Notes

Pre-Calculus:
Sec. 5.5 (Day 2)
Solving Trigonometric Equations

1) Solve: a) For principal solutions on the interval $[0, 2\pi)$.
b) For general solutions.

$$2 \sin x \cos x = \tan x$$

$$\sin 2x = \tan x$$

$$2 \sin x \cos x = \frac{\sin x}{\cos x}$$

$$\sin 2x - \tan x = 0$$

Different x's

$$\cos x \cdot 2 \sin x \cos x - \frac{\sin x}{\cancel{\cos x}} \cancel{\cos x} = 0 \cdot \cos x$$

factor

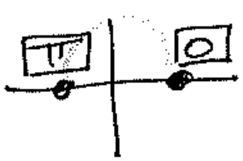
$$2 \sin x \cos^2 x - \sin x = 0$$

$$\sin x \, (2\cos^2 x - 1) = 0$$

a) For $[0, 2\pi)$:

$$\boxed{x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

$\sin x = 0$



QA Check

$\tan 0 = 0$ ✓

$\tan \pi = 0$ ✓

$$x = \cancel{0} + \pi n$$

$$2\cos^2 x - 1 = 0$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}}$$

$$|\cos x| = \frac{\sqrt{2}}{2}$$

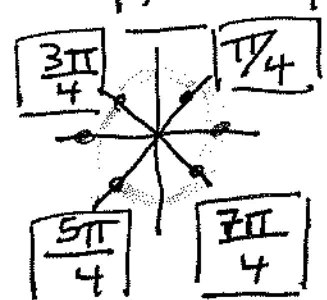
$$\cos x = \pm \frac{\sqrt{2}}{2}$$

$$\alpha = \frac{\pi}{4}$$

b) General:

$$x = \pi n, \; n \in \mathbb{Z}$$

$$x = \frac{\pi}{4} + \frac{\pi}{2} n, \; n \in \mathbb{Z}$$



2) Solve: a) For principal solutions on the interval $[0, 2\pi)$.
b) For general solutions.

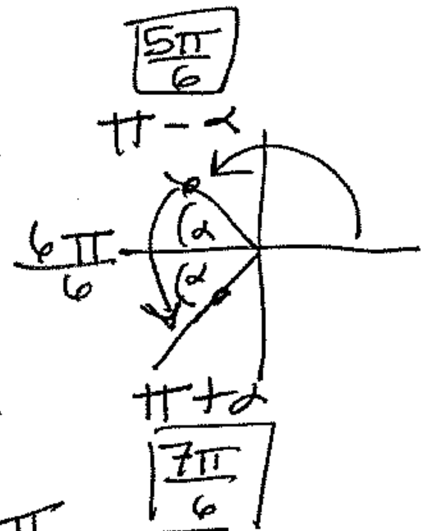
$$\sec\theta = -\frac{2\sqrt{3}}{3}$$

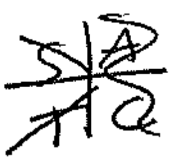
$\downarrow$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

neg

$$\alpha = \frac{\pi}{6}$$





a) For $[0, 2\pi)$: $\boxed{\theta = \frac{5\pi}{6}, \frac{7\pi}{6}}$

b) General:

$$\theta = \frac{5\pi}{6} + 2\pi n,\ n \in \mathbb{Z}$$

$$\theta = \frac{7\pi}{6} + 2\pi n,\ n \in \mathbb{Z}$$

3) Review:

Amp: $\boxed{1}$

Period: $\boxed{2\pi}$

Incre: $\boxed{\frac{\pi}{2}}$

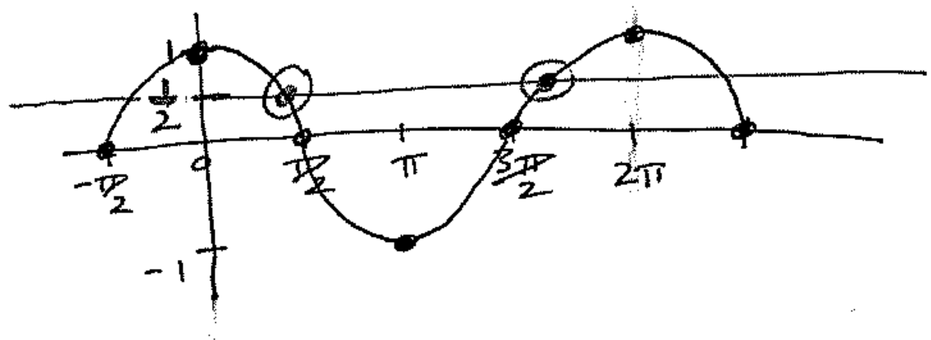
Graph LOF $y = \cos x$ and $y = \cos 2x$

