Pre-Calculus
Sec. 10.1
Sequences
and
Summation Notation

An infinite sequence is a function whose domain is the set of positive integers. The function values:

$$a_1, a_2, a_3, \ldots\ldots, \quad a_n, \ldots\ldots$$

Are the terms of the sequence. If the domain of a function consists of the first *n* positive integers only, the sequence is a finite (ends) sequence.

On occasion, it is convenient to begin subscripting a sequence with 0 instead of 1 so that the terms of the sequence become

$$a_0, a_1, a_2, \ldots\ldots, \quad a_n, \ldots\ldots$$

Ex1) Write the first five terms of the sequence whose general term formula is given (Assume *n* begins with 1).

$$a_n = \frac{2n}{n+1}$$

A recursive formula defines the nth term of a sequence as a function of the previous term (depending on the prior term or terms).

Ex2) Write the first four terms of the sequence using the given recursive formula.

a) $a_1 = 3$ and $a_{k+1} = 2(a_k - 1)$

b) $a_1 = 5$ and $a_n = 3a_{n-1} - 1$

Evaluating Factorials:

0! (zero factorial), by definition, is 1.

1! = 1

2! = (2)(1) = 2

3! = (3)(2)(1) = 6

4! = (4)(3)(2)(1) = 24

etc…

Ex3) Evaluate each factorial expression:

a) $\frac{8!}{2!6!}$

b) $\frac{2!6!}{3!5!}$

c) $\frac{10!3!}{4!6!}$

d) $\frac{n!}{(n+2)!}$

e) $\frac{(n-2)!}{n!}$

It is sometimes useful to find the sum of the first *n* terms of a sequence. We can use Summation Notation to express the sum.

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdots + a_n,$$

Where *i* is the index of summation, *n* is the upper limit of summation, and 1 is the lower limit of summation.

Any letter can be used for the index of summation. The letters *i, j,* and *k* are commonly used.

Ex4) Expand and evaluate the sum.

a. $\sum_{i=1}^{6} (i^2 + 1)$

b. $\sum_{k=4}^{7} [(-2)^k - 5]$

c. $\sum_{i=1}^{5} 3.$

d. $\sum_{i=0}^{3} (i + 1)^2$

Ex5) Express the sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

$$\frac{1}{3} + \frac{1}{2} + \frac{3}{5} + \cdots + \frac{6}{7}$$

Properties of Sums pg. 1009 of textbook

Property	Example
1. $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$, c any real number	$\sum_{i=1}^{4} 3i^2 = 3\cdot1^2 + 3\cdot2^2 + 3\cdot3^2 + 3\cdot4^2$ $3\sum_{i=1}^{4} i^2 = 3(1^2 + 2^2 + 3^2 + 4^2) = 3\cdot1^2 + 3\cdot2^2 + 3\cdot3^2 + 3\cdot4^2$ Conclusion: $\sum_{i=1}^{4} 3i^2 = 3\sum_{i=1}^{4} i^2$
2. $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$	$\sum_{i=1}^{4} (i + i^2) = (1 + 1^2) + (2 + 2^2) + (3 + 3^2) + (4 + 4^2)$ $\sum_{i=1}^{4} i + \sum_{i=1}^{4} i^2 = (1 + 2 + 3 + 4) + (1^2 + 2^2 + 3^2 + 4^2)$ $= (1 + 1^2) + (2 + 2^2) + (3 + 3^2) + (4 + 4^2)$ Conclusion: $\sum_{i=1}^{4} (i + i^2) = \sum_{i=1}^{4} i + \sum_{i=1}^{4} i^2$
3. $\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$	$\sum_{i=3}^{5} (i^2 - i^3) = (3^2 - 3^3) + (4^2 - 4^3) + (5^2 - 5^3)$ $\sum_{i=3}^{5} i^2 - \sum_{i=3}^{5} i^3 = (3^2 + 4^2 + 5^2) - (3^3 + 4^3 + 5^3)$ $= (3^2 - 3^3) + (4^2 - 4^3) + (5^2 - 5^3)$ Conclusion: $\sum_{i=3}^{5} (i^2 - i^3) = \sum_{i=3}^{5} i^2 - \sum_{i=3}^{5} i^3$