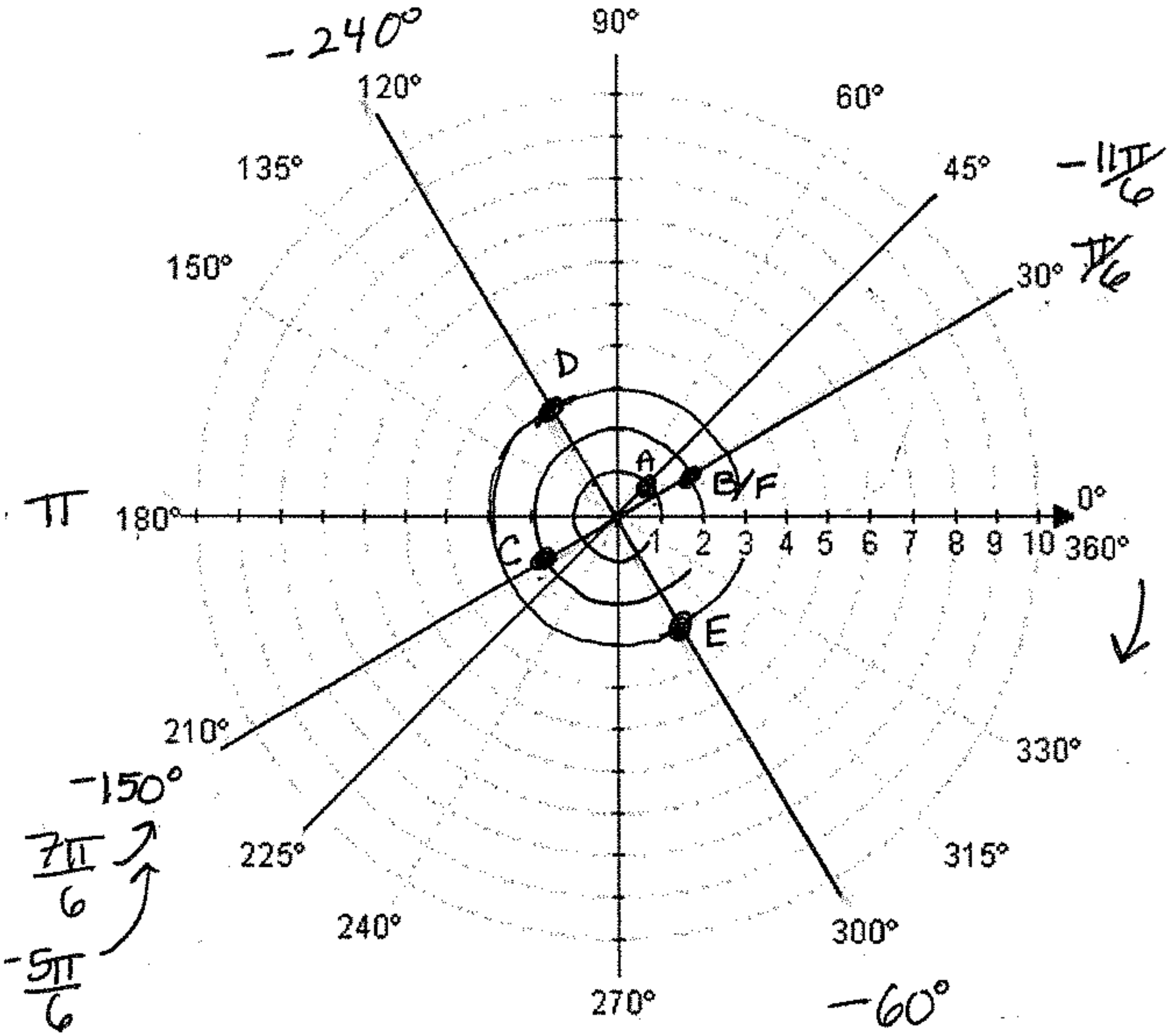
Sec. 6.3 Polar Coordinates

Components of Polar Coordinates: (r, θ) radius angle

$\boxed{\text{Center = pole}}$



Ex 1. Plot each point.

A = (1, 45°)

B = (2, π/6)

opposite way across the pole

C = (−2, π/6)

D = (3, 120°)

E = (−3, 120°)

F = (−2, −150°)

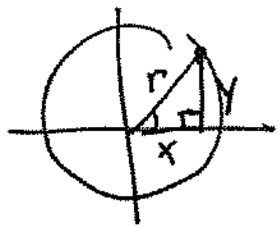
Ex. 2: Find three other coordinates (to have two positive and two negative) for point C and E from example 1.

* BTWN: $-360° < \Theta < 360°$ or $-2\pi < \Theta < 2\pi$

Keep degrees in degrees and radians in radians (as given).

