Sec. 6.1: Law of Sines

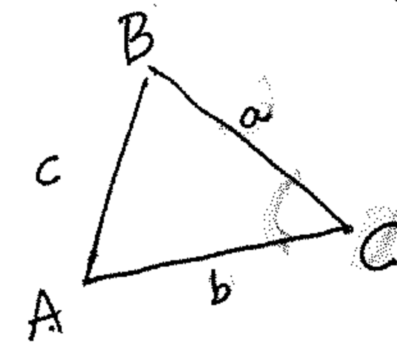
$K_\Delta = \frac{1}{2} b h$

Area of any Δ: Need 2 sides and their included angle.

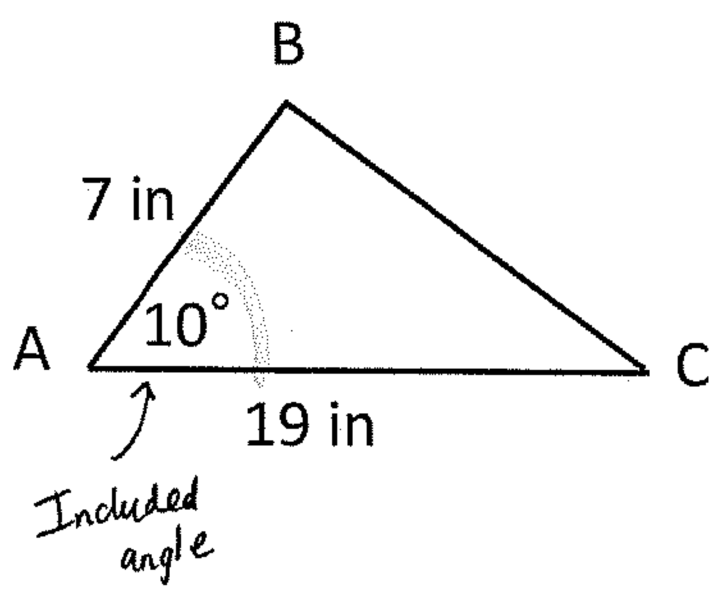
Btwn the 2 sides

Memorize

$$\boxed{K_\Delta = \frac{1}{2} \underbrace{ab}_{\text{sides}} \sin \underbrace{C}_{\text{Angle}}}$$



Ex. 1) Find the area of Δ ABC. Round to the tenths.



Mode → Degrees

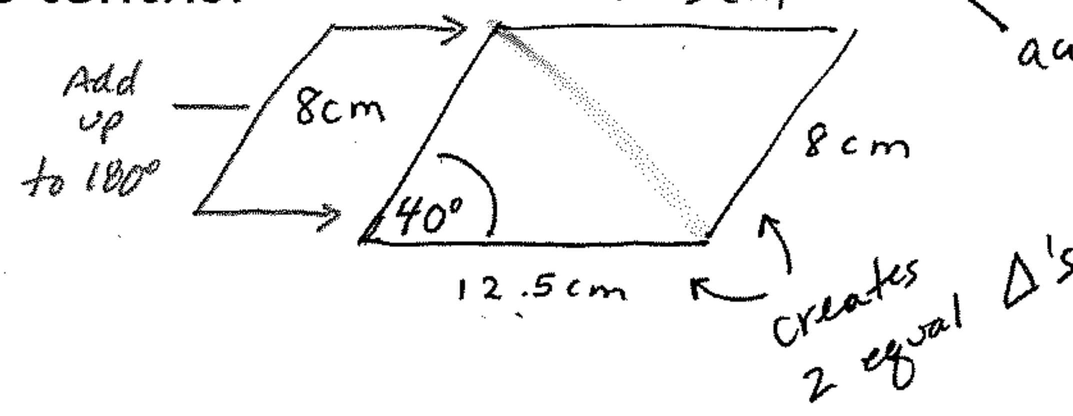
$K_\Delta = \frac{1}{2}(7)(19)\sin 10°$

$$\boxed{K_\Delta \approx 11.5 \text{ in}^2}$$

add units

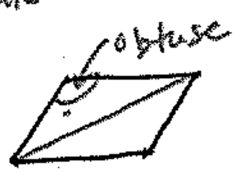
Ex.2) Adjacent sides of a parallelogram have lengths 12.5 cm and 8 cm. Their included angle is 40 degrees. Find its area. Round to the tenths.