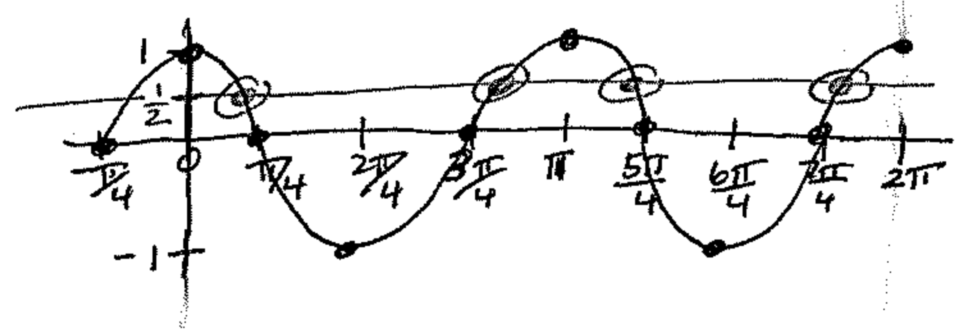
Amp: $\boxed{1}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \boxed{\pi}$

Incre: $\frac{1}{4}(\pi) = \boxed{\frac{\pi}{4}}$

PS: $\boxed{0}$





How many solutions does each of the following have on the interval $[0, 2\pi)$:

$$\cos\theta = \frac{1}{2}$$

2 solutions for $[0, 2\pi)$

$$\cos 2\theta = \frac{1}{2}$$

4 solutions for $[0, 2\pi)$

So…we can determine that when the period of the graph changes, so will the number of solutions in a given interval.

4) Solve the **multiple angle** equation on the interval $[0, 2\pi)$. Principal

Anytime you are solving a multiple angle problem, you must find the general solutions *first*.

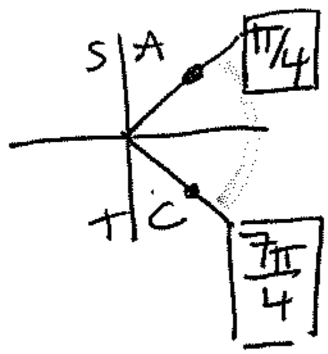
$$2\cos 2x - \sqrt{2} = 0$$

Let $\theta = 2x$

$2\cos\theta - \sqrt{2} = 0$

$2\cos\theta = \sqrt{2}$

$\cos\theta = \frac{\sqrt{2}}{2}$



$\alpha = \frac{\pi}{4}$

$\theta = \frac{\pi}{4} + 2\pi n$; $\theta = \frac{7\pi}{4} + 2\pi n$

Substitute Back

$\frac{1}{\cancel{2}} \cdot \cancel{2}x = \frac{1}{2} \cdot \frac{\pi}{4} + \frac{1}{\cancel{2}} \cancel{2}\pi n$; $\frac{1}{\cancel{2}} \cdot \cancel{2}x = \frac{1}{2} \cdot \frac{7\pi}{4} + \frac{1}{\cancel{2}} \cancel{2}\pi n$

$$\boxed{x = \frac{\pi}{8} + \pi n,\ n \in \mathbb{Z} ;\ x = \frac{7\pi}{8} + \pi n,\ n \in \mathbb{Z}}$$

$\frac{8\pi}{8}$

General Solutions

For $[0, 2\pi)$

$[0, \frac{16\pi}{8})$

$$\boxed{x = \frac{\pi}{8}, \frac{9\pi}{8}, \cancel{\frac{17\pi}{8}}, \frac{7\pi}{8}, \frac{15\pi}{8}}$$

