Notes

Sec. 4.7 Inverse Trigonometric Functions

Regular Trig. Function:

$\sin\theta = \text{value}$

Inverse Trig. Function:

$\sin^{-1}(\text{value}) = \theta$

$\arcsin(\text{value}) = \theta$ ← angle as answer

Inverse is used to find the measure of an angle.

ie. $\sin 30^\circ = \boxed{\tfrac{1}{2}}$ ⇒ $\sin^{-1}(\tfrac{1}{2}) = \boxed{30^\circ}$ or $\boxed{\frac{\pi}{6}}$

Value as answer (ratio)

Angle as answer

Trig. Function: | Inverse Trig. Functions:

$\underline{\underline{y}} = \tan \underline{\underline{x}}$ $\longrightarrow$ $y = \tan^{-1} x$

or Arctan x

CH1: Inverse
switch x ; y

$x = \tan y$

now solve for "y"

$y = \tan^{-1} x$

Read as "The inverse of tangent."

Means: tan y = x.

This can be done for all $\underline{\underline{6}}$ trig. functions.

Graphing Inverse Functions:

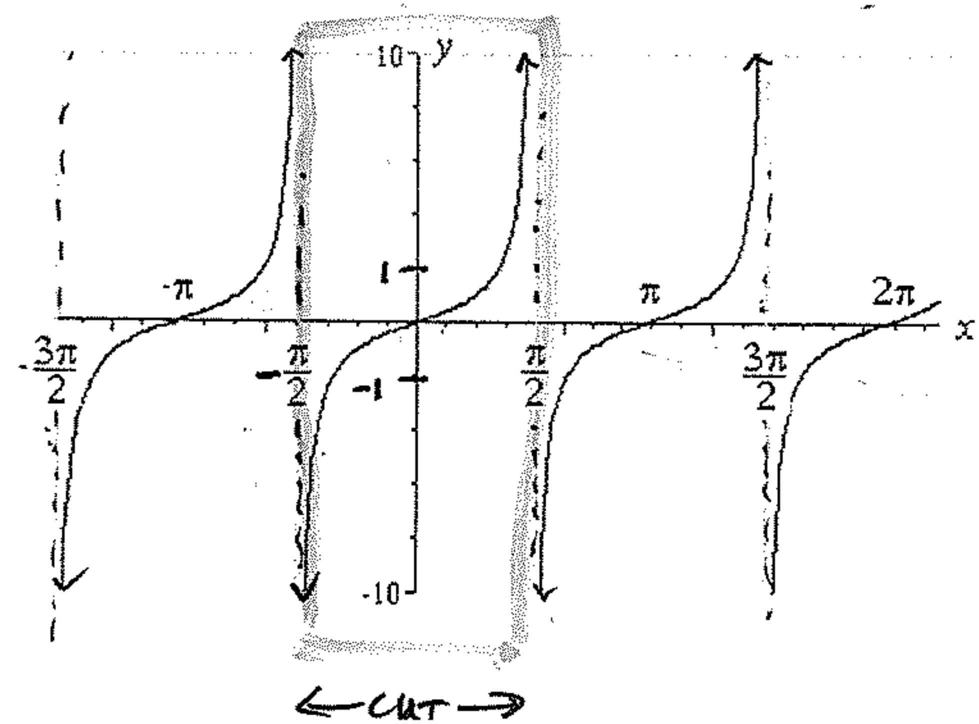
Recall: 1) x & y coordinates switch.
2) reflect about the line $\boxed{y = x.}$
3) domain and range interchange.

CH1

Graphing: $\boxed{y = \tan^{-1} x}$

1st: Let's look at the graph of y = tan x ← not inverse

To graph the inverse, the function must be one-to-one



Domain:

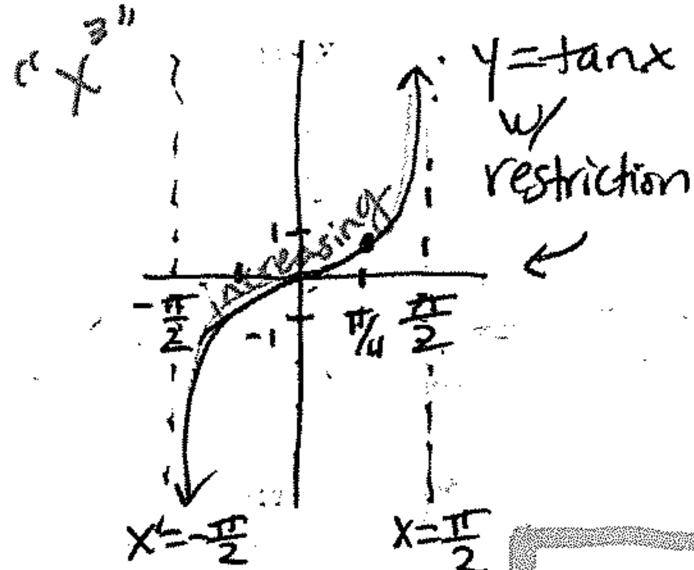
$(-\frac{\pi}{2}, \frac{\pi}{2})$ now one-to-one w/restriction

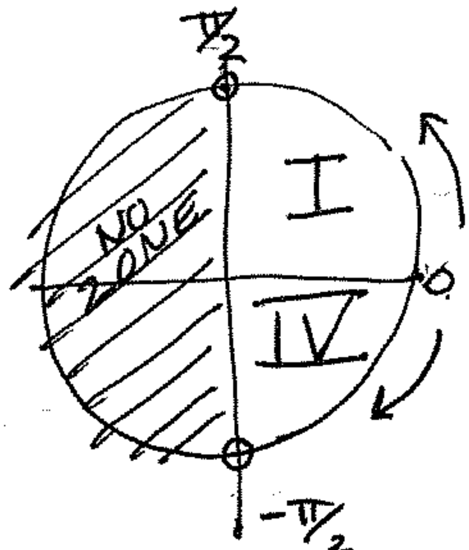
to pass VLT & HLT.

