Ellipse: Sec. 9.1

Notes

General Form:

positive positive

$$Ax^2 + Bx + Cy^2 + Dy + E = 0$$

different

Standard form:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$ ← This must be equal to 1.

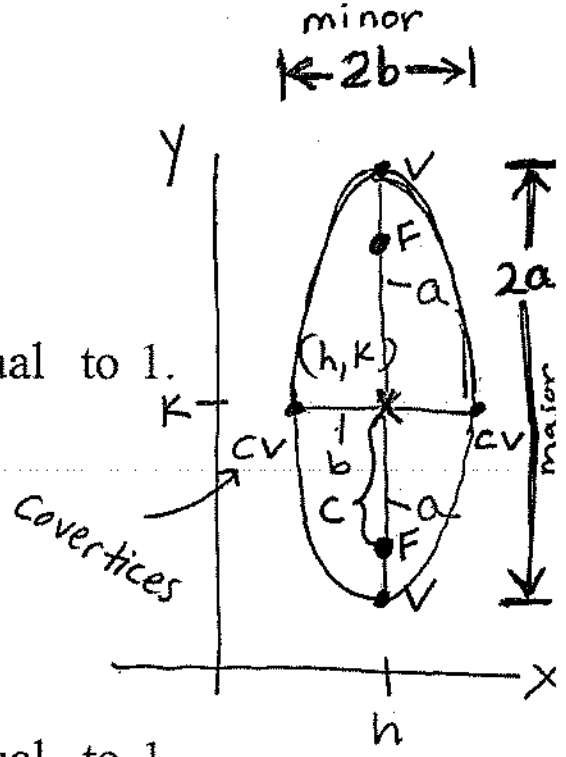
Major axis is horizontal .

OR

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$ ← This must be equal to 1.

Major axis is vertical.

The major axis for an ellipse is always the longer one.



center : (h,k)

Major and minor axes of lengths 2a and 2b, where $0 < b < a$.

The foci lie on the major axis, c units from the center,

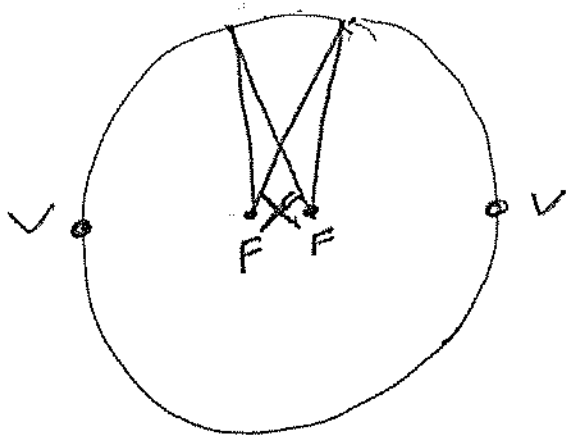
with $c^2 = a^2 - b^2$

To measure the ovalness of an ellipse, you can use the concept of eccentricity.

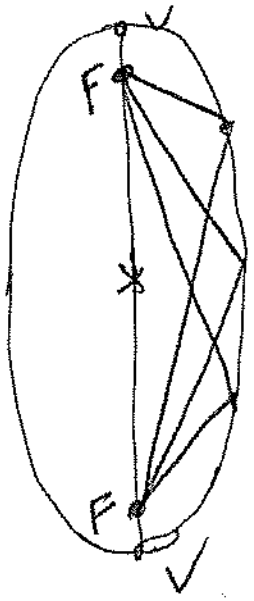
The eccentricity "*e*" of an ellipse is given by the ratio $e = \frac{c}{a}$

$0 < e < 1$ for every ellipse.

For a more circular ellipse,
the ratio is small (closer to 0).
The foci are close to the center.



For a more elongated ellipse,
the ratio is close to 1.
The foci are close to the vertices.



Ex. 1) Find the standard form of the equation, the center, vertices, foci, and eccentricity of the ellipse. Then sketch the ellipse, labeling these parts.

