Circles: Sec. 1.9

Notes

pos. pos

General Form: $Ax^2 + Bx + Ay^2 + Cy + D = 0$

same number (both positive)

Standard form: $(x-h)^2 + (y-k)^2 = r^2$

$center : (h, k)$

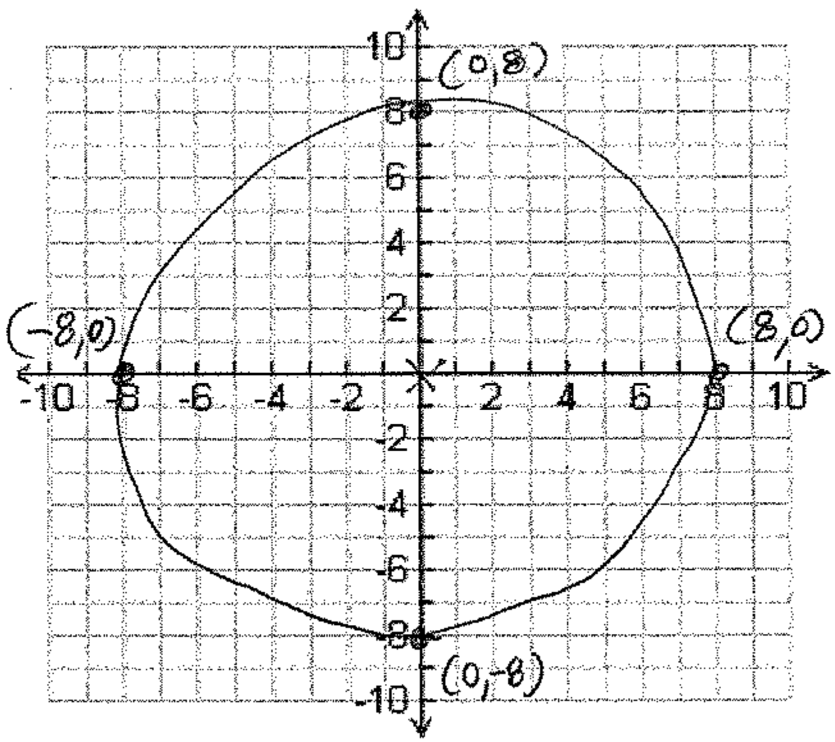
radius squared

1) Write the standard form of the equation of the circle with the given center and radius.

