Notes

Pre-Calculus:
Sec. 5.5 (Day 1)
Solving Trigonometric Equations Strategies:

1) Need the same angle (argument): ex. x, 2x, 3x, x/2, etc...
2) Try to get the **same function** if it will help.
3) Look for rules/identities.
4) Beware of the "temptation": ex. $\underline{\sin x} \tan x = \underline{\sin x}$

   (Do Not Divide out a trig. function!!!)

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0 \mid x = 1$

5) If you have a **MULTIPLE** angle: ex. 2x, 3x, x/2, etc...
   substitute in **θ**, solve, then substitute back.
6) Graphing calculator. Calculate intersections or zeros, depending on what you input.

Beware!!!

When solving trigonometric equations you sometimes will need to check for **extraneous solutions.**

1) Whenever you square both sides of the equation (or raise both sides to any even # power) check **ALL** solutions. Actually plug each answer into the original equation and check if the value works, for each angle found.

2) If the original equation has tan x, cot x, sec x or csc x (which means any function other than sin x or cos x) ***and*** the answer is a quadrantal angle. Only check the quadrantals because they may be undefined. (You do not need to do this for sin x and cos x because they are never undefined).

1) Find all solutions (also known as general solutions) of the equation, in radians. (No calculator)

↖ no interval given

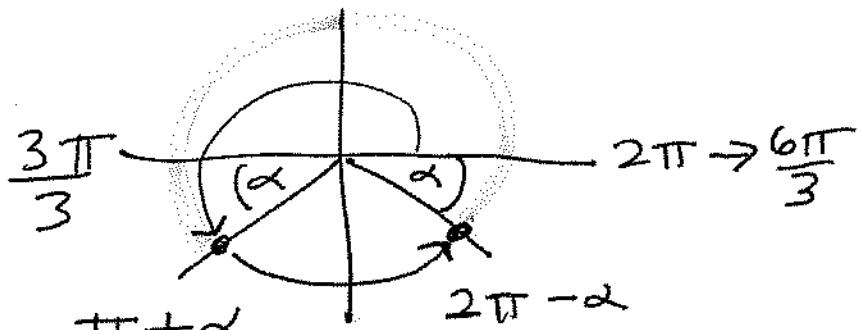
$$\sin x = -\frac{\sqrt{3}}{2}$$

neg

not QA

$\alpha = \frac{\pi}{3}$

Quad III & IV



$\frac{4\pi}{3}$ $\frac{5\pi}{3}$

General Solutions

$$x = \frac{4\pi}{3} + 2\pi n, n \in \mathbb{Z}$$

$$x = \frac{5\pi}{3} + 2\pi n, n \in \mathbb{Z}$$

"General"

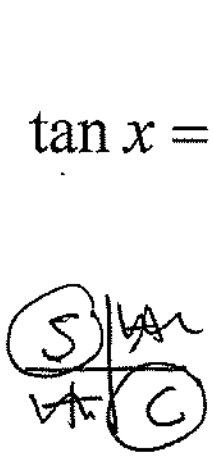
no interval given

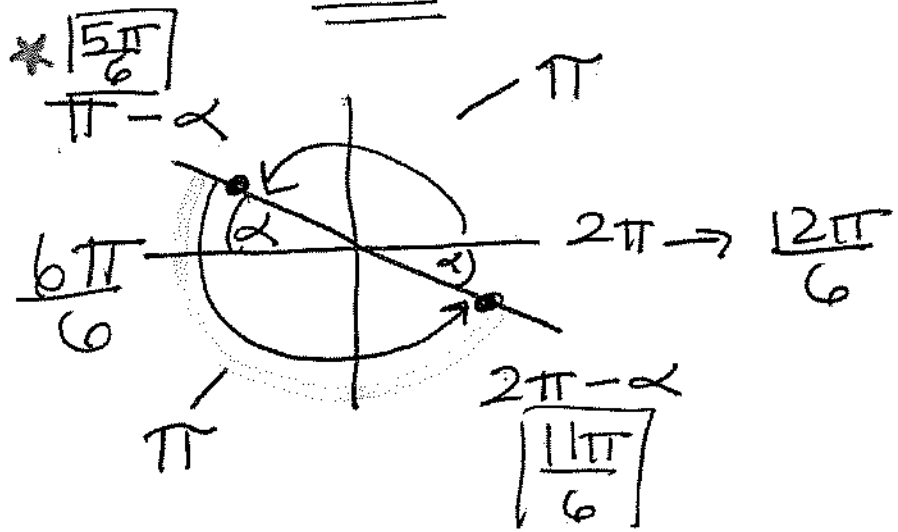
2) Find all solutions of the equation, in radians. (No calculator)

