yperbola: Sec. 9.2

eneral
rm:

$$\overset{\oplus}{A}x^2 + Bx - \overset{\ominus}{C}y^2 + Dy + E = 0$$

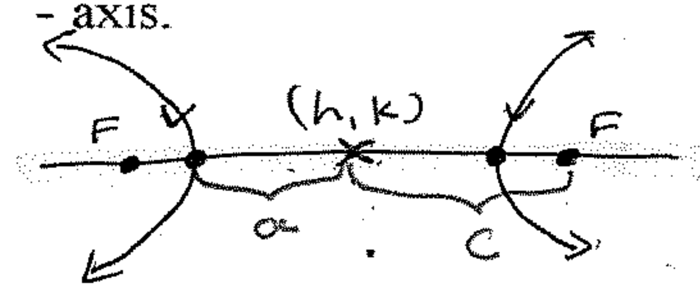
andard
rm:

$$\boxed{\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1} \leftarrow$$ This must be equal to 1.

The x^2 portion is positive,

so the hyperbola crosses over the x - axis.

Transverse axis is horizontal.
"major axis"



center : (h, k)

The line segment connecting the vertices is the transverse axis.

The foci are c units from the center,

where $\boxed{c^2 = a^2 + b^2}$
add

"rise" / run

"y-part over the x-part"

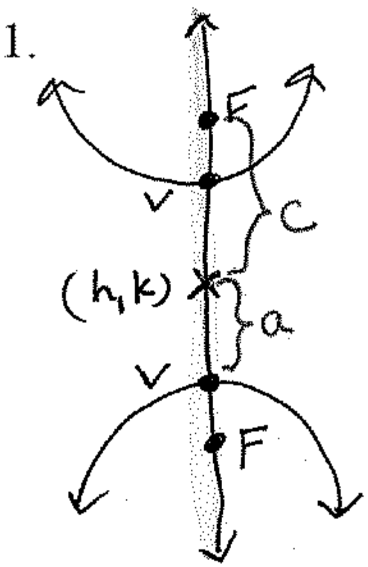
Asymptotes : $y = k \pm \frac{b}{a}(x-h)$

$\boxed{OR}$

Hyperbola: Sec. 9.3

General Form: $Ay^2 + By - Cx^2 + Dx + E = 0$

Standard form: $\boxed{\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1}$ ← This must be equal to 1.



The y^2 portion is positive,

so the hyperbola crosses over the y - axis.

Transverse axis is vertical.

center : (h, k)

The line segment connecting the vertices is the transverse axis.

The foci are c units from the center,

where $\boxed{c^2 = a^2 + b^2}$

"$\dfrac{\text{rise}}{\text{run}}$"

Asymptotes : $y = k \pm \dfrac{a}{b}(x-h)$

$$y - k = \pm \frac{a}{b}(x-h)$$

Ex. 1) Find the standard form of the equation, the center, vertices, foci, and asymptotes of the hyperbola. Then sketch the hyperbola, labeling these parts.