A) Center (0,0); r = 8

h k

$$\boxed{x^2 + y^2 = 64}$$

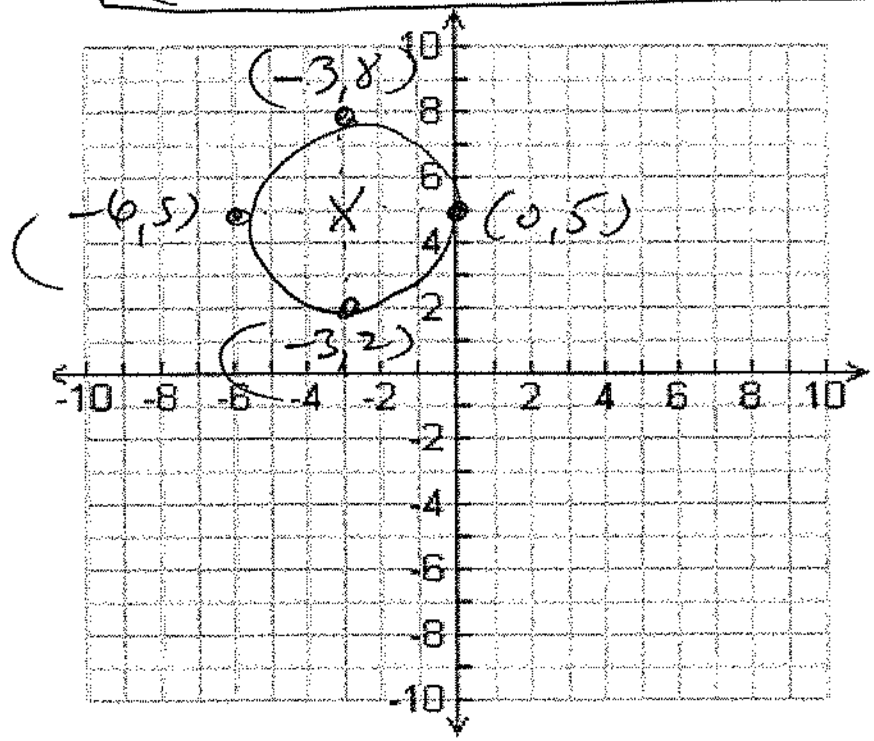


B) Center (-3,5); r = 3

h k

$$(x-(-3))^2 + (y-5)^2 = (3)^2$$

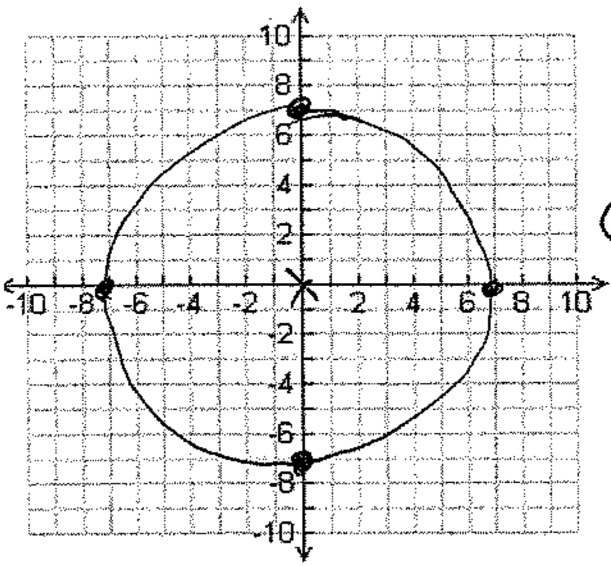
$$\boxed{(x+3)^2 + (y-5)^2 = 9}$$



2) Give the center and radius of the circle described by the equation and graph each equation.

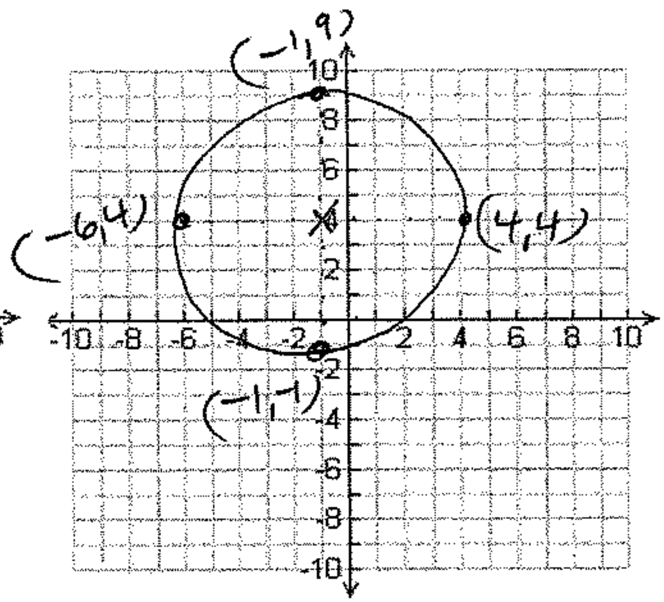
A) $x^2 + y^2 = 49$

$\boxed{\begin{array}{c} C: (0,0) \\ \hline r = 7 \end{array}}$



B) $(x+1)^2 + (y-4)^2 = 25$

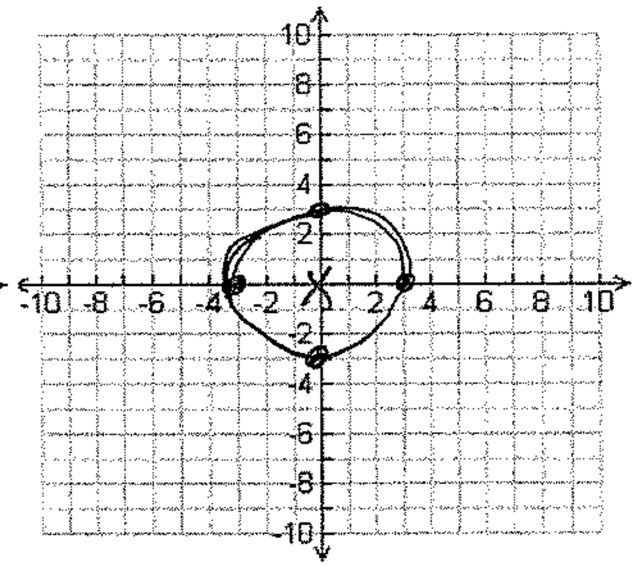
$\boxed{\begin{array}{c} C: (-1,4) \\ \hline r = 5 \end{array}}$