$$\tan x = -\frac{\sqrt{3}}{3}$$

not QA

$\alpha = \frac{\pi}{6}$





General:

$$x = \frac{5\pi}{6} + \pi n, \; n \in \mathbb{Z}$$

"General"

3) Find all solutions of the equation, in radians. (No calculator)

no interval given

$$\sqrt{2}\sin x + 3 = 2$$
$$-3 \quad -3$$

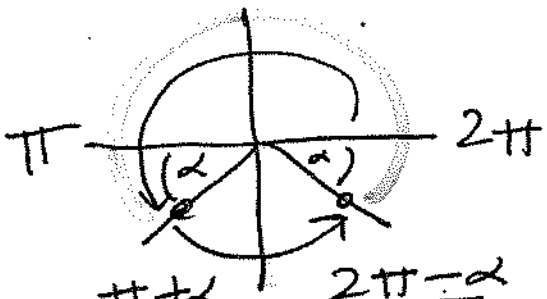
$$\frac{\sqrt{2}\sin x}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

neg

not QA

$\alpha = \frac{\pi}{4}$



$\pi + \alpha$

$\frac{5\pi}{4}$

$2\pi - \alpha$

$\frac{7\pi}{4}$

QA
T | C

General:

$$x = \frac{5\pi}{4} + 2\pi n, \; n \in \mathbb{Z}$$

$$x = \frac{7\pi}{4} + 2\pi n, \; n \in \mathbb{Z}$$

4) Solve on the interval $[0, 2\pi)$ ← open (No calculator)

(Also known as principal solutions).

$$2\sin\theta - \csc\theta = 1$$

$$\sin\theta \cdot 2\sin\theta - \frac{1 \cdot \sin\theta}{\sin\theta} = 1 \cdot \sin\theta$$

$$2\sin^2\theta - 1 = \sin\theta$$

$$2\sin^2\theta - \sin\theta - 1 = 0$$

factor

$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$

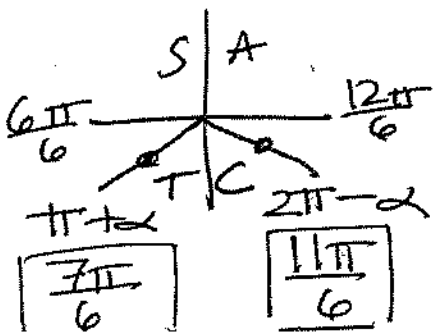
$$2\sin\theta + 1 = 0$$

$$2\sin\theta = -1$$

$$\sin\theta = -\frac{1}{2}$$

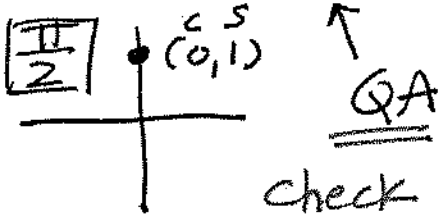
neg ↑ ↑ not QA

$\alpha = \frac{\pi}{6}$



$$\sin\theta - 1 = 0$$

$$\sin\theta = 1$$



QA

check

$\csc\theta$

$\csc\frac{\pi}{2} = 1$ ✓

$$2x^2 - x - 1 = 0$$

factor

$$(2x+1)(x-1) = 0$$

For $[0, 2\pi)$:

$$\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

principal solutions

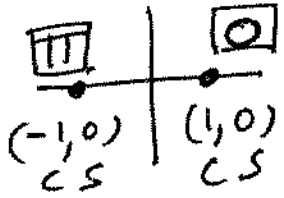
5) Solve on the interval $[0, 2\pi)$. (open) (No calculator)