Range:

$(-\infty, \infty)$

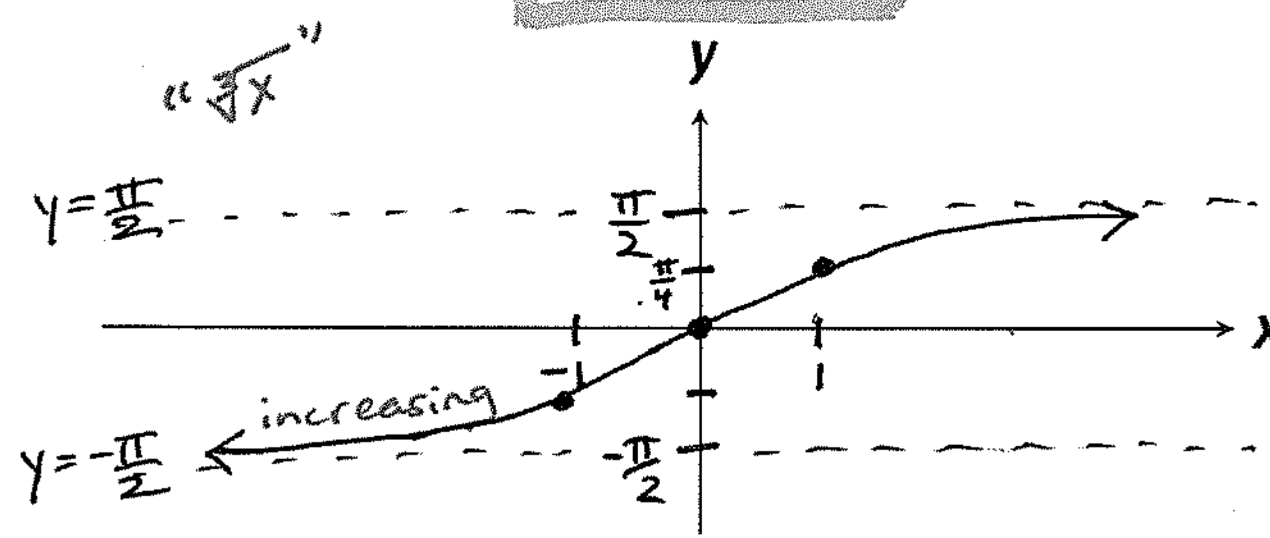
2nd: Since tan graph is not one-to-one, we need to restrict the domain. Take one "Chunk"!!! Now it is one-to-one.





$\tan x$:	$\rightarrow \tan^{-1} x$:
$(0,0)$	$(0,0)$
$(\frac{\pi}{4}, 1)$	$(1, \frac{\pi}{4})$
$(-\frac{\pi}{4}, -1)$	$(-1, -\frac{\pi}{4})$

