Pre-Calculus
Sec. 5.3
Double Angle
and
Power Reducing
Formulas

Double Angle Formulas

$$\sin 2x = 2\sin x \cos x$$

Double Angle Formulas

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

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$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

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Ex. 1: Given a right triangle with a side of 5, hypotenuse of 13, and θ the angle between them; find the following:

$a) \sin\theta$

$b) \cos\theta$

$c) \sin 2\theta$

$d) \cos 2\theta$

$e) \tan 2\theta$

$f) \cot 2\theta$

Ex. 2: Find the exact value of cos2u, using the double angle formulas.

$$Given: \quad \cot u = -6 \quad and \quad \frac{3\pi}{2} < u < 2\pi$$

Power Reduction Formulas

These new formulas are derived from the Cosine Double Angle Formulas

Recall :

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= 2\cos^2 u - 1$$

$$= 1 - 2\sin^2 u$$

The Power Reducing Formulas:

so...

$$2\cos^2 u - 1 = \cos 2u$$

$$2\cos^2 u = 1 + \cos 2u$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

The Power Reducing Formulas:

so...

$$1-2\sin^2 u = \cos 2u$$

$$-2\sin^2 u = -1+\cos 2u$$

$$2\sin^2 u = 1-\cos 2u$$

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

The Power Reducing Formulas:

so...

$$\tan^2 u = \frac{\sin^2 u}{\cos^2 u} = \frac{\frac{1-\cos 2u}{2}}{\frac{1+\cos 2u}{2}} = \frac{1-\cos 2u}{1+\cos 2u}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Ex.1:Rewrite the expression in terms of the 1st power of cosine.

$a)\cos^2 2x$

$b)\sin^2 \frac{x}{2}$

$c) \sin^4 x$

Ex.2: Write the trigonometric expression as an algebraic expression.

$$\sin\left(2\arccos x\right)$$

Ex. 3: Verify the Identity:

$$a) \qquad \sec 2\theta = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$$

$b)$ $\quad 1+\cos 10x \quad = \quad 2\cos^2 5x$

$c)$ $\cos 3x = 4\cos^3 x - 3\cos x$