$$^{\oplus}x^2 \,{}^{\ominus}\!-9y^2 + 36y - 72 = 0$$

factor

$x^2 - 9y^2 + 36y = 72$

$x^2 - 9(y^2 - 4y + \underline{4}) = 72 + (\underline{-36})$

negative

$c = \left(\frac{-4}{2}\right)^2 = (-2)^2$

$\frac{x^2}{36} - \frac{9(y-2)^2}{36} = \frac{36}{36}$

$$\boxed{\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1}$$

a^2 w/ positive

$a = 6$

transverse

Horizontal

$b = 2$

$\boxed{\text{Center: } (0,2)}$

$c^2 = a^2 + b^2$

$c^2 = 36 + 4$

$c^2 = 40$

$c = \sqrt{40}$ $\sqrt{36}$ $\sqrt{49}$ Btwn 6+7

$c = 2\sqrt{10}$

vertices: $(-6,2)$ + $(6,2)$

Foci: $(\pm 2\sqrt{10}, 2)$

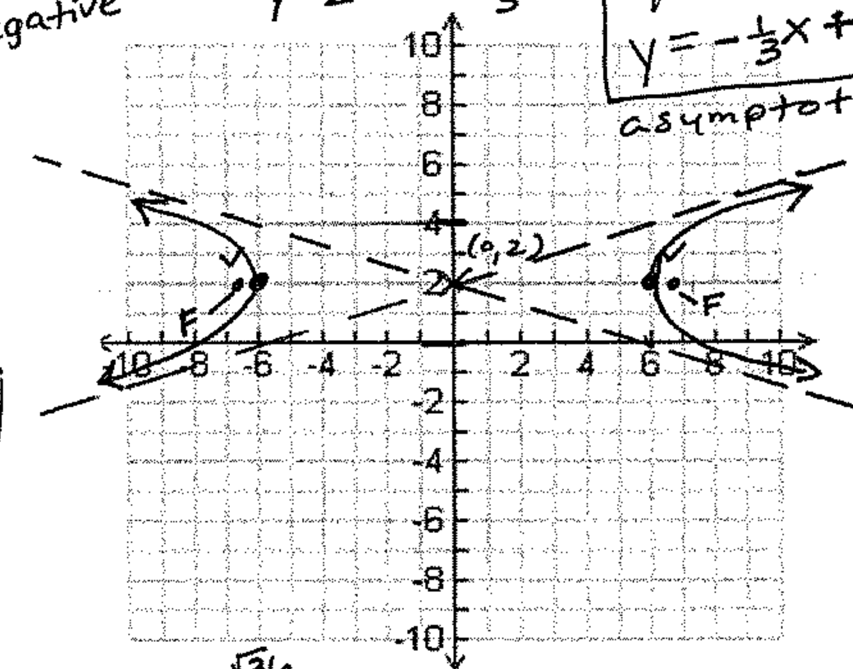
$y - k = \pm \frac{b}{a}(x - h)$

$y - 2 = \pm \frac{2}{6}(x - 0)$

$y - 2 = \pm \frac{1}{3}x$

$$\boxed{\begin{aligned} y &= \tfrac{1}{3}x + 2 \\ y &= -\tfrac{1}{3}x + 2 \end{aligned}}$$

asymptotes



Ex. 2) Find the standard form of the equation, the center, vertices, foci, and asymptotes of the hyperbola.
Then sketch the hyperbola, labeling these parts.

$$\oplus 9y^2 \ominus 4x^2 - 18y + 24x - 63 = 0$$

factor $\quad$ factor

$$9y^2 - 18y \quad -4x^2 + 24x \quad = 63$$

$$9(y^2 - 2y + \underline{1}) - 4(x^2 - 6x + \underline{9}) = 63 + \underline{9} + \underline{(-36)}$$

$$c = \left(\frac{-2}{2}\right)^2 = (-1)^2 \qquad c = \left(\frac{-6}{2}\right)^2 = (-3)^2$$

$$\frac{9(y-1)^2}{36} - \frac{4(x-3)^2}{36} = \frac{36}{36}$$

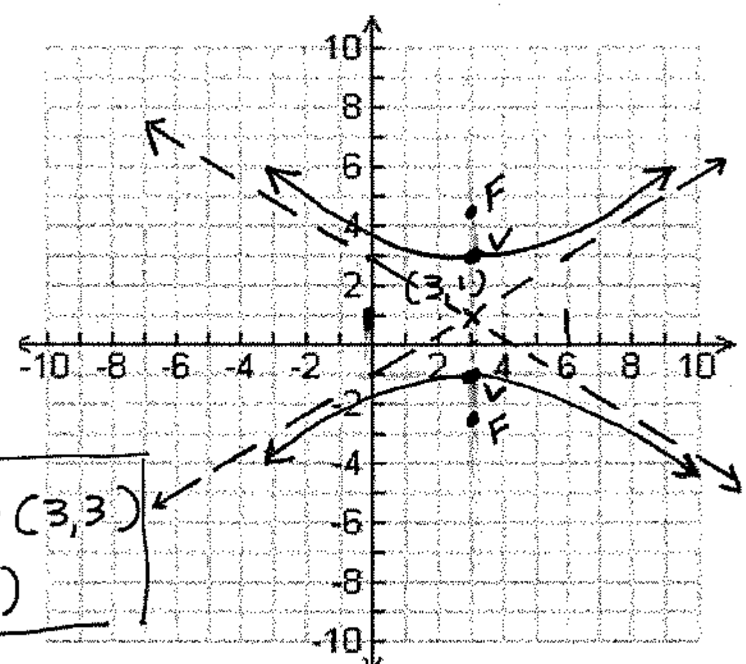
$$\boxed{\frac{(y-1)^2}{4} - \frac{(x-3)^2}{9} = 1}$$

$\boxed{\text{center: } (3, 1)}$

$a^2 = 4$
$a = 2$
transverse
vertical

$b = 3$

Vertices: $(3,-1)$ + $(3,3)$
Foci: $(3, 1 \pm \sqrt{13})$



$c^2 = a^2 + b^2$
$c^2 = 4 + 9$
$c^2 = 13$

$c = \sqrt{13}$ — $\sqrt{9}$, $\sqrt{16}$ Btwn 3+4

asymptotes ⟶

Ex2) continued

Asymptotes:

Center: (3, 1)
(h, k)

$$y-k = \pm \frac{a}{b}(x-h)$$

$$\boxed{y-1 = \pm \frac{2}{3}(x-3)}$$

point slope form

$$y-1 = \frac{2}{3}(x-3)$$

$$y-1 = \frac{2}{3}x - 2$$

$$+1 \qquad\qquad +1$$

$$\boxed{y = \frac{2}{3}x - 1}$$

$$y-1 = -\frac{2}{3}(x-3)$$

$$y-1 = -\frac{2}{3}x + 2$$

$$+1 \qquad\qquad +1$$

$$\boxed{y = -\frac{2}{3}x + 3}$$

Slope-intercept form.