3rd: **To graph $\boxed{y = \tan^{-1} x}$.**

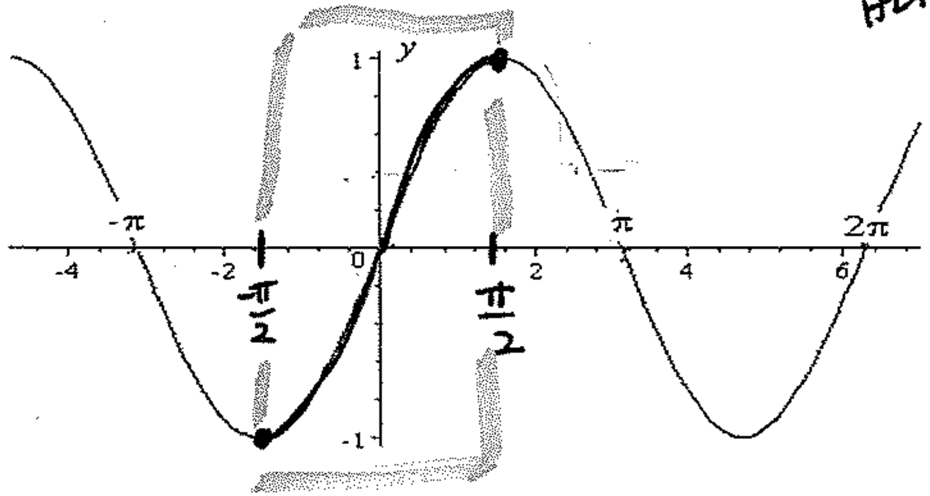


Domain:
$(-\infty, \infty)$

Range:
$(-\frac{\pi}{2}, \frac{\pi}{2})$

Graphing: $\boxed{y = \sin^{-1} x}$

1st: graph of $y = \sin x$ ← not inverse Fails HLT

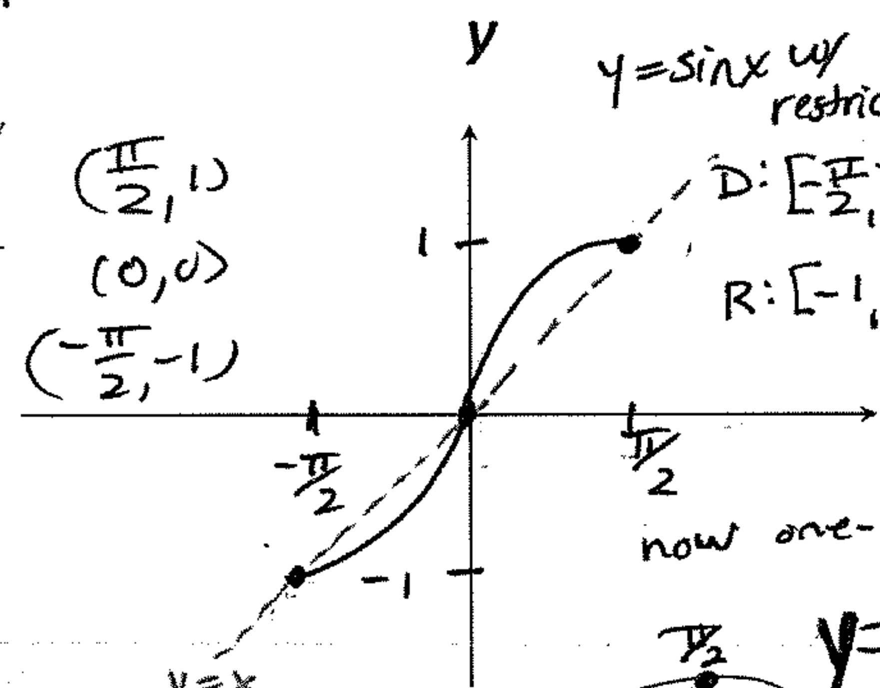


2nd: Cut a Chunk!!

$(\frac{\pi}{2}, 1)$

$(0, 0)$

$(-\frac{\pi}{2}, -1)$



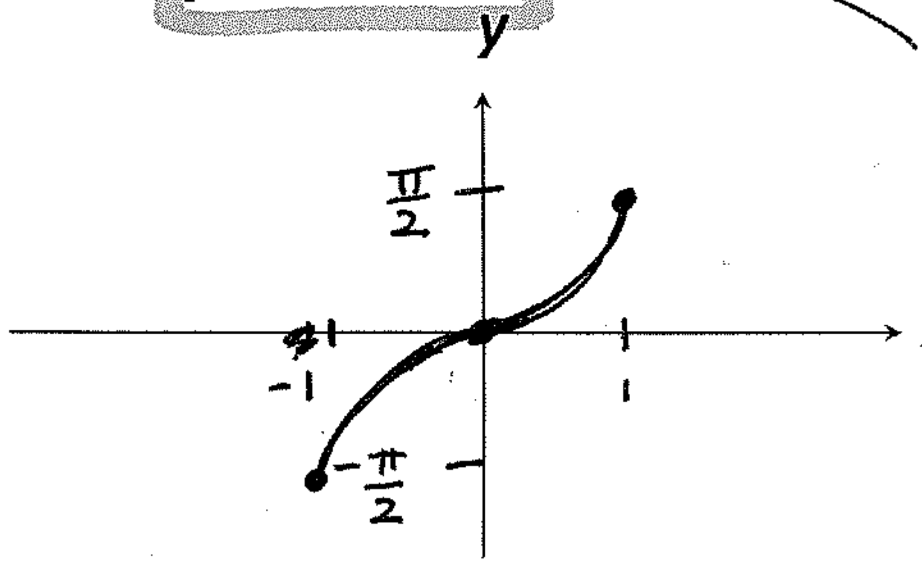
y = sin x w/ restriction

D: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

R: $[-1, 1]$

now one-to-one

3rd: $\boxed{y = \sin^{-1} x}$



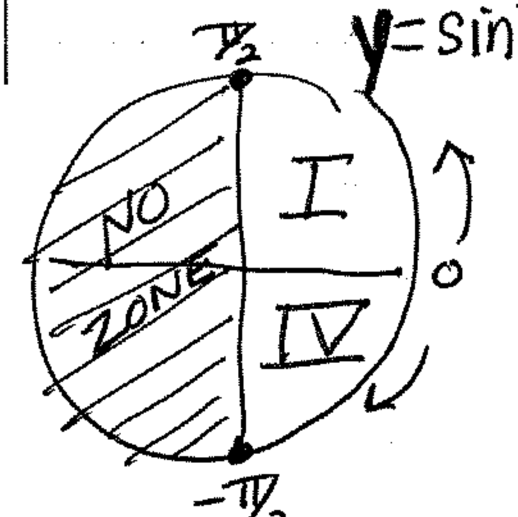
Domain:

$[-1, 1]$

Range:

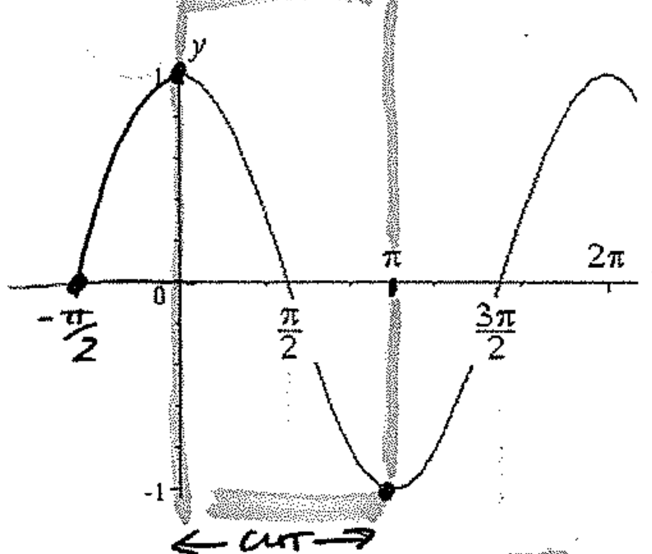
$[-\frac{\pi}{2}, \frac{\pi}{2}]$

y = $\sin^{-1}$ x



Graphing: $\boxed{y = \cos^{-1} x}$

1st: graph of $y = \cos x$ ← not inverse, fails HLT



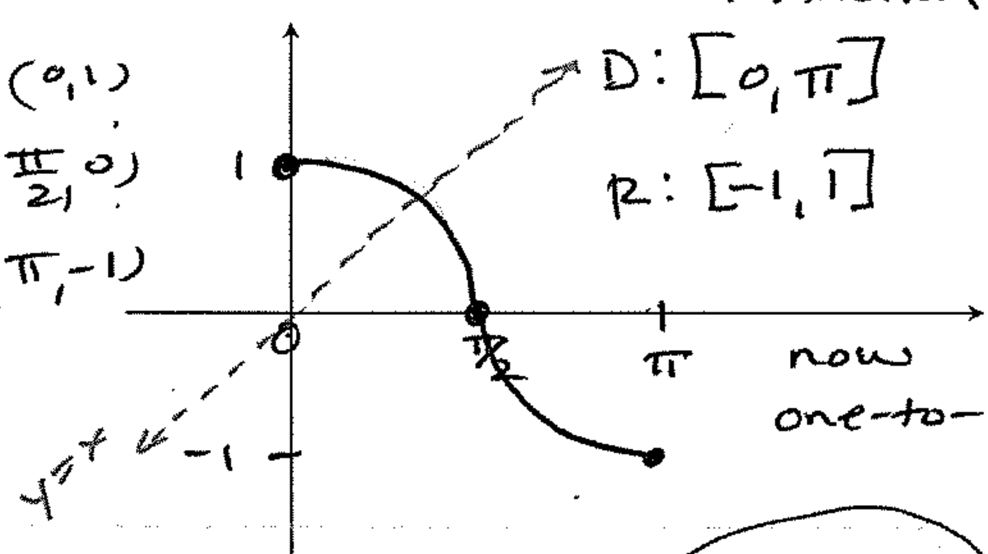
2nd: Cut a Chunk!!