A) $4x^2 + y^2 = 36$

diff = ellipse

$\frac{4x^2}{36} + \frac{y^2}{36} = \frac{36}{36}$

$\frac{x^2}{9} + \frac{y^2}{36} = 1$

$b^2 = 9$

$b = 3$

major axis

$a^2 = 36$

$a = 6$

"vertical"

$\sqrt{27}$ ($\sqrt{25}$, $\sqrt{36}$)

Btwn 5 + 6

Center: (0,0)

$c^2 = a^2 - b^2$

$c^2 = 36 - 9$

$c^2 = 27$

$c = \sqrt{27}$

$c = 3\sqrt{3}$

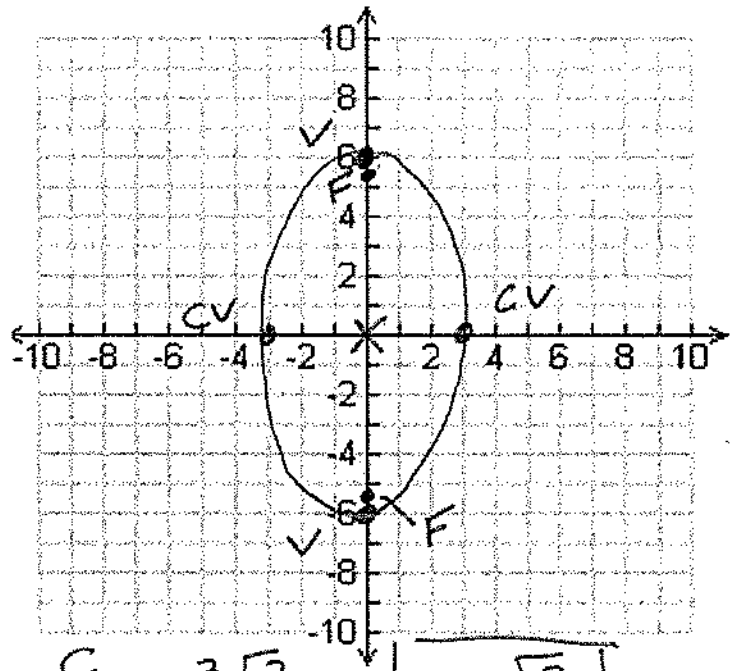
$e = \frac{c}{a} = \frac{3\sqrt{3}}{6}$

$e = \frac{\sqrt{3}}{2}$

vertices: (0,6) & (0,-6)

covertices: (3,0) & (-3,0)

Foci: $(0, 3\sqrt{3})$ & $(0, -3\sqrt{3})$



B) $9x^2 + 4y^2 - 54x + 40y + 37 = 0$

(+) (+) different

$e = \frac{c}{a} = \frac{2\sqrt{5}}{6}$

$\boxed{e = \frac{\sqrt{5}}{3}}$

$(9x^2 - 54x\ \ ) + (4y^2 + 40y\ \ ) = -37$

$9(x^2 - 6x + \underline{9}) + 4(y^2 + 10y + \underline{25}) = -37 + \underline{81} + \underline{100}$

$c = \left(\frac{-6}{2}\right)^2 = (-3)^2$ $\quad c = \left(\frac{+10}{2}\right)^2 = (+5)^2$