$C = (-2, \pi/6)$	$E = (-3, 120°)$
* $\left(2, \frac{7\pi}{6}\right)$	* $(3, 300°)$
$\left(2, -\frac{5\pi}{6}\right)$	$(3, -60°)$
$\left(-2, -\frac{11\pi}{6}\right)$	$(-3, -240°)$

Converting Polar to Rectangular



$$(r, \theta) \longrightarrow (x, y)$$

polar — rectangular

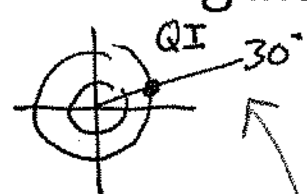
$$\frac{\cos\theta}{1} \neq \frac{x}{r} \quad \text{and} \quad \frac{\sin\theta}{1} \neq \frac{y}{r}$$

$$\boxed{x = r\cos\theta} \qquad \boxed{y = r\sin\theta}$$

Plot it!

Ex.3: Convert to rectangular form:

a) $(2, 30^\circ)$ — r, θ



$X = r\cos\theta$

$X = 2\cos 30^\circ$

$X = \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)$

$X = \sqrt{3}$

$Y = r\sin\theta$

$Y = 2\sin 30^\circ$

$Y = \cancel{2}\left(\frac{1}{\cancel{2}}\right)$

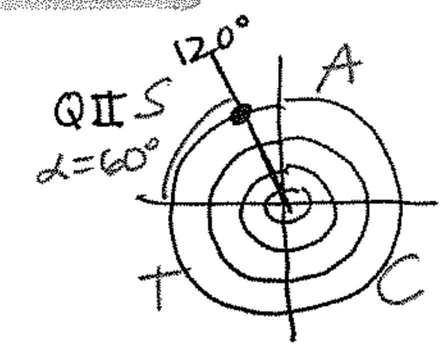
$Y = 1$

matches

rectangular form: $\boxed{(\sqrt{3}, 1)}$ ← QI

(X, Y)

b) $(4, 120^\circ)$



$X = r\cos\theta$

$X = 4\cos 120^\circ$

$X = 4\left(-\frac{1}{2}\right)$

$X = -2$

$Y = r\sin\theta$

$Y = 4\sin 120^\circ$

$Y = 4\left(+\frac{\sqrt{3}}{2}\right)$

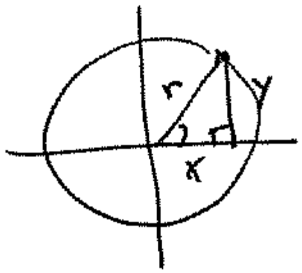
$Y = 2\sqrt{3}$

rectangular form: $\boxed{(-2, 2\sqrt{3})}$

QII

Converting Rectangular to Polar

$(x, y) \longrightarrow (r, \theta)$

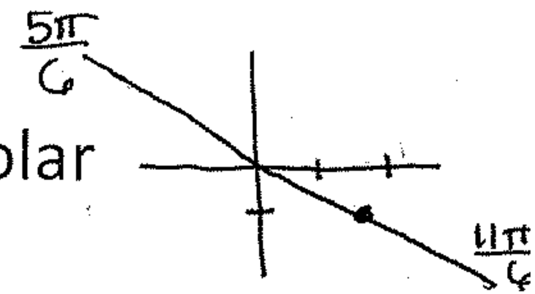


$$r^2 = x^2 + y^2$$

$$\boxed{\tan\theta = \frac{y}{x}}$$

prefer to choose $+$

$$\boxed{r = \pm\sqrt{x^2 + y^2}}$$

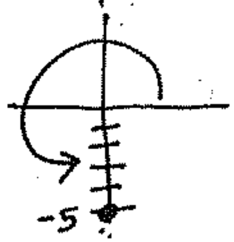


Ex.4: Convert to polar form. Find two sets of polar coordinates for the point for $0 \le \theta < 2\pi$.

a) (0, - 5)

x y

QA



$\circledast$ $\boxed{\begin{array}{l} \left(5, \frac{3\pi}{2}\right) \\ \left(-5, \frac{\pi}{2}\right) \end{array}}$

$\tan\theta = \frac{y}{x}$

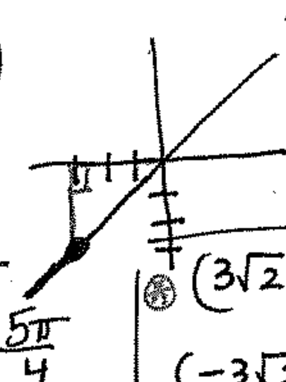
$= \frac{-5}{0}$

$\tan\theta = \text{Und}$

b) (- 3, - 3)

x y

QIII



$\circledast$ $\left(3\sqrt{2}, \frac{5\pi}{4}\right)$

$\left(-3\sqrt{2}, \frac{\pi}{4}\right)$

$r = \pm\sqrt{(-3)^2 + (-3)^2}$

$= \pm\sqrt{18} \lt_2^9$

$r = \pm 3\sqrt{2}$

c) $(\sqrt{3}, -1)$

QIV

$r = \pm\sqrt{(\sqrt{3})^2 + (-1)^2}$

$r = \pm\sqrt{4}$

$r = \pm 2$

$\tan\theta = \frac{y}{x} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\alpha = \frac{\pi}{6}$