C) $\frac{3x^2}{3} + \frac{3y^2}{3} = \frac{27}{3}$

$x^2 + y^2 = 9$

$\boxed{C: (0,0) \mid r = 3}$



3) The following circle equations are in general form. Complete the square and write the equation in standard form. Then give the center and radius of each circle and graph the equation.

A) $x^2+y^2+8x+4y+16=0$

Notice that the coefficients on the x^2 and y^2 are the same, and are positive....this info. Tells us we have a circle.

Now reorder the equation so the x and y terms are together in descending order. Move the constant to the other side of the equation. Then complete the square on each variable.

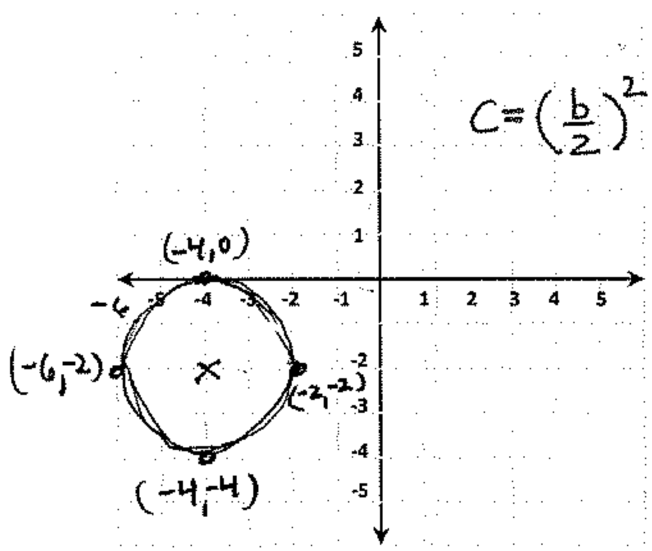
$(1x^2+8x+16)+(1y^2+4y+4)=-16+16+4$

$c=\frac{+8}{2}$ $\quad (+4)^2$ $\qquad c=\frac{+4}{2}$ $\quad (+2)^2$

$c=\left(\frac{b}{2}\right)^2$

$\boxed{(x+4)^2+(y+2)^2=4}$

$\boxed{\text{center: } (-4,-2) \quad r=2}$



D: [-6,-2]

R: [-4,0]