acute ∡



Side note:

a) 

b) 

$$\text{Area}_{\square} = 2 \cdot K_{\Delta}$$

$$= \cancel{2} \cdot \tfrac{1}{\cancel{2}} (8)(12.5) \sin 40^\circ$$

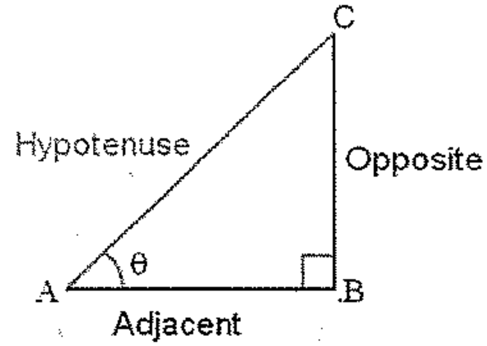
$$\boxed{\text{Area}_{\square} \approx 64.3 \text{ cm}^2}$$

which diagonal is longer? and why?

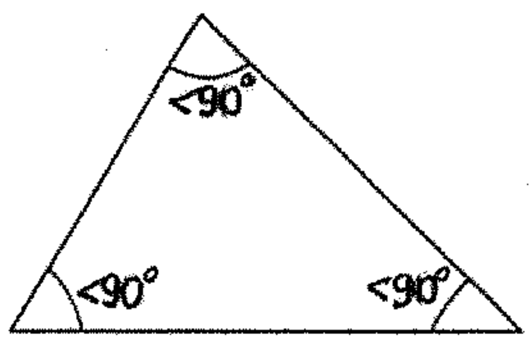
$\boxed{b}$ b/c it's opposite the obtuse (bigger) angle.

- Longest side is opposite biggest ∡
- Shortest side is opposite smallest ∡

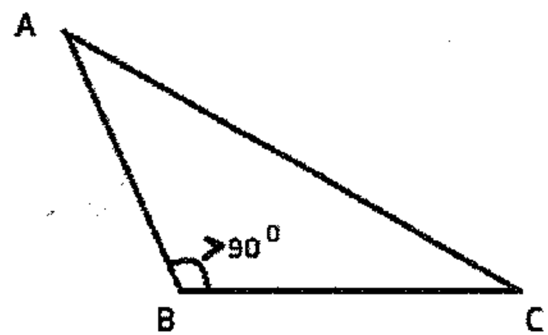
When solving triangles we already know we use SOH CAH TOA and the Pythagorean Theorem to solve right triangles.



An ***oblique triangle*** is a triangle that does not contain a right angle. An oblique triangle either has 3 acute angles or 2 acute angles and 1 obtuse.



Acute Angle Triangle



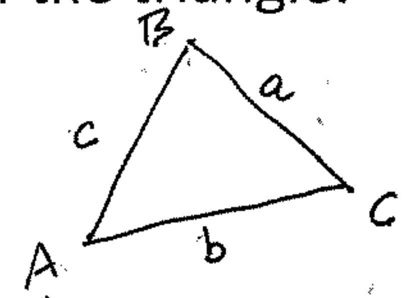
Obtuse angle triangle

We need other methods to solve oblique triangles....

