Notes

Pre-Calculus:
Sec. 5.5 (Day 3)
Solving Trigonometric Equations

1) Solve: a) For principal solutions on the interval $[0, 2\pi)$.
b) For general solutions.

use double angle

$$\sin 2x + \cos x = 0$$

factor

$$2\sin x\cos x + \cos x = 0$$

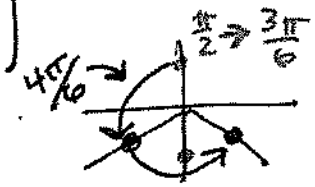
$$\cos x(2\sin x + 1) = 0$$

$\cos x = 0$ | $2\sin x + 1 = 0$

QA check

$\pi/2$, $\frac{3\pi}{2}$

$\sin x = -\frac{1}{2}$



$\alpha = \pi/6$

$\pi + \alpha$: $\frac{7\pi}{6}$ $\quad$ $2\pi - \alpha$: $\frac{11\pi}{6}$

Combine if possible.

$\frac{4\pi}{6} \to \frac{2\pi}{3}$

a) For $[0, 2\pi)$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

"Tricky combo"

b) General:

$$x = \frac{\pi}{2} + \frac{2\pi}{3}n, \; n \in \mathbb{Z}$$

$$x = \frac{3\pi}{2} + 2\pi n, \; n \in \mathbb{Z}$$

Combines to 2 math sentences.

2) Solve: a) For principal solutions on the interval $[0, 2\pi)$.
b) For general solutions.

$$\cos 2x = -\cos x$$

double angle

$\cos 2x + \cos x = 0$

$2\cos^2 x - 1 + \cos x = 0$

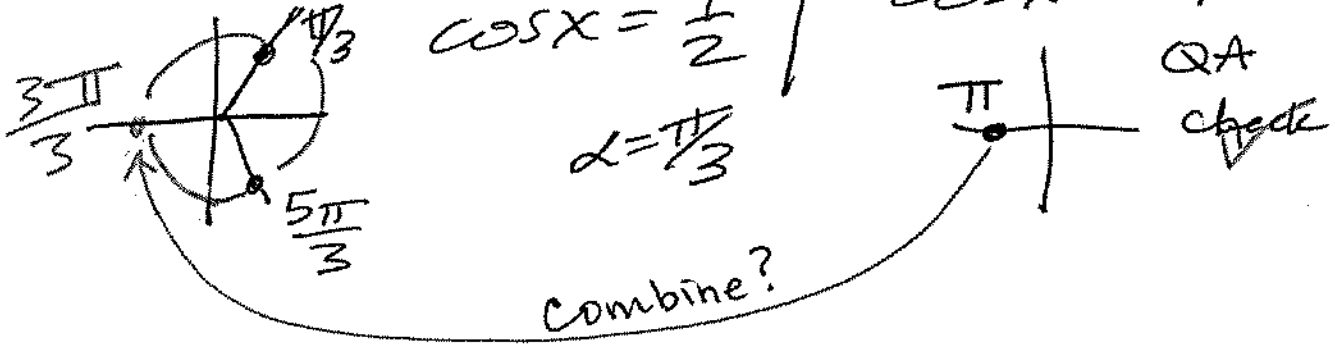
$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ | $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ | $\cos x = -1$

$\alpha = \frac{\pi}{3}$ | π QA check



a) For $[0, 2\pi)$:

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi}$

b) General:

$\boxed{x = \frac{\pi}{3} + \frac{2\pi}{3}n, n \in \mathbb{Z}}$

3) Solve: a) For principal solutions on the interval $[0, 2\pi)$.
b) For general solutions.