Extra Practice: Write the standard form of the conic:

1) Given a circle with center (2, -1) and a point on the circle at (5, 3).

(h, k)

x y

$(x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y+1)^2 = r^2$ ← need for standard form

plug in the given point → $(5-2)^2 + (3+1)^2 = r^2$

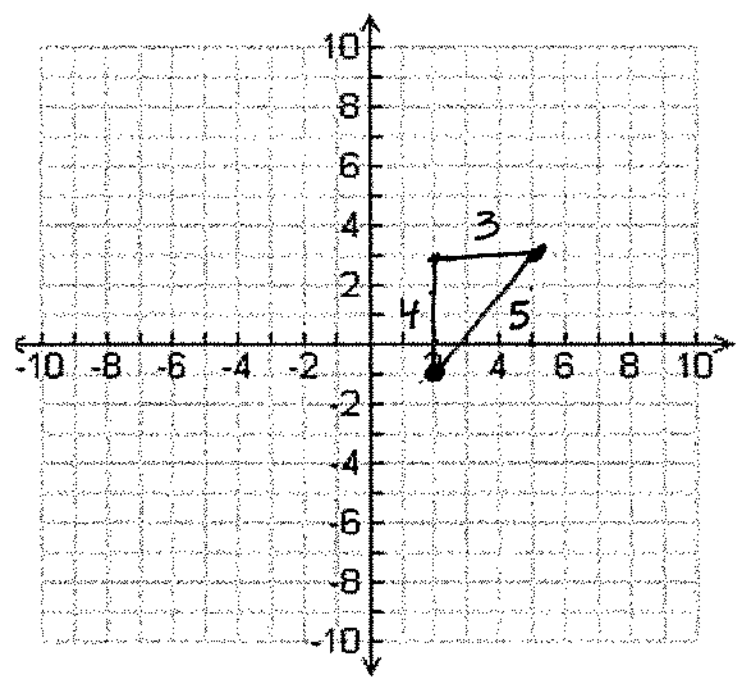
or use the distance formula.

$(3)^2 + (4)^2 = r^2$

$9 + 16 = r^2$

$25 = r^2$

$$\boxed{(x-2)^2 + (y+1)^2 = 25}$$



Extra Practice: Write the standard form of the conic:

2) Given a parabola with focus (4, -7) and directrix y=1.

Horizontal

$(x-h)^2 = 4p(y-k)$

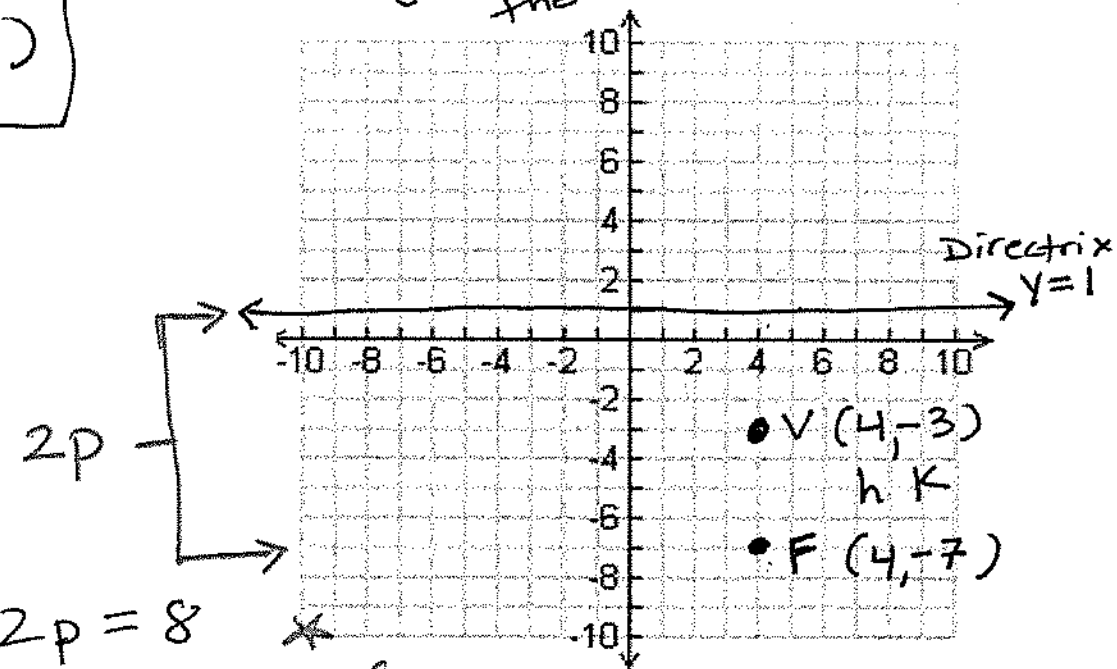
$(x-4)^2 = 4(-4)(y-(-3))$

$$\boxed{(x-4)^2 = -16(y+3)}$$

opens down

opens down (away from the directrix)

x^2 or y^2



$2p = 8$

$p = 4$

$p = -4$

opens down make p negative