The Law of Sines:

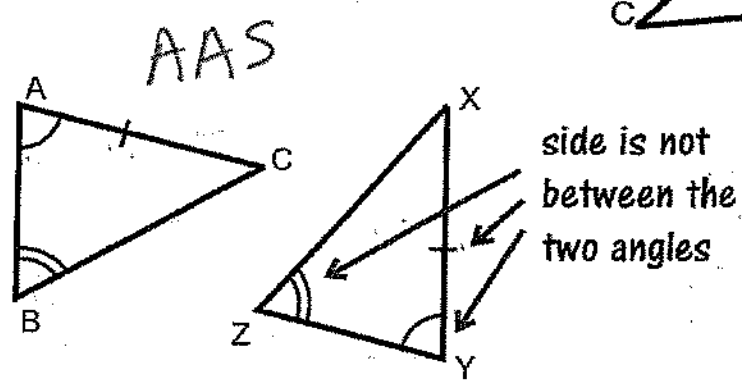
The ratio of the length of the side of any triangle to the sine of the angle opposite that side is the same for all three sides of the triangle.

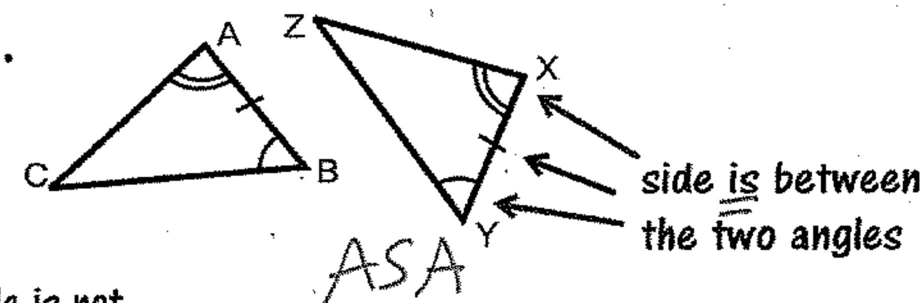
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

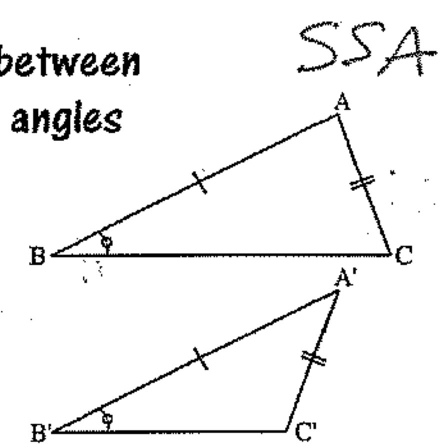


*This law can also be written in its reciprocal form.

This law is used when you have 2 angles and any side (AAS, ASA), or when you have 2 sides and an angle opposite one of them (SSA).





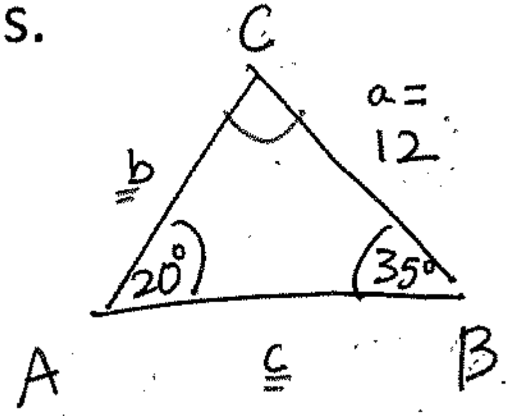


Ex.3) Solve Δ ABC, if <A = 20°, <B = 35° and $\overline{BC}$ = 12.

Round to the tenths.

$a = 12$

Find <u>all</u> the unknown sides and angles



$\boxed{AAS}$

→ use Law of sines

① $\measuredangle C = 180° - (20° + 35°)$

$$\boxed{\measuredangle C = 125°}$$

② $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$$\frac{\sin 20°}{12} = \frac{\sin 35°}{b}$$

$$b \sin 20° = 12 \sin 35°$$

$$\boxed{b = \frac{12 \sin 35°}{\sin 20°}} \quad \boxed{b \approx 20.1 \text{ units}}$$