$y = \cos x$ w/ restriction

(0, 1)
$(\frac{\pi}{2}, 0)$
$(\pi, -1)$

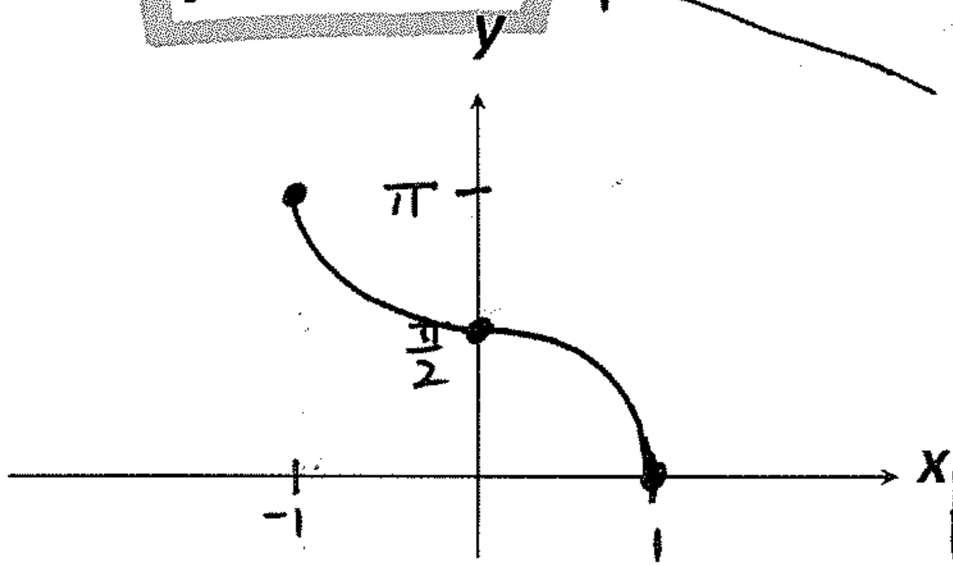
D: $[0, \pi]$

R: $[-1, 1]$



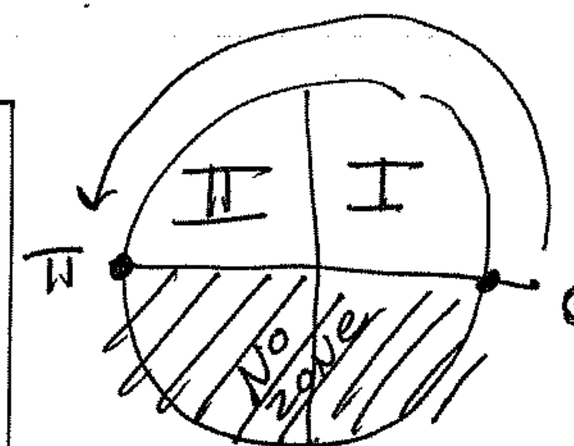
now one-to-one

3rd: $\boxed{y = \cos^{-1} x}$



Domain:
$[-1, 1]$

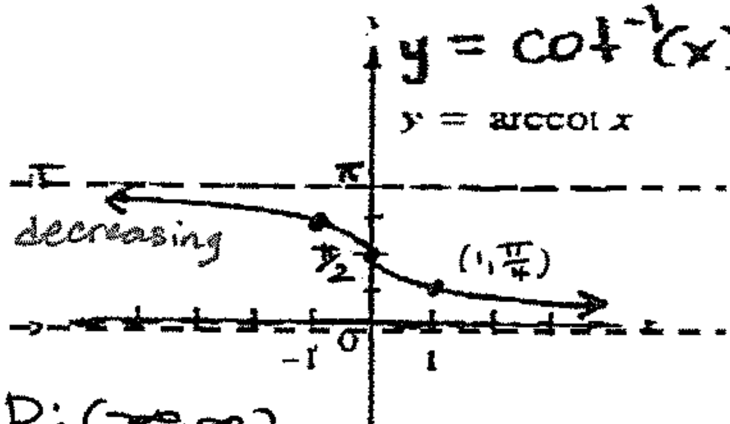
Range:
$[0, \pi]$



$y = \cos^{-1} x$

Graphs of the Other Inverse Trig. Functions

$y = \cot^{-1} x$



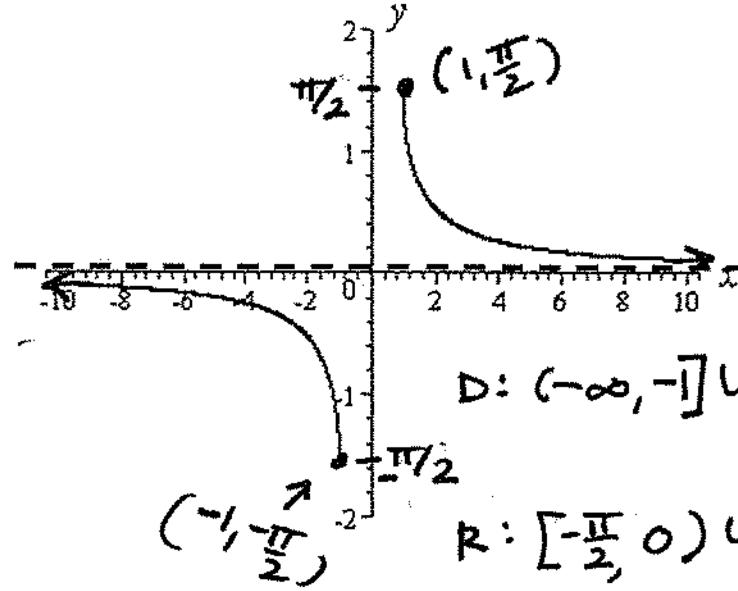
$y = \pi$

$y = 0$

D: $(-\infty, \infty)$

R: $(0, \pi)$

$y = \csc^{-1} x$

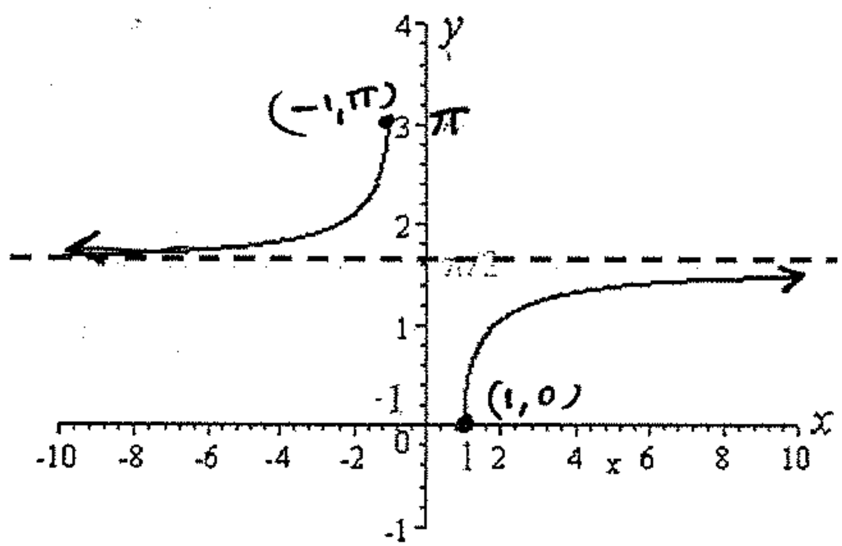


$y = 0$

D: $(-\infty, -1] \cup [1, \infty)$

R: $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$

$y = \sec^{-1} x$



$y = \pi/2$

D: $(-\infty, -1] \cup [1, \infty)$

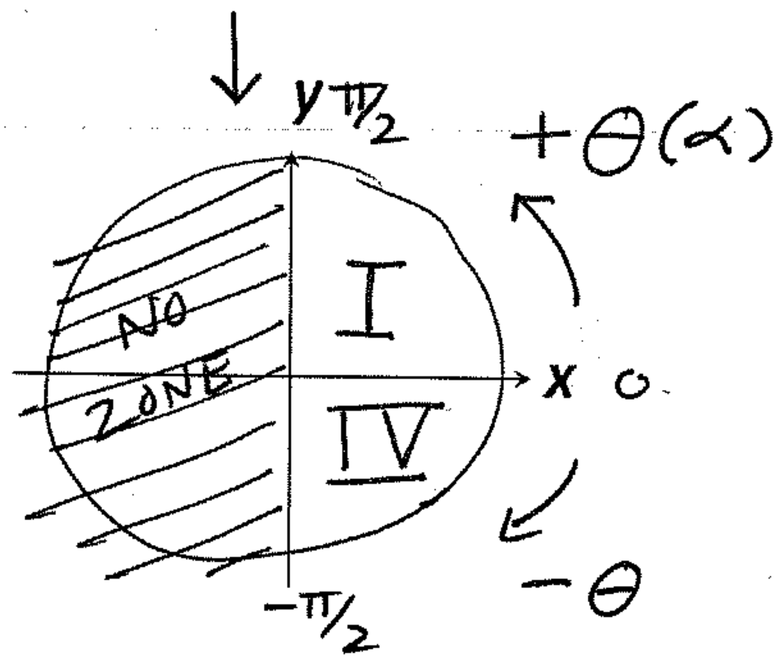
R: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

Since inverse trig. functions are used to find the measure of angles, let's look at the restrictions for each when expressed on the coordinate plane.

