Pre-Calculus
Sec. 10.2
Arithmetic
Sequences and Series

- Objective: To identify an arithmetic sequence and series. To find the explicit (general) formula for its nth term and to find the partial sum of an arithmetic series.

- Definition: A sequence is a set of numbers (terms) arranged in a particular order (separated by commas)

$\{0, 2, 4, 6, 8\}$ Finite Sequence $\{1, 3, 5, 7, 9, 11, ...\}$ Infinite Sequence

- All arithmetic sequences will look linear, where the slope is the common difference (d)

2 SPECIAL TYPES OF SEQUENCES:

1. Arithmetic – the next term is the previous one plus "d"
(**common difference = slope**)

2. Geometric – the next term is the previous one times "r"
(**common ratio**)

Ex1) State whether the given sequence is arithmetic, geometric, or neither. If arithmetic, give the common difference, and if geometric, give the common ratio.

a) $3, 8, 13, 18, \ldots\ldots$

b) $4, 8, 16, 32, \ldots$

c) $27, -18, 12, -8, \ldots$

d) $1, -3, 5, -7, \ldots$

Ex.2) Find the first five terms of the given sequences and state whether the sequence is arithmetic. If so , state the common difference.

$$a_n = 3 - 7n$$

Formulas to Memorize:

Finding the nth term of an arithmetic sequence:
$$a_n = a_1 + d(n-1)$$

Where: $a_1 = \text{first term}$

$a_n = n\text{th term}$

- Sequence: 2, 4, 6, 8, 10, ...
- Series: 2 + 4 + 6 + 8 + 10 + ...

More Formulas to Memorize:

Finding the sum of the first n terms of an Arithmetic Series (Partial Sum):

$$S_n = \frac{n}{2}\left(a_1 + a_n\right) \quad \text{where } a_1 = \text{first term} \text{ and } a_n = \text{last term}$$

OR

$$S_n = \frac{n}{2}\left[2a_1 + d(n-1)\right] \quad \text{Use when you don't know the last term.}$$

Sigma Notation Σ is another way to find the partial sum of an arithmetic series.

Ex.3) Find the general term formula (the nth term) for the arithmetic sequence, given that:

$a)$ $a_1 = 15$, $d = 4$

b) $a_1 = -4$, $a_5 = 16$

c) $a_5 = 190$, $a_{10} = 115$

Ex.4) Write the 1st five terms of the arithmetic sequence:

$a_4 = 16$, $a_{10} = 46$

Ex.5) In an arithmetic sequence:

$$a_n = a_{n-1} + 3 \; and \; a_1 = 17, \text{find the value of } a_{20}$$

Ex.6) Find the sum of the finite arithmetic series:

$$1 + 4 + 7 + 10 + 13 + 16 + 19 + 22 + 25$$

Ex.7) Find the sum of the first 100 positive odd integers:

Ex.8) Find the sum of the integers from -10 to 50.

Ex.9) Find the sum of the first 50 terms of the arithmetic sequence: $-6, -2, 2, 6, ...$

Ex.10) Find the sum:

$$\sum_{i=1}^{32} (-6i + 3)$$