Sec. 4.7 Inverse Trigonometric Functions

Regular Trig. Function: Inverse Trig. Function:

$\sin \theta = \text{value}$ $\sin^{-1}(\text{value}) = \theta$

Inverse is used to find the measure of an angle.

ie. $\sin 30^\circ = \frac{1}{2}$  $\sin^{-1}(\frac{1}{2}) = 30^\circ$

Trig. Function:

$y = \tan x$

Inverse Trig. Functions:

$y = \tan^{-1} x$

or Arctan x

Read as "The inverse of tangent."

Means: tan y = x.

This can be done for all 6 trig. functions.

Graphing Inverse Functions:

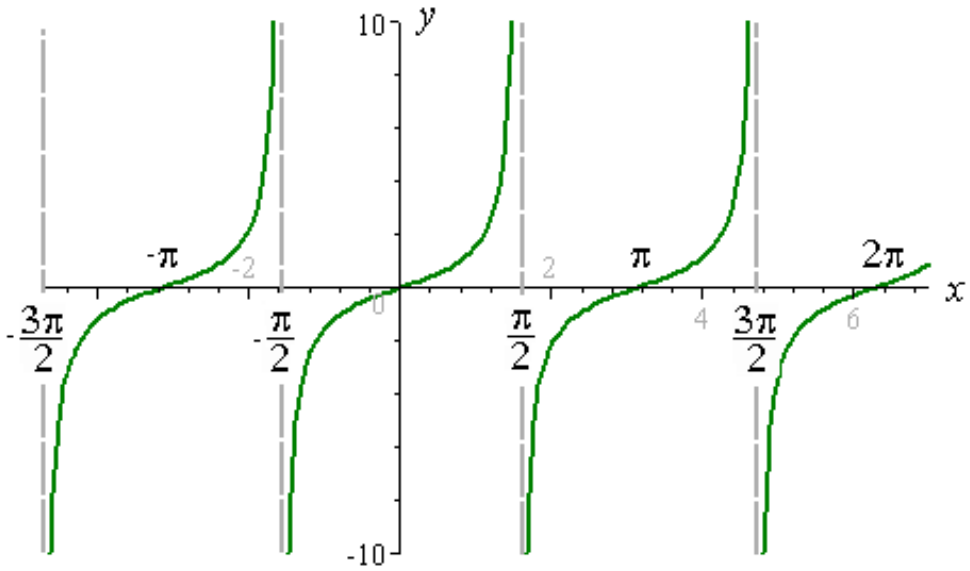
Recall: 1) x & y coordinates switch.

2) reflect about the line y = x.

3) domain and range interchange.

Graphing: $y = \tan^{-1} x$

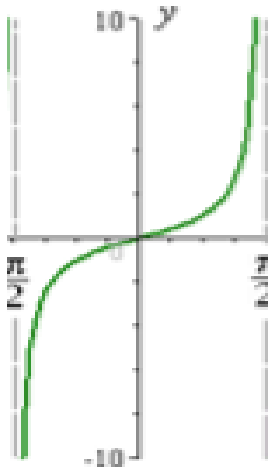
1st: Let's look at the graph of y = tan x



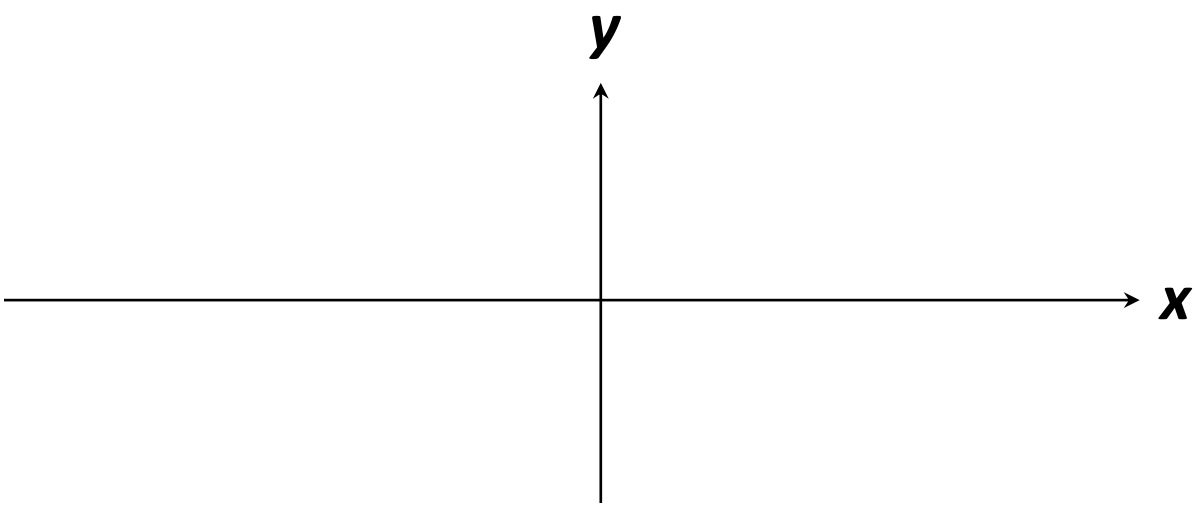
Domain:

Range:

2nd: Since tan graph is not one-to-one, we need to restrict the domain. Take one "Chunk"!!! Now it is one-to-one.



