Pre-Calculus
Sec. 5.2
Sum and Difference Formulas

The Sum and Difference Formula for Sine

Sum: $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

Difference: $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

The Sum and Difference Formula for Cosine

Sum: $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

Difference: $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

The Two Main Purposes for the Sum/Diff. Formulas:

1)Finding the **exact** values of trigonometric expressions with non common angles.

2)Simplifying expressions to obtain other identities.

Ex. 1: Find the exact value of each of the following:

$a)\ \cos 15^\circ$

$b)\ \sin 285°$

°

Ex.2: Simplify each of the following expressions.

Can you condense using a sum/difference formula?

$a)\ \cos 50^\circ \cos 40^\circ - \sin 50^\circ \sin 40^\circ$

$$b)\ \sin\frac{11\pi}{30}\cos\frac{\pi}{5} - \sin\frac{\pi}{5}\cos\frac{11\pi}{30}$$

The Sum and Difference Formula for Tangent

Sum: $\tan(\alpha + \beta)$

Difference: $\tan(\alpha - \beta)$

Ex. 3: Find the exact value of each of the following:

$a)\ \tan 195^\circ$

$b)\ \cot(-15°)$

Finding the Exact Value

Ex. 4: If $\sin\alpha = \frac{12}{13}$, $0 < \alpha < \frac{\pi}{2}$, and $\sin\beta = \frac{3}{5}$, $\frac{\pi}{2} < \beta < \pi$

a) Find cos(α + β)

b) Find $\tan(\alpha + \beta)$ for example 4.

Ex. 5: Find the value of the expression without a calculator.

$$a)\ \cos\left[\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}1\right]$$

$$b)\ \tan\left(\sin^{-1}\frac{4}{5}-\cos^{-1}\frac{5}{13}\right)$$

The sum/diff. formulas can be used to verify many identities that we have seen, such as the cofunction rule sin (90° – θ) = cos θ, and to derive new identities.

Ex. 6: Verify the identity: $\sec\left(\frac{\pi}{2} - x\right) = \csc x$

Ex. 7: Write the trigonometric expression as an algebraic expression.

$$\cos(\arccos x - \arcsin x)$$

Ex. 8: Find the solutions of the equation in the interval $[0,2\pi)$

$$\cos\left(x+\frac{\pi}{6}\right)-\cos\left(x-\frac{\pi}{6}\right)=1$$