③ $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$$\frac{\sin 20°}{12} = \frac{\sin 125°}{c}$$

$$c \sin 20° = 12 \sin 125°$$

$$\boxed{c = \frac{12 \sin 125°}{\sin 20°}}$$

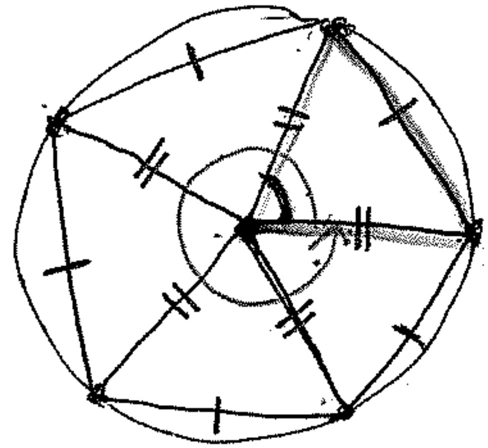
$$\boxed{c \approx 28.7 \text{ units}}$$

Ex.4) Find the area of a *regular* (all sides and angles are equal) pentagon inscribed in a circle of radius 9 inches. Round to the hundredths.

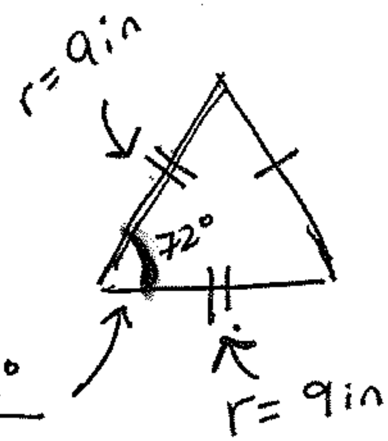
5 sides
+ 5 angles

$r = 9$ in.

*Draw the diagram today to understand; then you should not need to draw it in the future.



• created 5Δ's equal



$$5\overline{)360^\circ}\ \ 72^\circ$$

number of sides (Δ's)

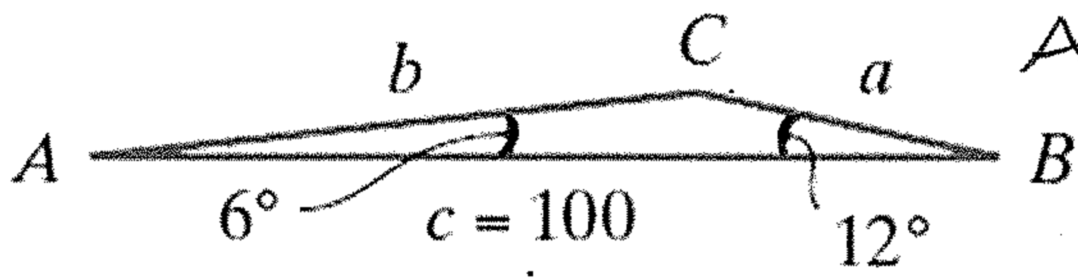
can do this for any regular polygon.
Hexagon
octagon
decagon
dodecagon
etc...

Area of pentagon $= 5 K_{\Delta}$

$= 5 \cdot \frac{1}{2}(9)(9)\sin 72^\circ$

$$\boxed{\text{Area of pentagon} \approx 192.59 \text{ in}^2}$$

Ex.5) Solve Δ ABC. Round to the tenths.



ASA → Use the Law of Sines.

① $\measuredangle C = 180° - (6° + 12°)$

$\boxed{\measuredangle C = 162°}$

② $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\dfrac{\sin 162°}{100} = \dfrac{\sin 6°}{a}$

$a \sin 162° = 100 \sin 6°$

$\boxed{a = \dfrac{100 \sin 6°}{\sin 162°}}$

$\boxed{a \approx 33.8 \text{ units}}$

③ $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

$\dfrac{\sin 162°}{100} = \dfrac{\sin 12°}{b}$

$b \sin 162° = 100 \sin 12°$

$\boxed{b = \dfrac{100 \sin 12°}{\sin 162°}}$

$\boxed{b \approx 67.3 \text{ units}}$