B) $x^2 + y^2 - 6y - 7 = 0$

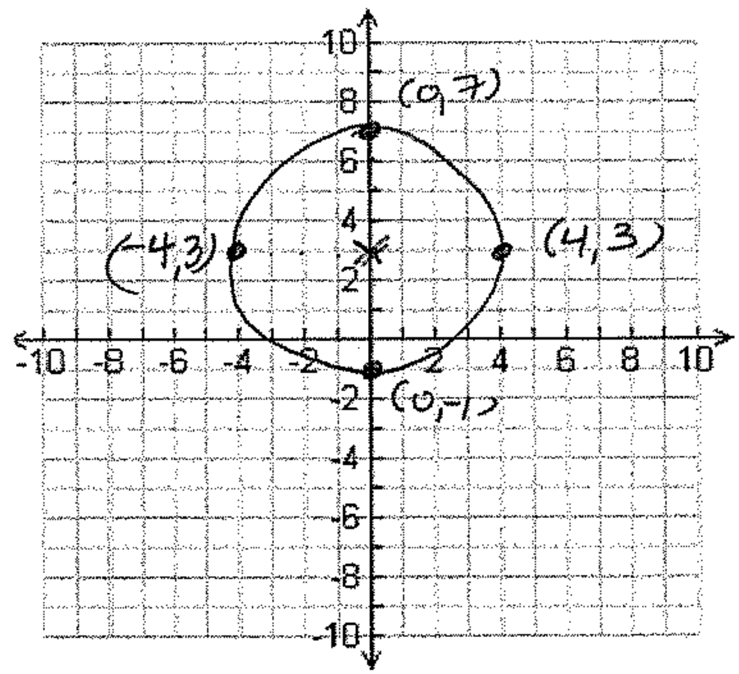
$x^2 + (y^2 - 6y + 9) = 7 + 9$

$c = \left(\frac{-6}{2}\right)^2 = (-3)^2$

$\boxed{x^2 + (y-3)^2 = 16}$

$\boxed{\begin{array}{c} C: (0,3) \\ \hline r = 4 \end{array}}$

$\boxed{\begin{array}{l} D: [-4, 4] \\ R: [-1, 7] \end{array}}$



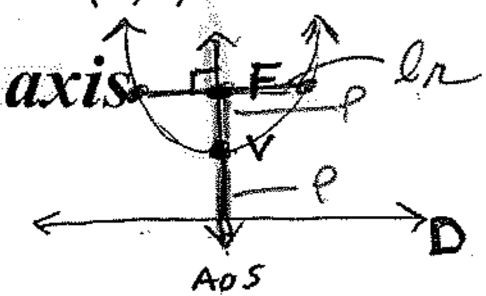
Parabolas: Sec. 9.3

LOF
$y=x^2$

Standard form of the equation of a parabola with vertex (h,k) is:

$$(x-h)^2 = \underline{4p}(y-k), p \neq 0$$

vertical axis



directrix : y = k − p   focus : (h, k + p)

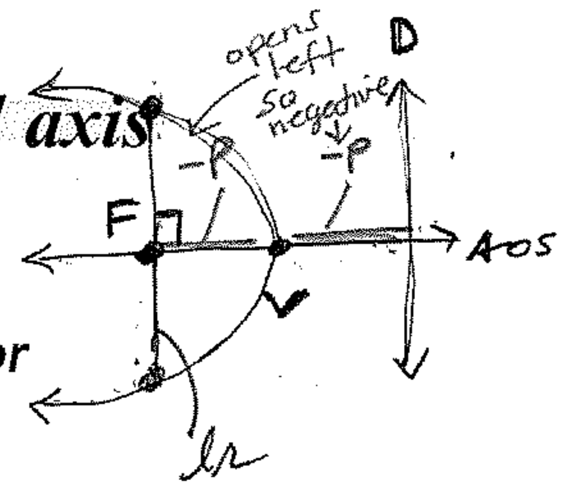
or ⊕ ⊖

sign on 4p

or

$$(y-k)^2 = \underline{4p}(x-h), p \neq 0$$

horizontal axis



directrix : x = h − p   focus : (h + p, k)

or ⊖ ⊕

sign on 4p

P is the directed distance from the vertex to the focus ***or*** from the directrix to the vertex.

The parabola can not pass through the directrix.

Latus Rectum = |4p| Forms a right angle to the axis of symmetry.

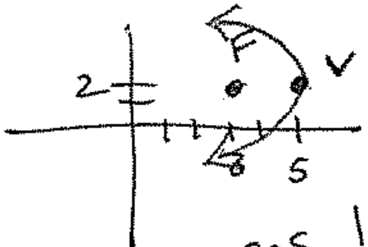
passes through the focus.

"PLOT WHAT YOU'VE GOT!!!"

Ex.1) Find the standard form of the equation of the parabola with the given information. Sketch the parabola

a) Vertex: (5,2) focus: (3,2)