$$\sin 4x = \sin 2x$$

$\sin 4x - \sin 2x = 0$

double angle $\sin 2(2x) - \sin 2x = 0$

$2\sin 2x\cos 2x - \sin 2x = 0$

factor $\sin 2x(2\cos 2x - 1) = 0$

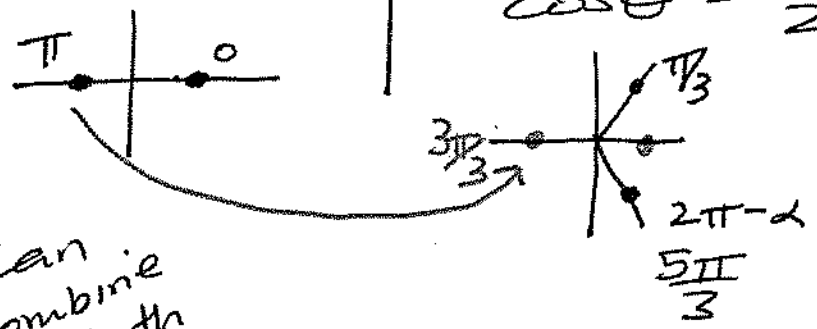
Let $\theta = 2x$

$\sin 2x = 0$ | $2\cos 2x - 1 = 0$

$\sin\theta = 0$ | $2\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$

$\alpha = \pi/3$



Can combine b/c both θ's $= 2x$

$\theta = \frac{\pi}{3} + \frac{2\pi}{3}n$ $\theta = \cancel{0} + 2\pi n$

$\frac{1}{2}\cdot 2x = \frac{1}{2}\cdot\frac{\pi}{3} + \frac{1}{2}\cdot\frac{2\pi}{3}n$ $\frac{1}{2}\cdot 2x = \frac{1}{2}\cdot 2\pi n$

b) General:

$x = \frac{\pi}{6} + \frac{\pi}{3}n, n\in\mathbb{Z}$ ← $\frac{2\pi}{6}$

$x = \pi n, n\in\mathbb{Z}$

Written as 2 math sentences (not 3)

a) For $[0, 2\pi)$ ↑ $\frac{12\pi}{6}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}, 0, \pi$

Principal

4) Solve: a) For principal solutions on the interval $[0, 2\pi)$.
b) For general solutions.

$$\tan 2x = 3\tan x$$

use double angle: $\tan 2x - 3\tan x = 0$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$, $3\tan x = 3\tan x\,\dfrac{(1-\tan^2 x)}{(1-\tan^2 x)}$, $0 = 0\,\dfrac{(1-\tan^2 x)}{(1-\tan^2 x)}$

$$\frac{2\tan x}{1-\tan^2 x} - 3\tan x = 0$$

$$\frac{2\tan x - 3\tan x(1-\tan^2 x)}{1-\tan^2 x} = 0$$

combine like terms: $2\tan x - 3\tan x + 3\tan^3 x = 0$

$$3\tan^3 x - \tan x = 0$$

factor: $\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0$ | $3\tan^2 x - 1 = 0$

QA check

$\pi \mid 0$

$x = \emptyset + \pi n$

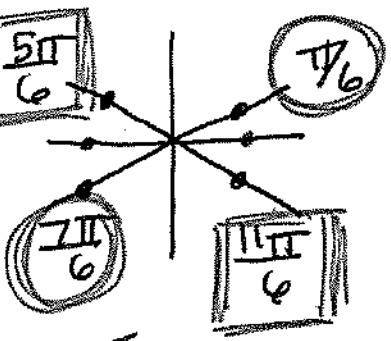
$3\tan^2 x = 1$

$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$

$|\tan x| = \frac{\sqrt{3}}{3}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$\alpha = \pi/6$



a) For $[0, 2\pi)$:

$x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

b) General:

$x = \pi n,\ n \in \mathbb{Z}$

$x = \frac{\pi}{6} + \pi n,\ n \in \mathbb{Z}$

$x = \frac{5\pi}{6} + \pi n,\ n \in \mathbb{Z}$

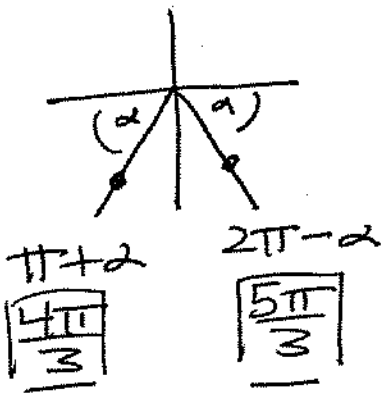
5) Solve: a) For principal solutions on the interval $[0, 2\pi)$.
b) For general solutions.