$y = \sin^{-1} x$

$y = \csc^{-1} x$

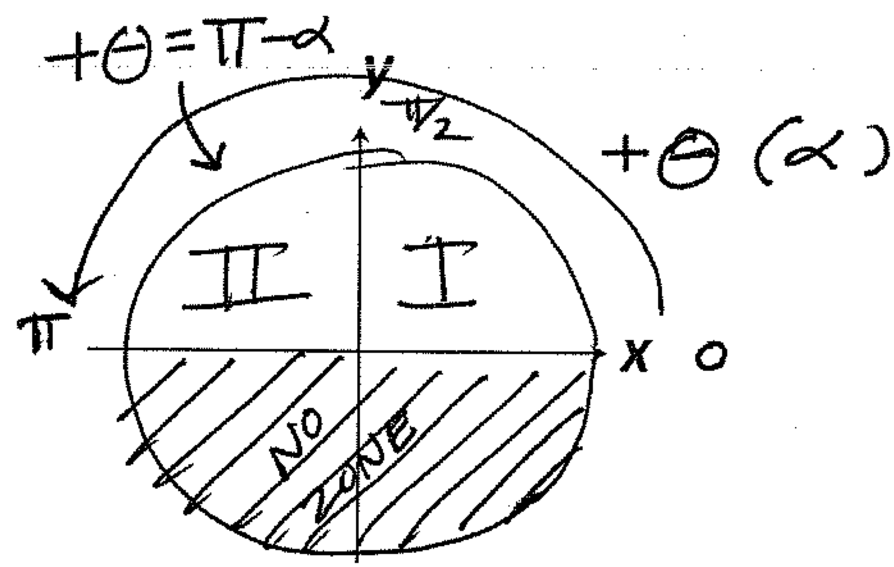
$y = \tan^{-1} x$



$y = \cos^{-1} x$

$y = \sec^{-1} x$

$y = \cot^{-1} x$



Ex. 1: Find the angle for each inverse function. (no calc.)

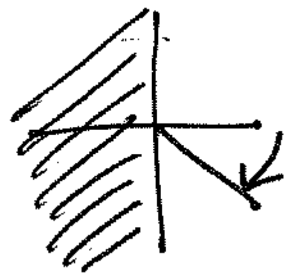
a) $\tan^{-1}\frac{\overset{+}{\sqrt{3}}}{3} = \boxed{\frac{\pi}{6}}$

not QA

$\alpha = \frac{\pi}{6}$

b) $Arc\sin\left(\underset{\text{neg}}{-}\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{\pi}{3}}$

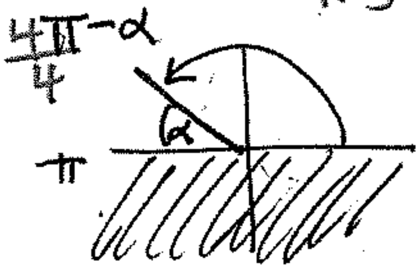
not QA



$\alpha = \frac{\pi}{3}$

NOT $\cancel{\frac{5\pi}{3}}$

c) $\cos^{-1}\left(\underset{\text{neg}}{-}\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$

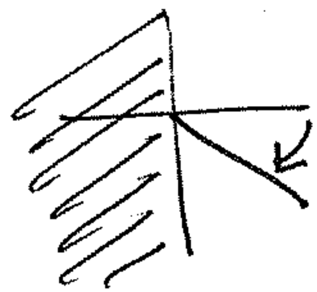


not QA

$\alpha = \frac{\pi}{4}$

d) $Arc\tan(\underset{\text{neg}}{-}1) = \boxed{-\frac{\pi}{4}}$

QA (not for tanx)



$\alpha = \frac{\pi}{4}$

NOT $\cancel{\frac{7\pi}{4}}$

Ex. 2: Without calculator, find...

Ⓐ $\underbrace{\tan\left(\sin^{-1}\left(-\frac{3}{7}\right)\right)}_{\theta} = \boxed{\frac{-3\sqrt{10}}{20}}$

not inverse

ratio

$\sin^{-1}\left(-\frac{3}{7}\right) = \theta$ angle

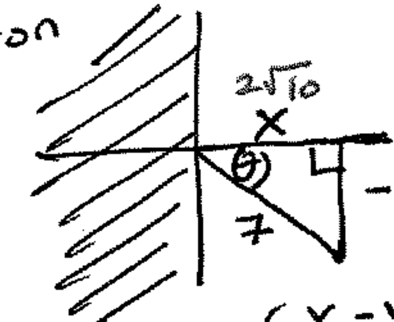
Don't recognize so you need a right △

So ... $\sin\theta = -\frac{3}{7}$

"SOH"

neg quad IV

restriction



$(x, -y)$

$(x, -3)$

$\tan\theta = \frac{-3}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$

"TOA"

10

$x^2 + y^2 = r^2$

$x^2 + 9 = 49$

$x^2 = 40$

$x = \oplus\sqrt{40}$

4 10

Quad IV

$x = 2\sqrt{10}$

Ⓑ $\cos^{-1}\left(\tan\frac{3\pi}{4}\right) = \boxed{\pi}$

angle

$\cos^{-1}(-1)$

QA

S A
T C

$\tan\frac{3\pi}{4}$

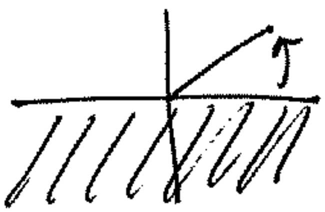
$= -\tan\frac{\pi}{4}$

$= -1$

Ex. 3: Evaluate. (no calc.)

