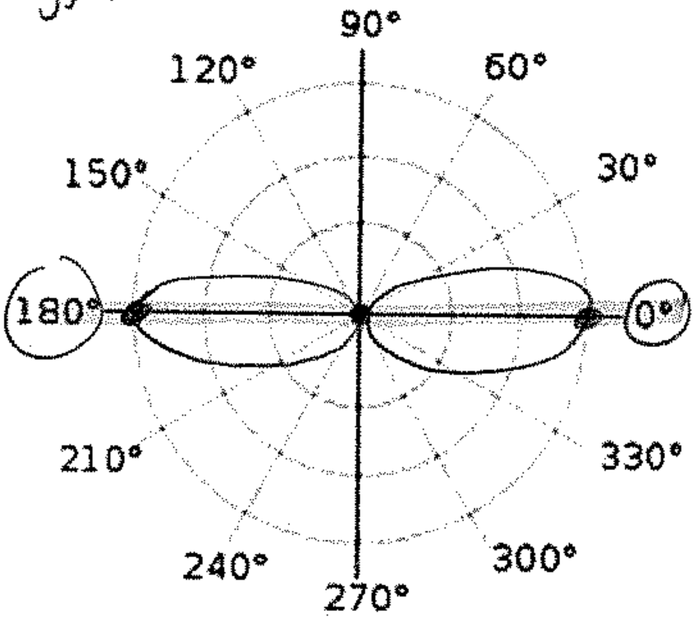
Ex. $r^2 = 9\cos 2\theta$

↑ positive cosine → x-axis on polar axis

$a^2 = 9$

$a = 3$

length



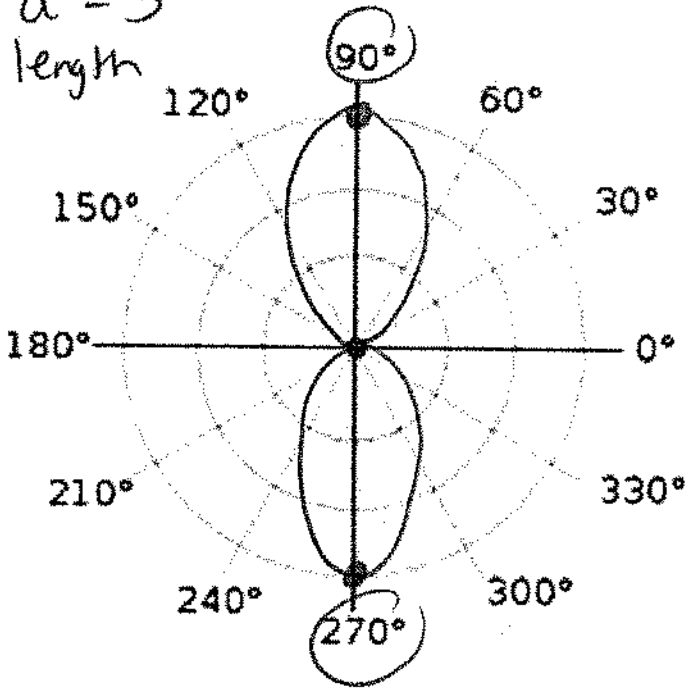
Ex. $r^2 = -9\cos 2\theta$

↑ negative cosine → y-axis on line $\theta = \frac{\pi}{2}$

$a^2 = 9$

$a = 3$

length



Review: Convert the equation to rectangular form:

$$r^2 = 10\cos 2\theta \quad \text{double angle}$$

$$r^2 = 10(\cos^2\theta - \sin^2\theta)$$

$$r^2 = 10\left[(\cos\theta)^2 - (\sin\theta)^2\right]$$

$$r^2 = 10\left[\left(\frac{x}{r}\right)^2 - \left(\frac{y}{r}\right)^2\right]$$

$$r^2 \cdot \frac{r^2}{1} = \frac{10x^2 - 10y^2}{\cancel{r^2}} \cdot \cancel{r^2}$$

$$\enclose{circle}{r}^4 = 10x^2 - 10y^2$$

$$\left(\sqrt{x^2+y^2}\right)^4 = 10x^2 - 10y^2$$

$$(x^2+y^2)^{\enclose{circle}{4/2}} = 10x^2 - 10y^2$$

$$\boxed{(x^2+y^2)^2 = 10x^2 - 10y^2}$$

Do not square because power is evenly divisible