$\sin x \tan x = \sin x$ (principal solutions)

factor $\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0$



QA

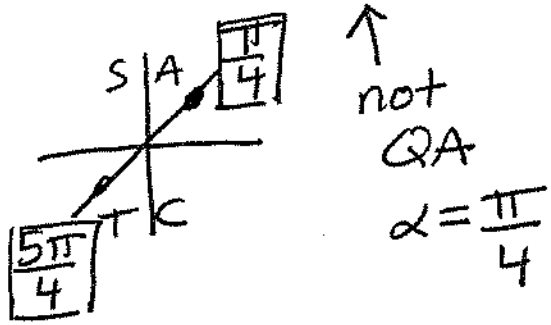
Check $\tan x$

$\tan 0 = 0$ ✓

$\tan \pi = 0$ ✓

$\tan x - 1 = 0$

$\tan x = 1$



not QA

$\alpha = \frac{\pi}{4}$

For $[0, 2\pi)$: $\boxed{x = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}}$

principal

6) Solve on the interval $[0, 2\pi)$. (No calculator)

$$\sin^2 x - \cos^2 x = 0$$

pyth

$$1 - \cos^2 x - \cos^2 x = 0$$

$$1 - 2\cos^2 x = 0$$

$$-1 \qquad\qquad -1$$

$$\frac{-2\cos^2 x}{-2} = \frac{-1}{-2}$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}}$$

$$|\cos x| = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$|\cos x| = \frac{\sqrt{2}}{2} \qquad \alpha = \frac{\pi}{4}$$

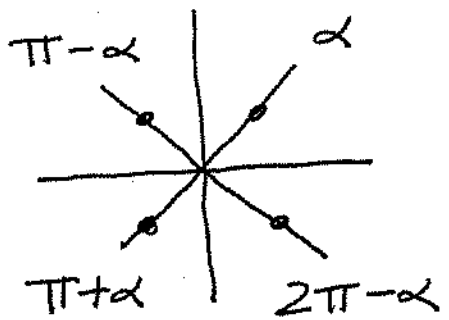
$$\cos x = \pm \frac{\sqrt{2}}{2}$$

all quadrants

For $[0, 2\pi)$:

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

principal



8) Solve: a) For principal solutions on the interval $[0, 2\pi)$.
b) For general solutions.

$$\cos^2 x + 2\sin x = -2$$

$\cos^2 x + 2\sin x + 2 = 0$

pyth

$1 - \sin^2 x + 2\sin x + 2 = 0$

Do not factor → w/negative.

$-\sin^2 x + 2\sin x + 3 = 0$

Now factor

$\sin^2 x - 2\sin x - 3 = 0$

$(\sin x - 3)(\sin x + 1) = 0$

$a^2 - 2a - 3 = 0$

$(a-3)(a+1) = 0$

a) For $[0, 2\pi)$:

$\boxed{x = \frac{3\pi}{2}}$

principal

b) $\boxed{x = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}}$

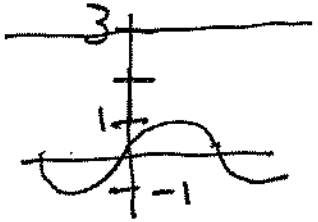
General

$\sin x - 3 = 0$

$\sin x = 3$

$\varnothing$

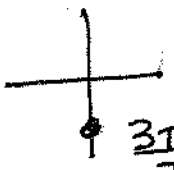
b/c range $[-1, 1]$



$\sin x + 1 = 0$

$\sin x = -1$

QA



check ✓