Pre-Calculus: Sec. 5.1 Trigonometric Identities

Reciprocal Identities:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\boldsymbol{\csc\theta = \frac{1}{\sin\theta}} \qquad \boldsymbol{\sec\theta = \frac{1}{\cos\theta}} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities (Dividing two trig. functions.):

$$\boldsymbol{\tan\theta = \frac{\sin\theta}{\cos\theta}} \qquad \boldsymbol{\cot\theta = \frac{\cos\theta}{\sin\theta}}$$

From the Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

$$x^2 + y^2 = r^2 \text{ (circle)}$$

$$x^2 + y^2 = 1 \text{ (unit circle)}$$

since $\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{r}$ and in a unit circle r = 1

Then.....$x^2 + y^2 = 1$ can be written as:

Pythagorean Identity #1:

More from the Pythagorean Theorem

From: $\cos^2 \theta + \sin^2 \theta = 1$, we can derive two others…

Pythagorean Identity #2:

Pythagorean Identity #3:

Pythagorean Identities:

$$cos^2\theta + \ sin^2\theta = 1$$

$$1 + \ tan^2\theta = \ sec^2\theta$$

$$cot^2\theta + 1 = \ csc^2\theta$$

Ex.1) Simplify the expression:

a) $\sin x(\csc x - \sin x)$

b) $\cot(\pi/2 - x)\cos x$

Ex.2) Factor and simplify the expression.

$$\sec^2 x \tan^2 x + \sec^2 x$$

Ex.3) Simplify the expression.

$$\sec^2 x(1 - \sin^2 x)$$

Guidelines for Proving Trigonometric Identities

- **Work** with each side of the equation independently.
- **Start** with the more complicated side and **transform** it step–by–step until both sides look the same.
- **Look** for opportunities to **apply** the **identities**.
- **Rewriting** the more complicated side of the equation in terms of **sine** and **cosine** is often helpful.
- **Simplify** algebraically: factor, combine 2 fractions, distribute, ….ect.
- You may **not move across** the = or × and ÷ on both sides.
- Always keep this in mind "Your answer is already there!!!"

Ex. 4) Prove (Transform one side of the equation into the other.)

$$\frac{1}{1-\sec\theta}+\frac{1}{1+\sec\theta} = -2\cot^2\theta$$

Ex. 5) Prove (Verify the identity algebraically.)

$$\frac{\cos x}{\cos\left(\frac{\pi}{2}-x\right)} = \cot x$$

Ex.6) Prove:

$$(\sec\theta - \tan\theta)(\csc\theta + 1) \quad = \quad \cot\theta$$

Ex. 7) Prove:

$$\frac{\tan\theta}{1+\sec\theta}+\frac{1+\sec\theta}{\tan\theta} = 2\csc\theta$$

Ex. 8) Prove:

$$\frac{\csc(\frac{\pi}{2}-\theta)}{\tan(-\theta)} = -\csc\theta$$

Ex. 9) Prove:

$$\sec^3 x - \sec^2 x - \sec x + 1 = (\sec x - 1)\tan^2 x$$

Ex. 10) Prove:

$$\sec^4 x - \tan^4 x \quad = \quad 2\sec^2 x - 1$$

Ex.11) Prove. Hint: Use the conjugate.

$$\frac{5}{tanx+secx} \quad = \quad 5(secx - tanx)$$

Ex.12) Prove.

$$\ln\left|\csc\theta\right| + \ln\left|\tan\theta\right| = \ln|sec\theta|$$