

# Pre-Calculus: Sec. 5.1

## Trigonometric Identities (Day 2)

Ex.1) Use a trigonometric substitution to write the algebraic expression as a trig. function of  $\theta$ .

1a)  $\sqrt{64 - 16x^2}$ ,  $x = 2 \cos \theta$  where  $0 < \theta < \frac{\pi}{2}$ .

1b)  $\sqrt{9x^2 + 4}$ ,  $3x = 2 \tan \theta$  where  $\frac{\pi}{2} < \theta < \pi$ .

**1c)**

Use  $a = 7\cos\theta$  to rewrite  $\frac{1}{\sqrt{49 - a^2}}$  as a trigonometric function involving  $\theta$ ,

where  $\frac{3\pi}{2} < \theta < 2\pi$ .

Ex.2) Use *cofunction* identities to evaluate without a calculator.

a)  $\cos^2 14^\circ + \cos^2 76^\circ$

b)  $\sin^2 18^\circ + \sin^2 40^\circ + \sin^2 50^\circ + \sin^2 72^\circ$

Ex. 3) Simplify:  $(\cos^2 x - 1)(1 + \cot^2 x)$

Ex.4) Verify the following identities:

$$a) \frac{1 + \csc \theta}{\cot \theta + \cos \theta} = \sec \theta$$

$$b) \quad \frac{\cot^2 \theta}{1 + \csc \theta} = \frac{1 - \sin \theta}{\sin \theta}$$

$$c) \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$$

$$d) 1 + \sec^2 \theta = 2 + \tan^2 \theta$$



$$e) \sec^4 x \tan^2 x = (\tan^2 x + \tan^4 x) \sec^2 x$$

$$f) \sin^4 x + \cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x$$

**Pre-Calculus**  
**Lesson 3**  
**Sec. 5.2**  
**Sum and Difference**  
**Formulas**

## The Sum and Difference Formula for Sine

**Sum:**  $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

**Difference:**  $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

## The Sum and Difference Formula for Cosine

**Sum:**  $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

**Difference:**  $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

### The Two Main Purposes for the Sum/Diff. Formulas:

- 1) Finding the **exact** values of trigonometric expressions with non common angles.
- 2) Simplifying expressions to obtain other identities.

Ex. 1: Find the exact value of each of the following:

a)  $\cos 15^\circ$

o

$$b) \sin 285^\circ$$

o

Ex.2: Simplify each of the following expressions.

Can you condense using a sum/difference formula?

a)  $\cos 50^\circ \cos 40^\circ - \sin 50^\circ \sin 40^\circ$

$$b) \sin \frac{11\pi}{30} \cos \frac{\pi}{5} - \sin \frac{\pi}{5} \cos \frac{11\pi}{30}$$