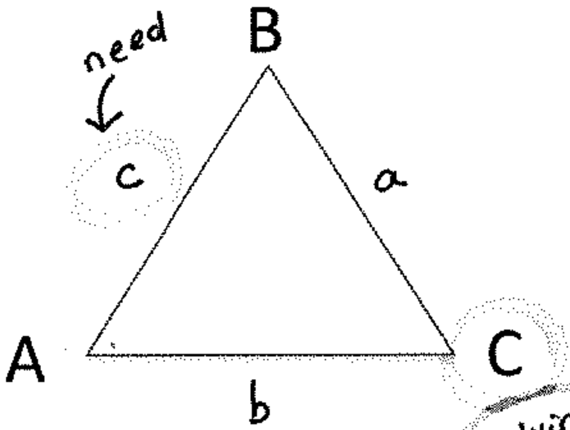
Sec. 6.2 Law of Cosines

$\boxed{\text{NOT SSA}}$
* Law of Sines

no rule for AAA

When the given is SAS or SSS, we use the **Law of Cosines.**



In Δ ABC,

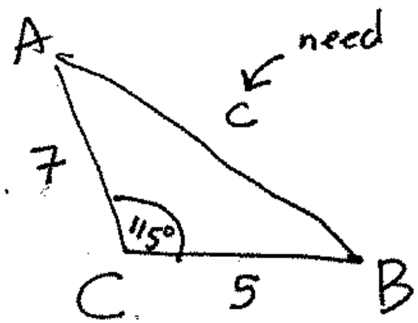
Memorize

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

side, Side, Side, Sides, Angle

solving for a side → $c = \sqrt{\text{all of the above}}$

Ex.1) A Δ has sides 5 cm and 7 cm. The <u>included</u> angle is 115° Find the length of the 3rd side. (round to the nearest 10th)



$\boxed{\text{SAS}} \rightarrow$ L. of Cosines

$c^2 = 5^2 + 7^2 - 2(5)(7)\cos 115°$ ← degree mode

$\sqrt{c^2} = \sqrt{103.583\ldots}$

$$\boxed{c \approx 10.2 \text{ cm}}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\underbrace{-(c^2 - a^2 - b^2)}_{+2ab} = \frac{-2\cancel{ab}\cos C}{\cancel{-2ab}}$$

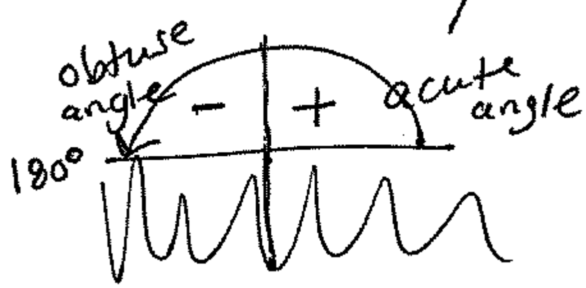
$$\frac{-c^2 + a^2 + b^2}{2ab} = \cos C$$

memorize

$$\boxed{\cos C = \frac{a^2 + b^2 - c^2}{2ab}}$$

Solving for an angle

$$\measuredangle C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

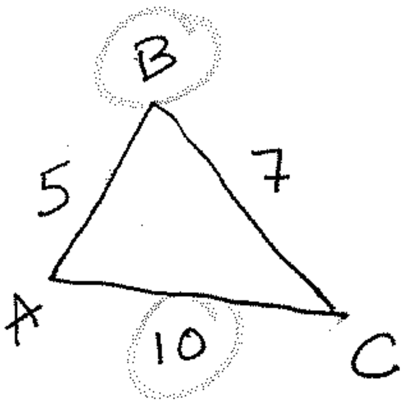


$\boxed{SSS} \to$ L. Cosines

Ex.2) A Δ has sides measuring 5, 7, and 10 cm. Find the measure of the 3 angles. (round to the nearest 10^{th})

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



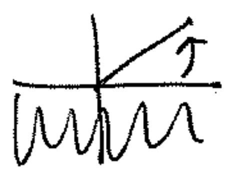
② $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

① $\cos C = \dfrac{7^2 + 10^2 - 5^2}{2(7)(10)}$

$\cos B = \dfrac{7^2 + 5^2 - 10^2}{2(7)(5)}$

$\measuredangle C = \cos^{-1}\left(\dfrac{124}{140}\right)$

⊕ acute

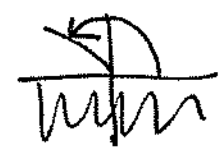


$\measuredangle B = \cos^{-1}\left(\dfrac{-26}{70}\right)$

neg. Obtuse

$\boxed{\measuredangle C \approx 27.7^\circ}$

$\boxed{\measuredangle B \approx 111.8^\circ}$



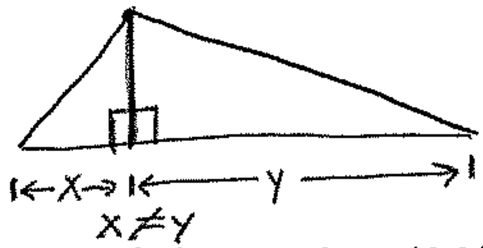
$\measuredangle A = 180^\circ - (\measuredangle B + \measuredangle C)$

$\boxed{\measuredangle A = 40.5^\circ}$

Altitude vs Median

Angle 90°

Middle



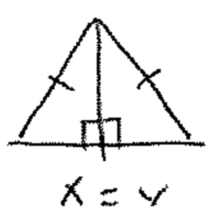
Creates right angles (90°).