a) $\sec^{-1}(2) = \boxed{\frac{\pi}{3}}$ or $\boxed{60^\circ}$

not QA



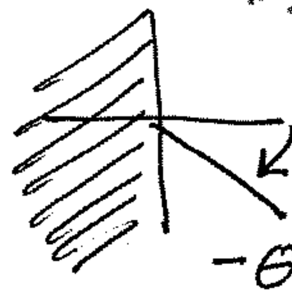
use $\cos\theta = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$

b) $\csc^{-1}\left(-\frac{2\sqrt{3}}{3}\right) = \boxed{-\frac{\pi}{3}}$

neg

no QA



use

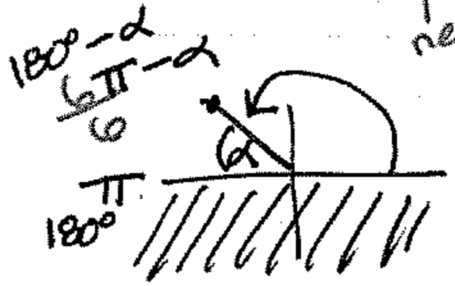
$\sin\theta = -\frac{\sqrt{3}}{2}$

$\alpha = \frac{\pi}{3}$

c) $\cot^{-1}(-\sqrt{3}) = \boxed{\frac{5\pi}{6}}$ or $\boxed{150^\circ}$

neg

not QA



use

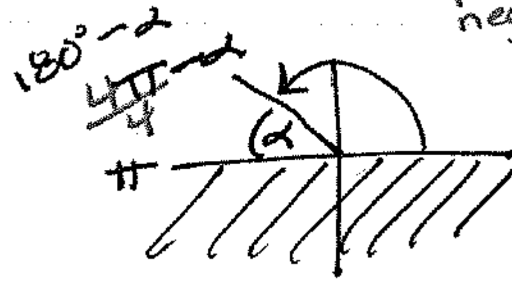
$\tan\theta = -\frac{\sqrt{3}}{3}$

$\alpha = \frac{\pi}{6}$ or 30°

d) $\sec^{-1}(-\sqrt{2}) = \boxed{\frac{3\pi}{4}}$ or $\boxed{135^\circ}$

neg

no QA



use

$\cos\theta = -\frac{\sqrt{2}}{2}$

$\alpha = \frac{\pi}{4}$ or 45°

Ex. 4: Without a calculator, find the exact value of...

a) $\cos(\underbrace{\csc^{-1}(-1.5)}_{\theta}) = \boxed{\frac{\sqrt{5}}{3}}$

not inverse

$\cos\theta = \frac{\sqrt{5}}{3}$

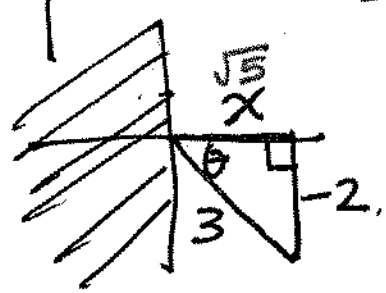
"CAH"

$\theta = \csc^{-1}(-1.5)$

$\theta = \csc^{-1}\left(-\frac{3}{2}\right)$

Don't recognize
need right ▲

use $\sin\theta = -\frac{2}{3}$ "SOH"



$x^2+y^2=r^2$

$x^2+4=9$

$x^2=5$

$x = \pm\sqrt{5}$

QIV

b) $\sin(\underbrace{\cot^{-1}(-0.5)}_{\theta}) = \boxed{\frac{2\sqrt{5}}{5}}$

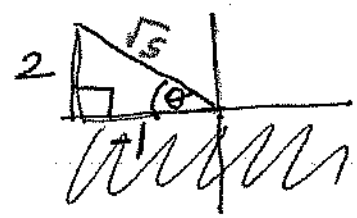
not inverse

$\sin\theta = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

"SOH"

$\theta = \cot^{-1}(-0.5)$

neg



so ... $\cot\theta = -0.5$

$= -\frac{5}{10}$

$\cot\theta = -\frac{1}{2}$

then $\tan\theta = -\frac{2}{1}$

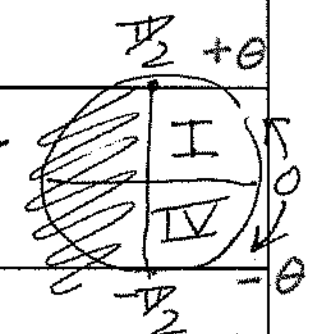
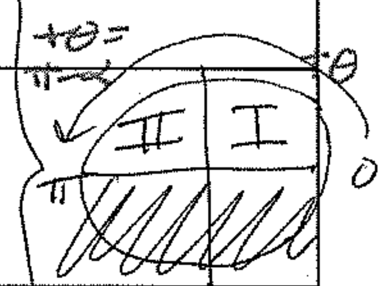
"TOA"

$x^2+y^2=r^2$

$1+4=r^2$

$5=r^2 \rightarrow r=\sqrt{5}$

Summary of the **Inverse** Trigonometric Functions

Function	Domain	Range	Quadrant of Range	No Zone
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	I and IV	
$y = \csc^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$	I and IV	
$y = \tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	I and IV	
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	I and II	
$y = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$	I and II	
$y = \cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$	I and II	

Ex. 5) *Use a calculator to find the value of the following: Express answer in degrees. Round to 3 decimal places.*

a) $\sin^{-1}(-0.852) = \theta$

$\theta \approx \boxed{-58.430^\circ}$

b) $\sec^{-1}(-1.325) = \theta$

so... $\sec\theta = -1.325$

$\cos\theta = -\frac{1}{1.325}$

↑ same angle ↑ flip value

$\theta = \cos^{-1}\left(-\frac{1}{1.325}\right)$

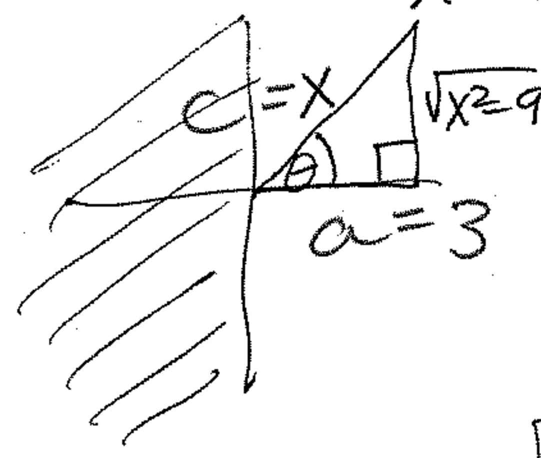
$\boxed{\theta \approx 139.001^\circ}$

Ex6) Write the trigonometric expression as an algebraic expression.