$$2\sin 3x + \sqrt{3} = 0$$

$2\sin\theta + \sqrt{3} = 0$

Let $\theta = 3x$

$\sin\theta = -\frac{\sqrt{3}}{2}$



$\alpha = \frac{\pi}{3}$

$\theta = \frac{4\pi}{3} + 2\pi n$; $\theta = \frac{5\pi}{3} + 2\pi n$

$\frac{1}{3}\cdot 3x = \frac{1}{3}\cdot\frac{4\pi}{3} + \frac{1}{3}\cdot 2\pi n$; $\frac{1}{3}\cdot 3x = \frac{1}{3}\cdot\frac{5\pi}{3} + \frac{1}{3}\cdot 2\pi n$

b) General:

$$x = \frac{4\pi}{9} + \frac{2\pi}{3}n, n\in\mathbb{Z}$$

$$x = \frac{5\pi}{9} + \frac{2\pi}{3}n, n\in\mathbb{Z}$$

$\frac{6\pi}{9}$

a) For $[0, 2\pi)$:

$\frac{18\pi}{9}$

Principal:

$$x = \frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9}, \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}$$

6) Solve for principal solutions on the interval $[0, 2\pi)$.

$$3\cos\frac{x}{2} = -1$$

Let $\theta = \frac{x}{2}$

$3\cos\theta = -1$

$\cos\theta = -\frac{1}{3}$ Btwn $[-1, 1]$

Solve w/ graphing utility

$3\cos\frac{x}{2} + 1 = 0$

Amp: 3 — up: 1

Y_1 Y_2

$X \approx 3.821$

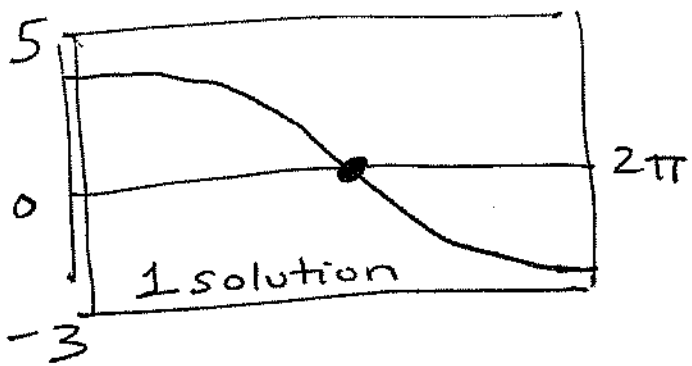
Radians (mode)

$X_{min} = -0.1$

$X_{max} = 2\pi$

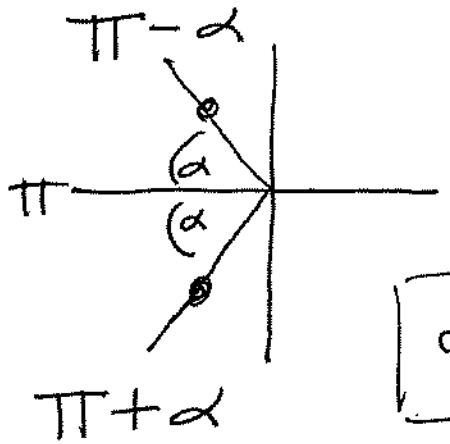
$Y_{min} = -3$

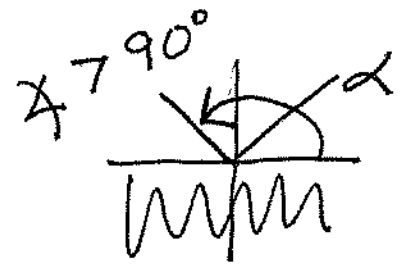
$Y_{max} = 5$



Let $\theta = \frac{x}{2}$

$\cos\theta = -\frac{1}{3}$ ← neg





$\alpha = \cos^{-1}\left(+\frac{1}{3}\right)$ ← make positive

$\alpha \approx 1.2396$

Solve w/out the graphing utility

$\theta = \pi - \alpha + 2\pi n$; $\theta = \pi + \alpha + 2\pi n$

$2\cdot\frac{x}{2} = 2\cdot\pi - 2\cdot\alpha + 2\cdot 2\pi n$; $2\cdot\frac{x}{2} = 2\cdot\pi + 2\cdot\alpha + 2\cdot 2\pi n$

$x = 2\pi - 2\alpha + 4\pi n$; $x = 2\pi + 2\alpha + 4\pi n$, $n \in \mathbb{Z}$

$n \in \mathbb{Z}$

For $[0, 2\pi)$ ← ≈ 6.28 $x \approx 3.821$, ~~8.745~~ too big