$\left(2, \frac{11\pi}{6}\right)$

$\left(-2, \frac{5\pi}{6}\right)$

When converting an equation from polar to rectangular:

1) Leave no <u>radicals</u> with <u>variables</u> under them

ex: $\sqrt{x}$ or $x^{1/2}$

2) Leave no rational exponents (fractions as exponents)

ex: $x^{3/2}$

get rid of the denominator (hidden radical) by squaring both sides.

3) If there are ( ) with a <u>product</u> inside, distribute the exponent

ex: $(2 \cdot x^3 \cdot y)^2 = \boxed{4x^6y^2}$

You can <u>not</u> do this for a sum or difference.
No need to expand; stop there.

ex: $\boxed{\underbrace{(x^2+2)}_{\text{sum}}{}^2} \neq x^4+4$

$= (x^2+2)(x^2+2)$

no need to expand

4) Rules used for converting :

$$\cos\theta = \frac{x}{r} \longrightarrow x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \longrightarrow y = r\sin\theta$$

$$\tan\theta = \frac{y}{x}$$

$$r^2 = x^2 + y^2$$

$$r = \pm\sqrt{x^2+y^2}$$

only positive in the rectangular form

An equation in terms of r and θ is called a polar equation. (r, θ)

Ex.5: Change the polar equation to a rectangular equation. (x, y)

(only in terms of x and y, use identities where necessary)

a) $r = 2\cos\theta$

$r = 2\left(\frac{x}{r}\right)$

$r \cdot r = \frac{2x}{\cancel{r}} \cdot \cancel{r}$

$r^2 = 2x$

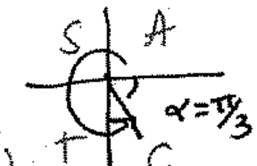
$\boxed{x^2 + y^2 = 2x}$

$\boxed{x^2 + y^2 - 2x = 0}$

Circle

b) $\theta = 5\pi/3$

$\tan\theta = \tan\frac{5\pi}{3}$



$\frac{y}{x} = \tan\frac{5\pi}{3}$ replace w/value

$\cancel{x} \cdot \frac{y}{\cancel{x}} = -\sqrt{3} \cdot x$

$\boxed{y = -\sqrt{3}\,x}$

$y = mx + b$

Linear

c) $r = \sin 2\theta$ double angle

$r = 2\sin\theta\cos\theta$

$r = 2\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$

$r^2 \cdot r = \frac{2xy}{\cancel{r^2}} \cdot \cancel{r^2}$

$r^3 = 2xy$

$\left[\left(\sqrt{x^2+y^2}\right)^3\right]^2 = \underbrace{(2 \cdot x \cdot y)^2}_{\text{product}}$

$\boxed{\underbrace{(x^2+y^2)^3}_{\text{Sum}} = 4x^2y^2}$

d) $\frac{r}{1} = \frac{5}{3\cos\theta - 2\sin\theta}$

$r(3\cos\theta - 2\sin\theta) = 5$

$3r\cos\theta - 2r\sin\theta = 5$

$\boxed{3x - 2y = 5}$

Linear

$3r\cos\theta$

$3\cancel{r}\left(\frac{x}{\cancel{r}}\right)$

Ex.6: Change the rectangular equation to a polar equation (in terms of r and θ). ⊛ usually you will solve for "r"

a) $x^2 + y^2 = 16$

circle
c: (0,0)
r=4

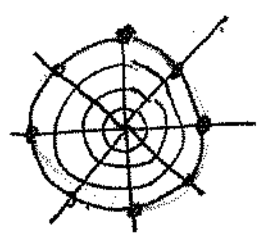
$r^2 = 16$

$\sqrt{r^2} = \sqrt{16}$

$|r| = 4$

$r = \pm 4$

$\boxed{r = -4}$ or $\boxed{r = 4}$



b) y = 4

Horizontal line

$r\sin\theta = 4$

⊛ $\boxed{r = \dfrac{4}{\sin\theta}}$

$r = 4 \cdot \dfrac{1}{\sin\theta}$

⊛ $\boxed{r = 4\csc\theta}$

c) 4x + 7y – 2 = 0

Linear

$4r\cos\theta + 7r\sin\theta = 2$

factor

$r(4\cos\theta + 7\sin\theta) = 2$

$\boxed{r = \dfrac{2}{4\cos\theta + 7\sin\theta}}$