$$\underline{\cot}\Big(\underbrace{\sin^{-1}\frac{\sqrt{x^2-9}}{x}}_{\theta}\Big) = \boxed{\frac{3\sqrt{x^2-9}}{x^2-9}}$$

not an inverse

$$\theta = \sin^{-1}\frac{\sqrt{x^2-9}}{x}$$



$$\cot\theta = \frac{3}{\sqrt{x^2-9}} \cdot \frac{\sqrt{x^2-9}}{\sqrt{x^2-9}}$$

use $\tan\theta = \dfrac{\sqrt{x^2-9}}{3}$
"TOA"

So... $\sin\theta = \dfrac{\sqrt{x^2-9}}{x}$ $\begin{matrix}\text{opp}\\ \text{hyp}\end{matrix}$
"SOH"

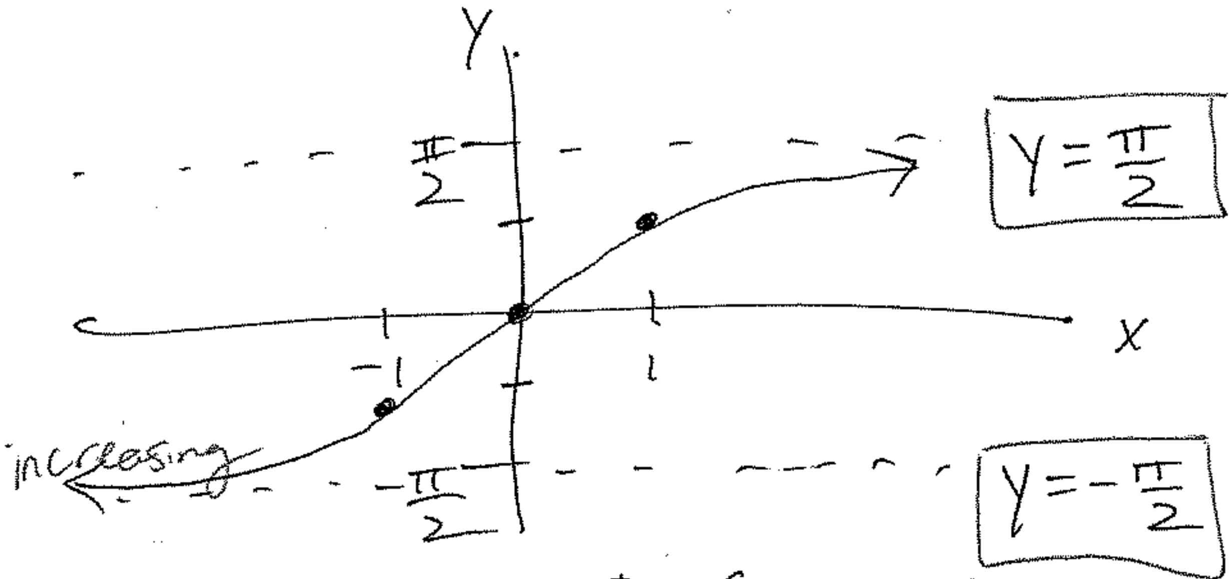
$$a^2+b^2=c^2$$
$$a^2+\left(\sqrt{x^2-9}\right)^2=(x)^2$$
$$a^2+\cancel{x^2}-9=\cancel{x^2}$$
$$-\cancel{x^2} \qquad -\cancel{x^2}$$
$$a^2-9=0 \qquad a^2=9 \qquad a=\pm 3$$

Reivew Again:

$y = \tan^{-1} x$

$x^3 \leftrightarrow \sqrt[3]{x}$

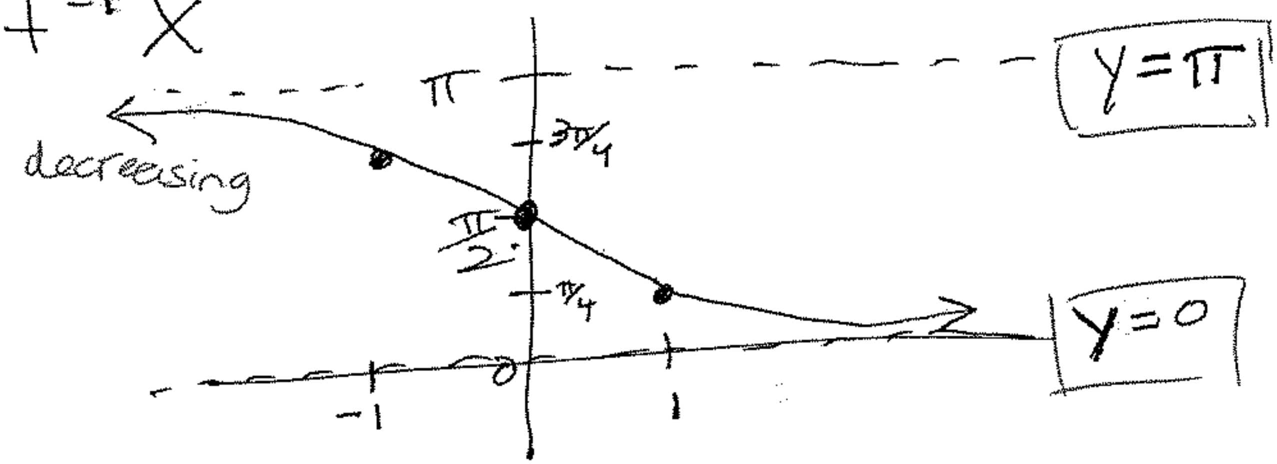
inverses



D: $(-\infty, \infty)$

R: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$y = \cot^{-1} x$



D: $(-\infty, \infty)$

R: $(0, \pi)$