h K



opens left

x^2 or y^2

$$(y-\overset{(2)}{k})^2 = 4\overset{(-2)}{p}(x-\overset{(5)}{h})$$

$$\boxed{(y-2)^2 = -8(x-5)}$$

$|r = |4p|$

$= |-8|$

$\boxed{|r = 8}$

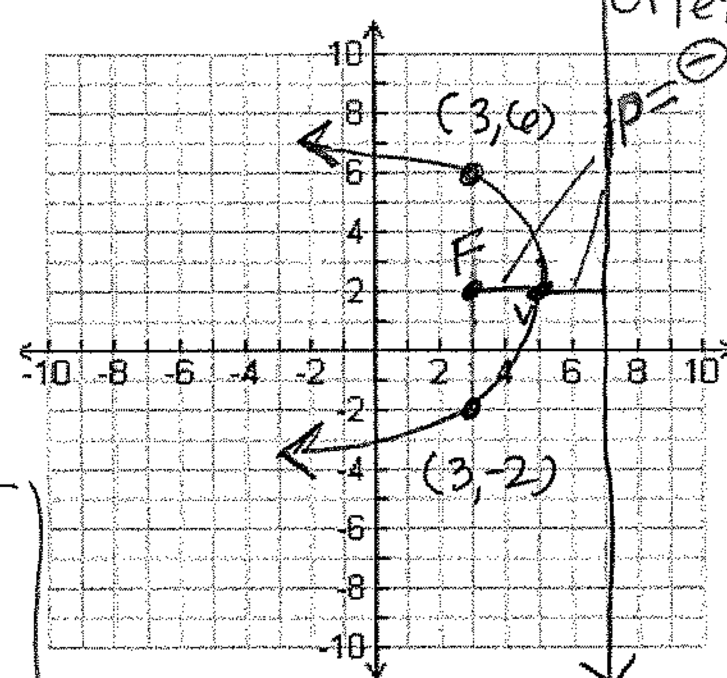
$$\boxed{\begin{array}{l} D: (-\infty, 5] \\ R: (-\infty, \infty) \end{array}}$$

Directrix

$\boxed{x = 7}$

opens left

$p = -2$



$\boxed{\begin{array}{l} AOS \\ y = 2 \end{array}}$

"PLOT WHAT YOU'VE GOT!!!"

b) Vertex: (- 2,1) directrix: x = 1

h K

Vertical

$(y-k)^2 = 4p(x-h)$

opens left

$(y-1)^2 = 4(-3)(x-(-2))$

$\boxed{(y-1)^2 = -12(x+2)}$

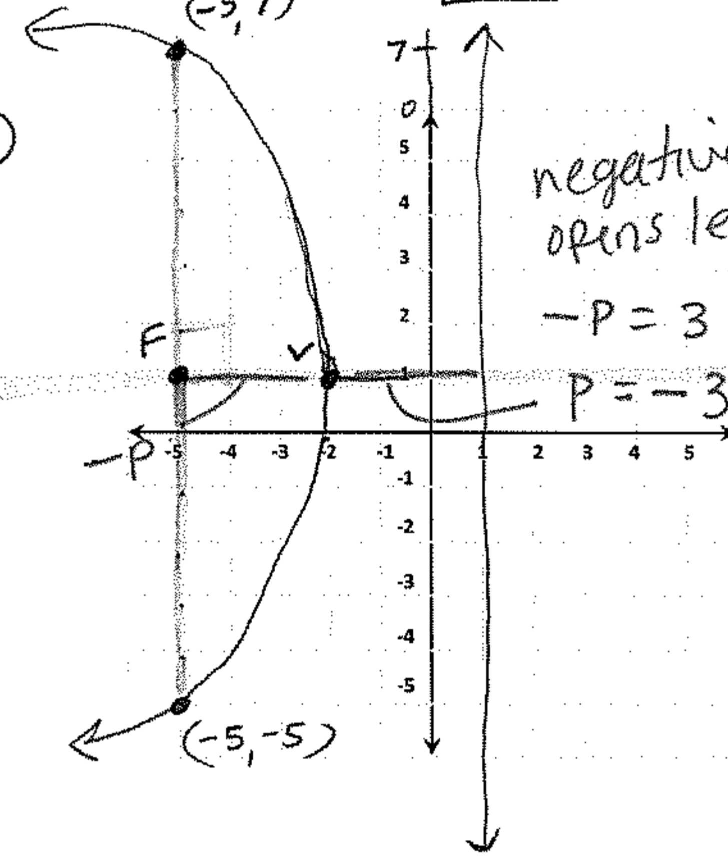
$\boxed{\text{Focus: } (-5,1)}$

$lr = |4p|$

$= |-12|$

$\boxed{lr = 12}$

D: $\boxed{x = 1}$



negative b/c opens left

$-p = 3$

$p = -3$

$\boxed{\begin{array}{l} \text{A.o.s.} \\ y = 1 \end{array}}$

$\boxed{\begin{array}{l} D: (-\infty, -2] \\ R: (-\infty, \infty) \end{array}}$

"PLOT WHAT YOU'VE GOT!!!"