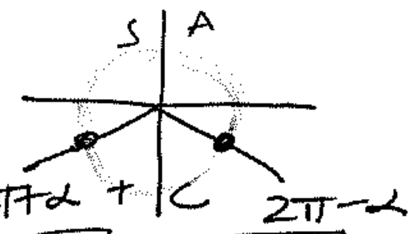
Principal solutions

5) Solve $\sin 3x = -\frac{1}{2}$ for $0 \le x < 2\pi$ Principal

Let $\theta = 3x$

$\sin\theta = -\frac{1}{2}$ neg

$\alpha = \frac{\pi}{6}$



$\boxed{\frac{7\pi}{6}}$ $\boxed{\frac{11\pi}{6}}$

$\theta = \frac{7\pi}{6} + 2\pi n \quad ; \quad \theta = \frac{11\pi}{6} + 2\pi n$

Substitute BACK

$\frac{1}{\cancel{3}} \cdot \cancel{3}x = \frac{1}{3} \cdot \frac{7\pi}{6} + \frac{1}{3} \cdot 2\pi n \quad ; \quad \frac{1}{\cancel{3}} \cdot \cancel{3}x = \frac{1}{3} \cdot \frac{11\pi}{6} + \frac{1}{3} \cdot 2\pi n$

$$\boxed{x = \frac{7\pi}{18} + \frac{2\pi}{3}n, n \in \mathbb{Z} \quad ; \quad x = \frac{11\pi}{18} + \frac{2\pi}{3}n, n \in \mathbb{Z}}$$

General Solutions $\frac{12\pi}{18}$

For $[0, 2\pi)$

$[0, \frac{36\pi}{18})$

$$\boxed{x = \frac{7\pi}{18}, \frac{19\pi}{18}, \frac{31\pi}{18}, \frac{11\pi}{18}, \frac{23\pi}{18}, \frac{35\pi}{18}}$$

principal solutions

6) Solve $\cos 2x - \sin x = 0$ for $0 \le x < 2\pi$

Different x's

change w/ a double angle formula

$1 - 2\sin^2 x - \sin x = 0$

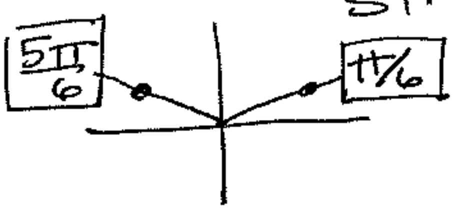
make into positive

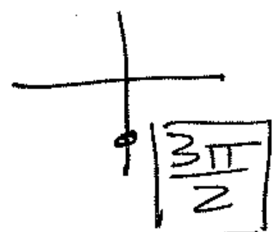
$-2\sin^2 x - \sin x + 1 = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$	$\sin x + 1 = 0$
$\sin x = \frac{1}{2}$	$\sin x = -1$
$\alpha = \frac{\pi}{6}$	





QA check ✓

For $[0, 2\pi)$:

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

Side note:

$\cos 2x(\sin x - 1) = 0$

$\cos 2x = 0$	$\sin x - 1 = 0$
use multiple angle solving	$\sin x = 1$

7) Solve $4\sin\frac{1}{2}x = 3$ *for* $0 \le x < 2\pi$

radians

Let $\theta = \frac{1}{2}x$

$4\sin\theta = 3$

$\sin\theta = \frac{3}{4}$ BTWN $[-1, 1]$

* need a calc.

$X_{min}: -0.1$

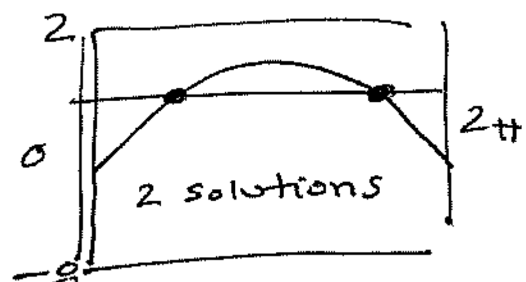
$X_{MAX}: 2\pi$

$Y_{min}: -8$

$Y_{MAX}: 2$

Solve w/ graphing utility

$\underbrace{4\sin\frac{1}{2}x - 3}_{Y_1} = \underbrace{0}_{Y_2}$



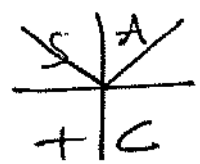
* We do not get a value we recognize here, so we must use another method. We will solve graphically with the graphing utility feature of our calculator.