$\frac{9(x-3)^2}{144} + \frac{4(y+5)^2}{144} = \frac{144}{144}$

$\boxed{\frac{(x-3)^2}{16} + \frac{(y+5)^2}{36} = 1}$

minor
$b = 4$

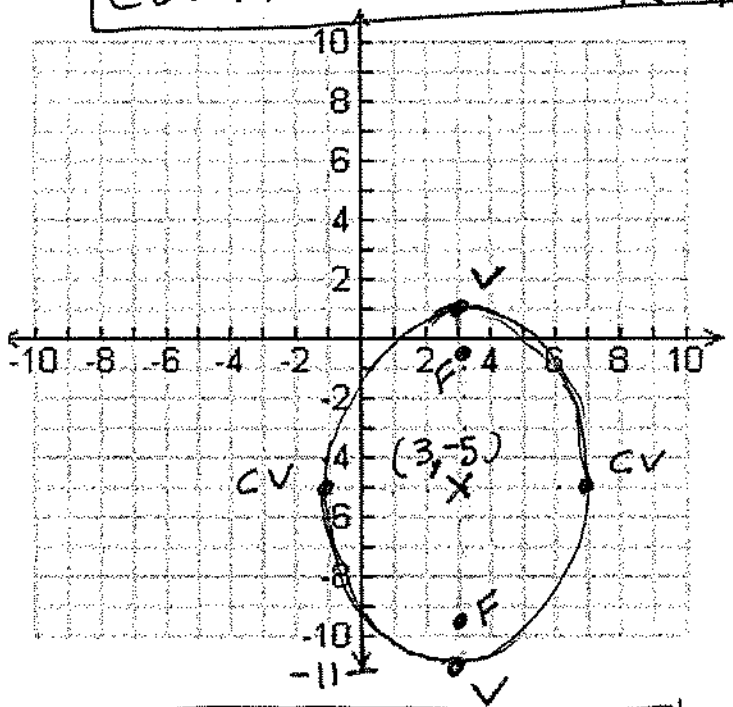
major axis
vertical
$a = 6$

$c^2 = a^2 - b^2$
$c^2 = 36 - 16$
$c^2 = 20$
$c = \sqrt{20} < \frac{4}{5}$
$c = 2\sqrt{5}$

$\sqrt{20} < \sqrt{25}$ (from $\sqrt{16}$)
Btwn 4 + 5

$\boxed{\text{Center: } (3, -5)}$

vertices: $(3, 1)$ & $(3, -11)$
covertices: $(-1, -5)$ & $(7, -5)$



$\boxed{\text{Foci: } (3, -5 \pm 2\sqrt{5})}$

C) $x^2+4y^2-6x+20y-2=0$

$e=\frac{c}{a}=\frac{3\sqrt{3}}{6}$

$\boxed{e=\frac{\sqrt{3}}{2}}$

$(x^2-6x\quad)+(4y^2+20y\quad)=2$

$(x^2-6x+\underline{9}\,)+4(y^2+5y+\underline{\frac{25}{4}}\,)=\underline{2+9+25}$

$c=\left(\frac{-6}{2}\right)^2=(-3)^2$ $\qquad c=\left(\frac{+5}{2}\right)^2=\frac{25}{4}$