Ex.2) Find the vertex, focus, directrix, and latus rectum of the parabola and sketch the graph.

$y^2 = 3x$

or

opens right

$y^2 = 3(x)$

positive $4p$

$\boxed{\text{Vertex: } (0,0)}$

$4p = 3$

$\boxed{p = \frac{3}{4}}$

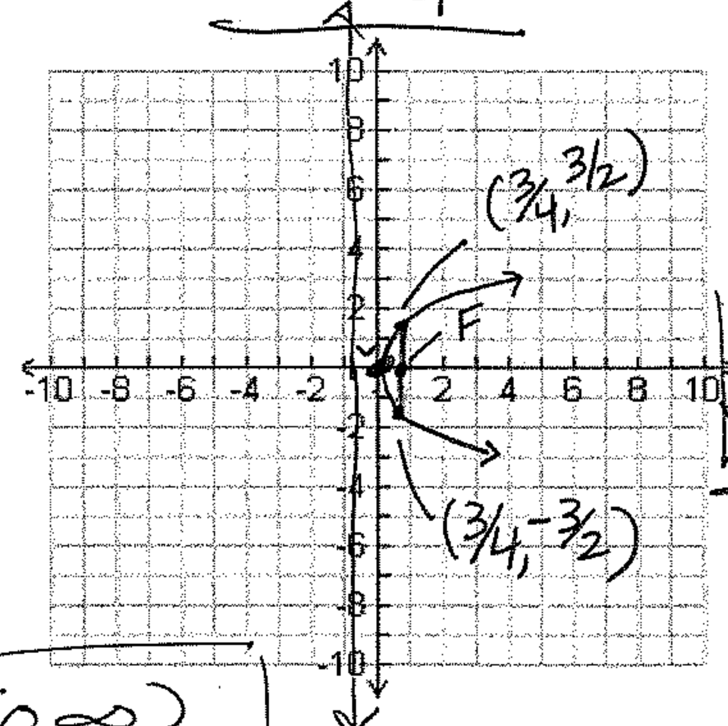
$\boxed{\text{Focus: } (\frac{3}{4}, 0)}$

$lr = |4p|$

$= |3|$

$\boxed{lr = 3}$

$\boxed{\text{Directrix } x = -\frac{3}{4}}$



$\boxed{\text{AOS } y = 0}$

$\boxed{D: [0, \infty) \quad R: (-\infty, \infty)}$

↺ or ↻

Ex4) $y^2+2y+12x-23=0$ need to change to $(y-k)^2=4p(x-h)$

$(1y^2+2y+\underline{\underline{1}})=-12x+23\ \underline{\underline{+1}}$

$c=\left(\frac{+2}{2}\right)^2=(+1)^2 \quad = \underbrace{-12}_{\text{factor}}x \ \underline{+24}$

↺ or ↻ opens left

$\boxed{(y+1)^2=-12(x-2)}$

negative (opens left)