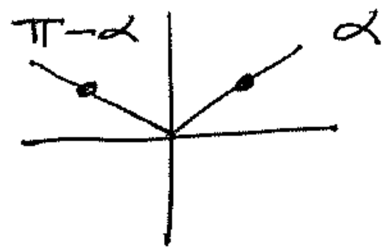
Round solutions to the nearest thousandth. $\boxed{X \approx 1.696,\ 4.587}$

$4 \sin \frac{1}{2} x = 3$

Let $\theta = \frac{1}{2} x$

$\sin \theta = \frac{3}{4}$





Solve w/out graphing utility.

$\alpha = \sin^{-1}\left(+\frac{3}{4}\right)$

$\boxed{\alpha \approx 0.84806}$ Store it.

$\theta = \alpha + 2\pi n$ $\quad\quad$ $\theta = \pi - \alpha + 2\pi n$

$\downarrow$ $\quad\quad$ $\downarrow$

$2 \cdot \frac{1}{2} x = 2 \cdot \alpha + 2 \cdot 2\pi n$ $\quad\quad$ $2 \cdot \frac{1}{2} x = 2 \cdot \pi - 2 \cdot \alpha + 2 \cdot 2\pi n$

General: $\boxed{x = 2\alpha + 4\pi n, \; n \in \mathbb{Z}}$ $\boxed{x = 2\pi - 2\alpha + 4\pi n, \; n \in \mathbb{Z}}$

for $[0, 2\pi)$ $\boxed{x \approx 1.696, \; 4.587}$

≈ 6.28

8) Solve $1.5\cos 2x = \frac{1}{2}$ *for* $0 \le x < 2\pi$

Let $\theta = 2x$

$\cancel{\frac{2}{3}} \cdot \frac{3}{2} \cos\theta = \frac{1}{2} \cdot \frac{2}{3}$

$\cos\theta = \frac{1}{3}$ BTWN $[-1, 1]$

$X_{min} = -0.1$

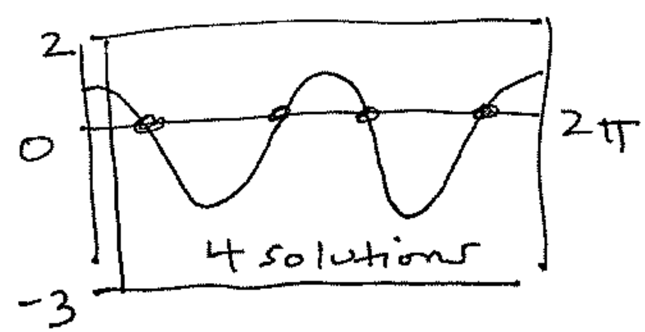
$X_{MAX} = 2\pi$

$Y_{min} = -3$

$Y_{MAX} = 2$

Solve w/ graphing utility

$\underbrace{1.5\cos 2x - 0.5}_{Y_1} = \underbrace{0}_{Y_2}$



* We do not get a value we recognize here, so we must use another method. We will solve graphically with the graphing utility feature of our calculator.

Round solutions to the nearest thousandth. $\boxed{X \approx 0.615, 2.526, 3.757, 5.668}$

9) Solve $2\cos^2 x - 1 = 3\cos x \quad \textit{for} \quad 0 \le x < 2\pi$

$2\cos^2 x - 3\cos x - 1 = 0$

$\cancel{(2\cos x - 1)(\cos x - 1)} = 0$ would need CPF

$X_{min} = {}^{-}0.1$

$X_{MAX} = 2\pi$

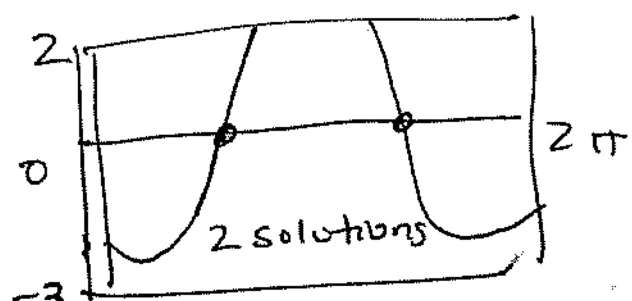
solve w/ graphing utility

$\underbrace{2\cos^2 x - 3\cos x - 1}_{Y_1} = \underbrace{0}_{Y_2}$

$Y_{min} = -3$

$Y_{MAX} = 2$

try graphing then decide



$\boxed{X \approx 1.855, 4.428}$