vertices: $(-3,-\frac{5}{2})$ & $(9,-\frac{5}{2})$

covertices: $(3,\frac{1}{2})$ & $(3,-\frac{11}{2})$

$\frac{(x-3)^2}{36}+\frac{4(y+\frac{5}{2})^2}{36}=\frac{36}{36}$

$\boxed{\frac{(x-3)^2}{36}+\frac{(y+\frac{5}{2})^2}{9}=1}$

major axis is horizontal

$a=6$

minor

$b=3$

$c^2=a^2-b^2$

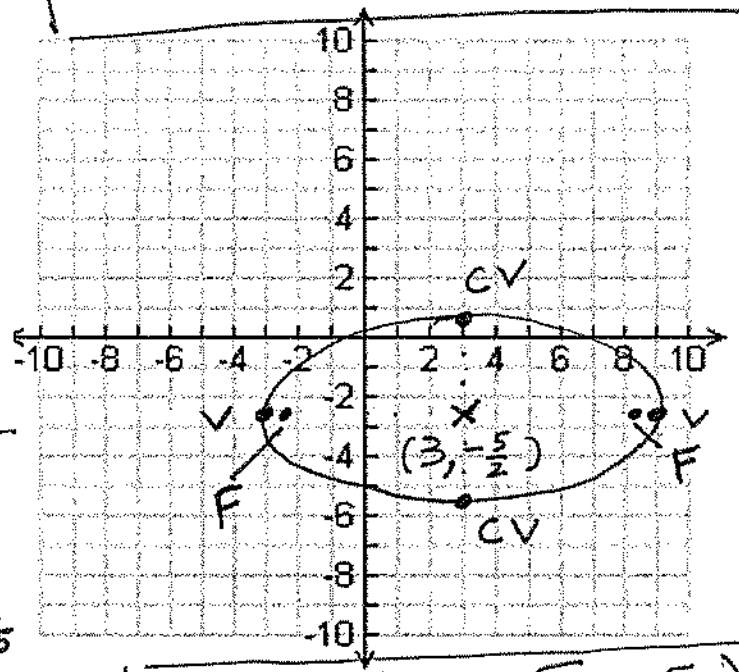
$c^2=36-9$

$c^2=27$

$c=\sqrt{27}$ $\quad\sqrt{25}<\sqrt{27}<\sqrt{36}$

$c=3\sqrt{3}$ Btwn 5 & 6

center: $(3,-\frac{5}{2})$



Foci: $(3\pm3\sqrt{3},-\frac{5}{2})$

Ex. 2) Find the equation, in standard form, of the specified ellipse. **"PLOT WHAT YOU'VE GOT!"**

A) vertices: (0, ± 8); foci: (0, ± 4)

Center: (0, 0)

* must decide where to put the a^2...

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

need b^2

* Vertical major axis → a^2 with y^2

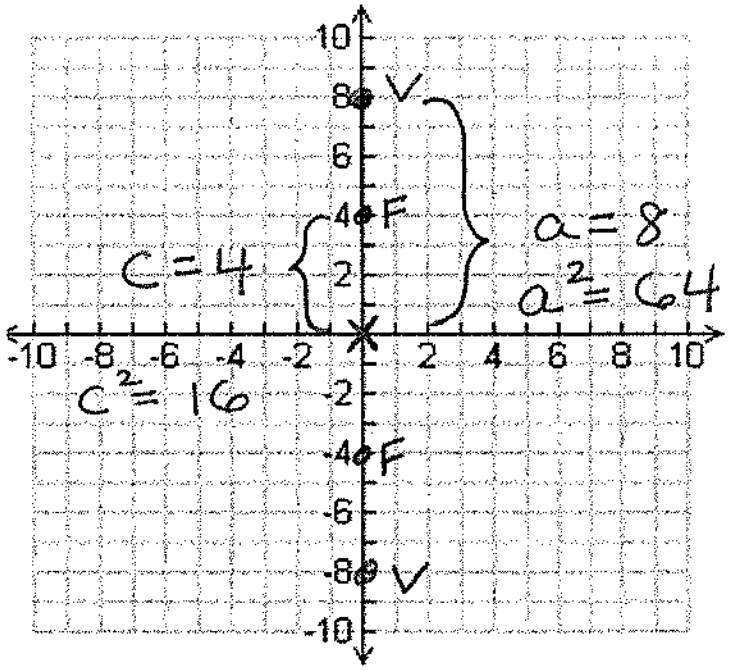
$c^2 = a^2 - b^2$

$16 = 64 - b^2$

$b^2 = 64 - 16$

$b^2 = 48$

$$\frac{x^2}{48} + \frac{y^2}{64} = 1$$



"PLOT WHAT YOU'VE GOT!"

B) foci: (± 2, 0); major axis is 12.