$4p=-12$

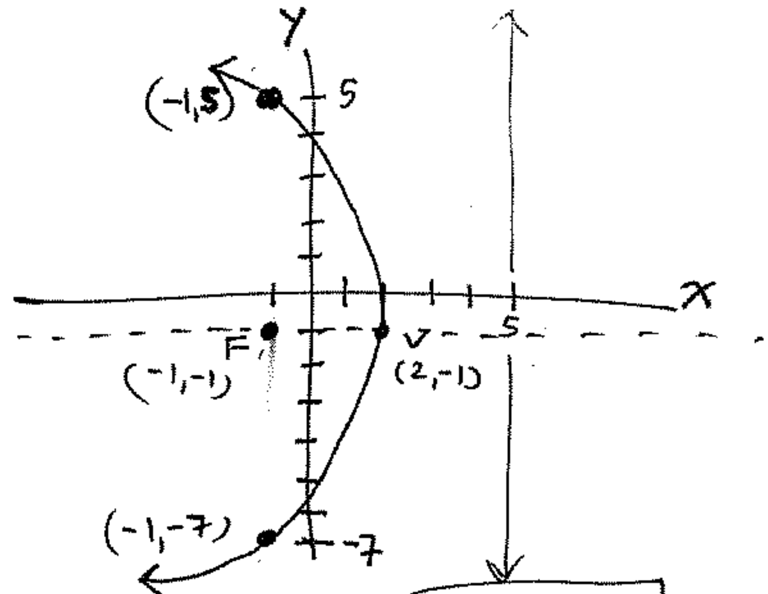
$\boxed{p=-3}$

* $\boxed{V:(2,-1)}$

$\boxed{F:(-1,-1)}$

$lr=|4p|=|-12|$

$\boxed{lr=12 \text{ total}}$



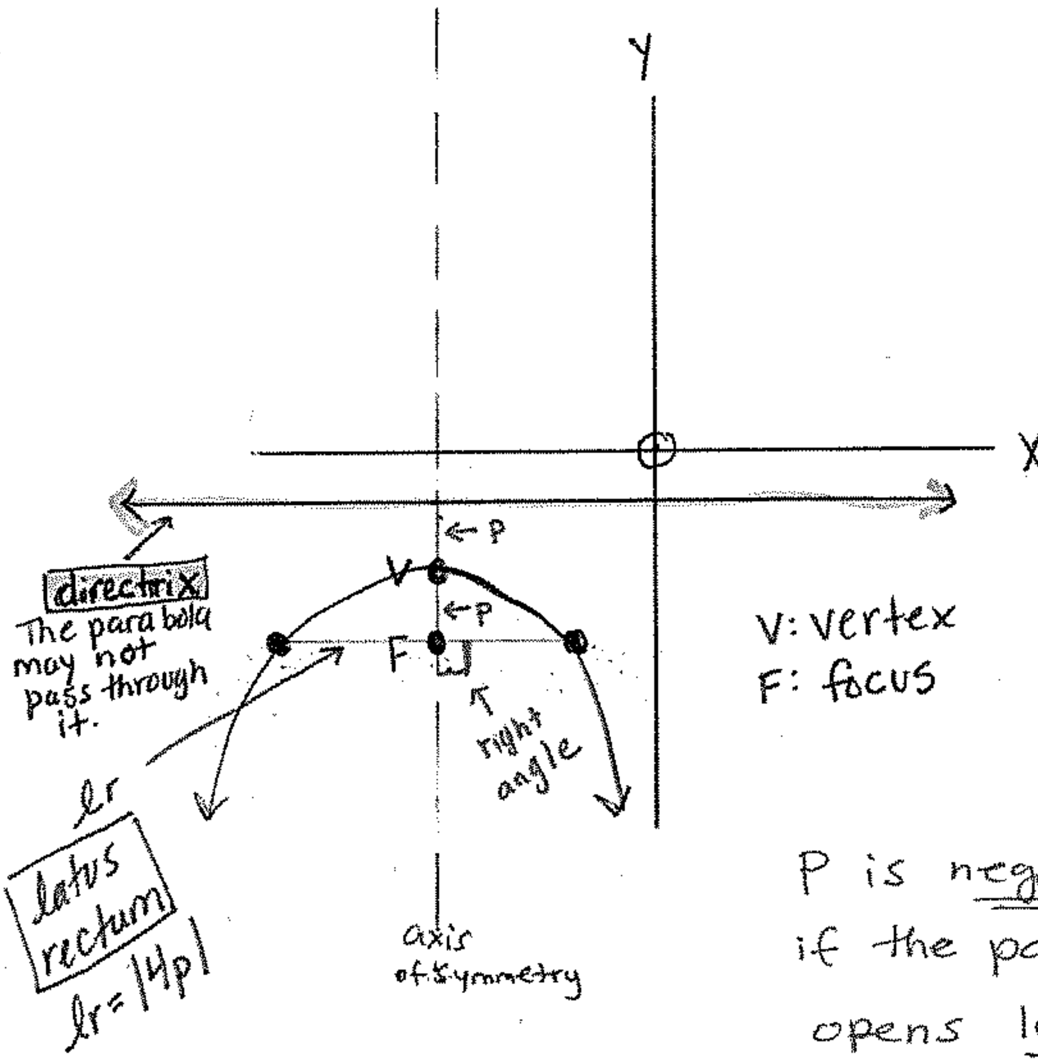
A.O.S.
$y=-1$

Directrix
$x=5$

$D:(-\infty,2]$

$R:(-\infty,\infty)$

Parabolas



p is __negative__ if the parabola opens __left__ or __down__.