Solve with SOH CAH TOA and/or Pythagorean Theorem: $a^2 + b^2 = c^2$

Usually x ≠ y….but sometimes can, for example in an isosceles or equilateral triangle.

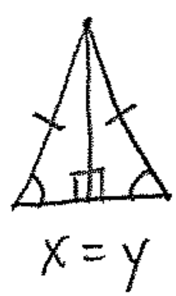
2 sides + 2 angle are equal.

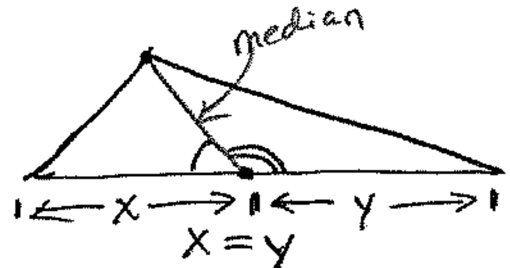
3 sides + 3 angles are equal



Special cases

Altitude = median





Splits the side its drawn to into 2 equal pieces.

x = y always.

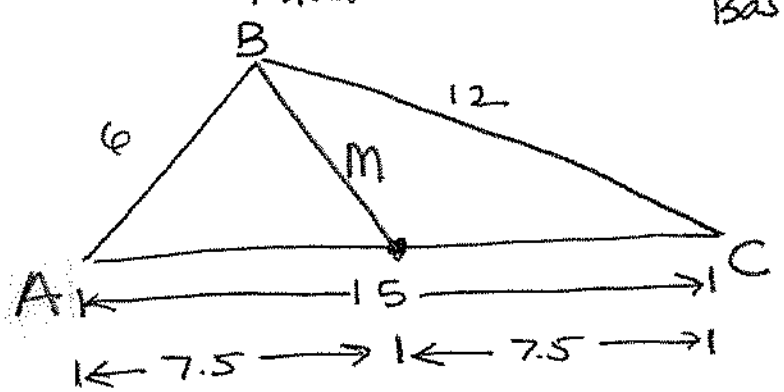
The angles are usually not 90° ….
but sometimes can be, for example in an isosceles or equilateral triangle.

$\boxed{SSS} \rightarrow$ L. Cosines

Ex.3) A Δ has sides 6, 12, and 15 meters. Find the length of the median to the longest side. (round to the nearest 100th)

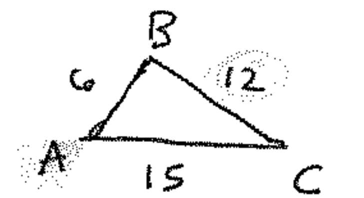
Middle

Base



m = median

① The large Δ



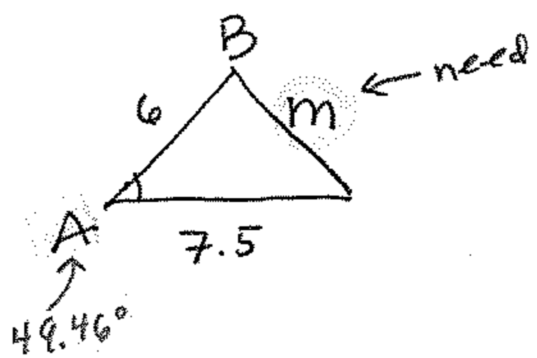
$\boxed{SSS} \rightarrow$ Cosines

$$\cos A = \frac{6^2 + 15^2 - 12^2}{2(6)(15)}$$

$$\measuredangle A = \cos^{-1}\left(\frac{117}{180}\right)$$

$$\measuredangle A \approx 49.46^\circ$$

② Use Small Δ



$\boxed{SAS} \rightarrow$ L. Cosines

$$m^2 = 6^2 + 7.5^2 - 2(6)(7.5)\cos \overset{A}{49.46^\circ}$$

$$\sqrt{m^2} \approx \sqrt{33.7519\ldots}$$

$$\boxed{m \approx 5.81 \text{ meters}}$$

Heron's Area Formula for Area of a Triangle

*Use when you have SSS (NO angles)

Already know:

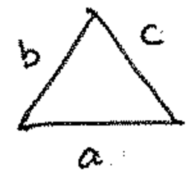
$K_\Delta = \frac{1}{2}bh$

$K_\Delta = \frac{1}{2}ab\sin C$

memorize

$$Area_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

Where $s = \frac{a+b+c}{2}$ (the semi-perimeter)

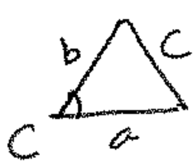


Ex4) Find the area of the triangle with sides 5 cm, 8 cm, and 10 cm. Round to the nearest 100th (2 decimal places).

$\boxed{SSS} \rightarrow$ Heron's for Area

$s = \frac{5+8+10}{2}$ $\quad s = 11.5$

$K_\Delta = \frac{1}{2}ab\sin C$



* Not as quick (or accurate) as Heron's for SSS.

$Area_\Delta = \sqrt{11.5(11.5-5)(11.5-8)(11.5-10)}$

$Area_\Delta = \sqrt{11.5(6.5)(3.5)(1.5)}$

$\boxed{Area_\Delta \approx 19.81\,cm^2}$

To decide which rules to use…

Law of Sines: AAS

ASA

SSA **(Ambiguous case: 0, 1, or 2 Δ's)**

Law of Cosines: SSS

SAS

no AAA

Right Triangles: Use "SOH-CAH-TOA" and $a^2 + b^2 = c^2$