$2a = 12$

$a = 6$

$a^2 = 36$

center: (0, 0)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

* major axis is horizontal = a^2 with x^2

need b^2

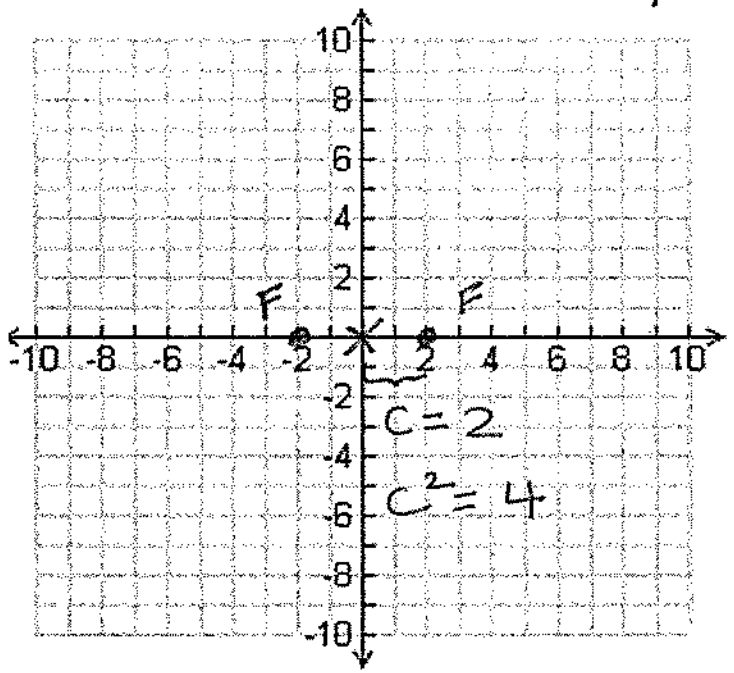
$c^2 = a^2 - b^2$

$4 = 36 - b^2$

$b^2 = 36 - 4$

$b^2 = 32$

$$\frac{x^2}{36} + \frac{y^2}{32} = 1$$



"PLOT WHAT YOU'VE GOT!"

C) foci: (-3, 4) and (-3, 0); major axis is 8

$2a = 8$

$a = 4$

$a^2 = 16$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$h = -3$, $k = 2$

* major axis is vertical $= a^2$ with y^2

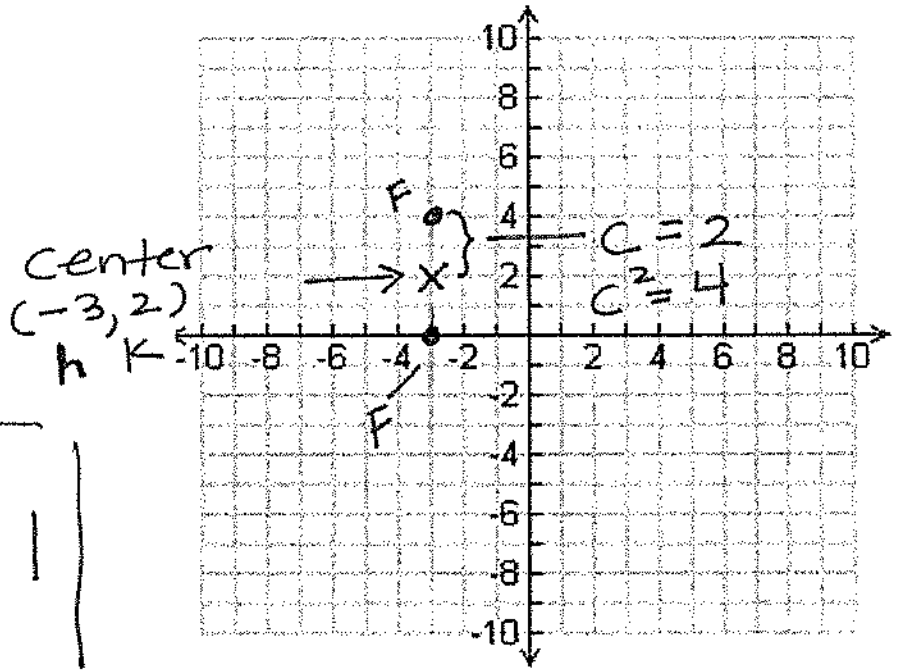
need b^2

$c^2 = a^2 - b^2$

$4 = 16 - b^2$

$b^2 = 16 - 4$

$b^2 = 12$



$$\boxed{\frac{(x+3)^2}{12} + \frac{(y-2)^2}{16} = 1}$$