3rd: **To graph $y = \tan^{-1} x$.**

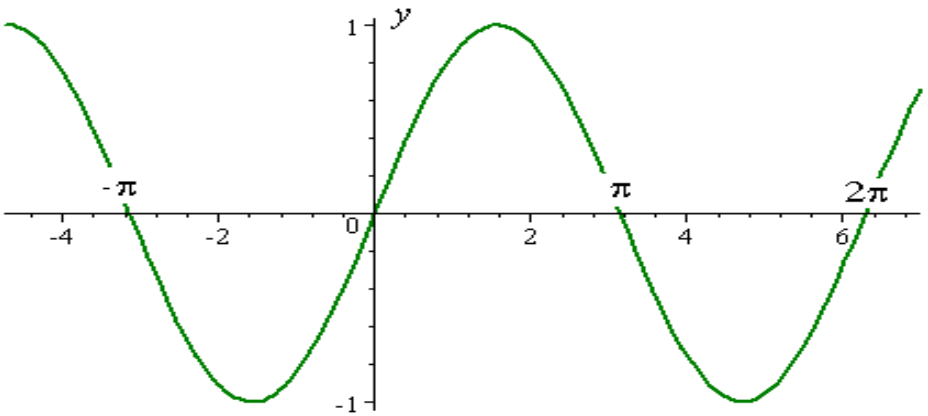


Domain:

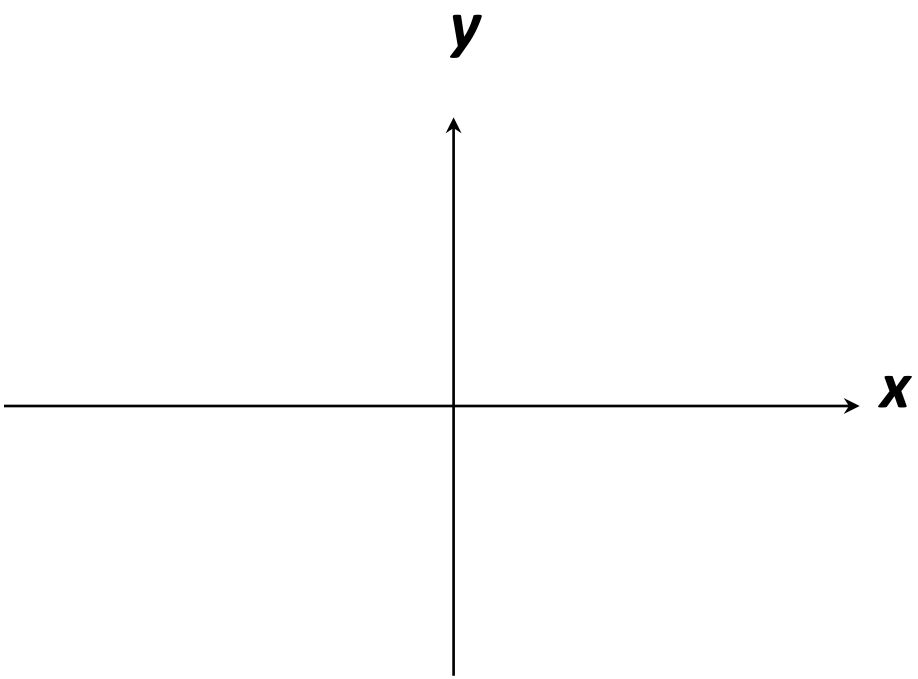
Range:

Graphing: $y = \sin^{-1} x$

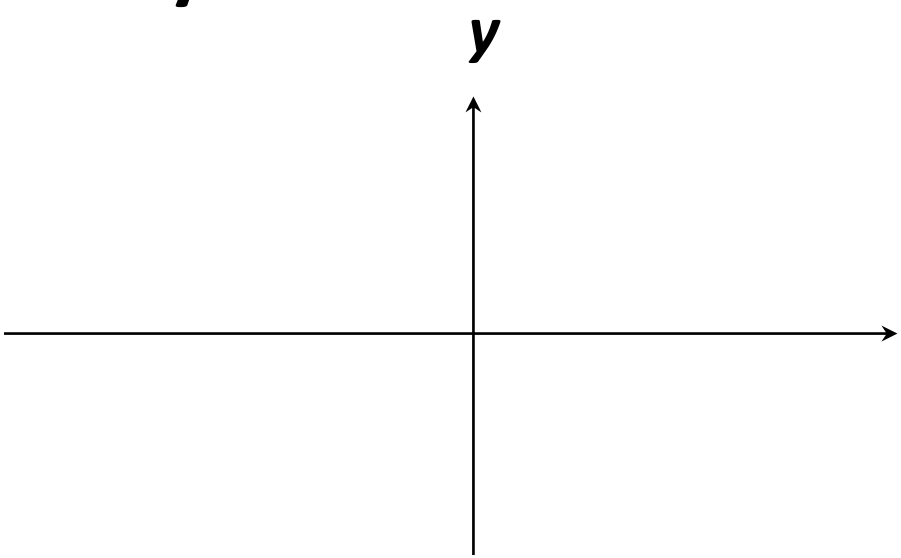
1st: graph of $y = \sin x$



2nd: Cut a Chunk!!



3rd: $\mathbf{y = \sin^{-1} x}$

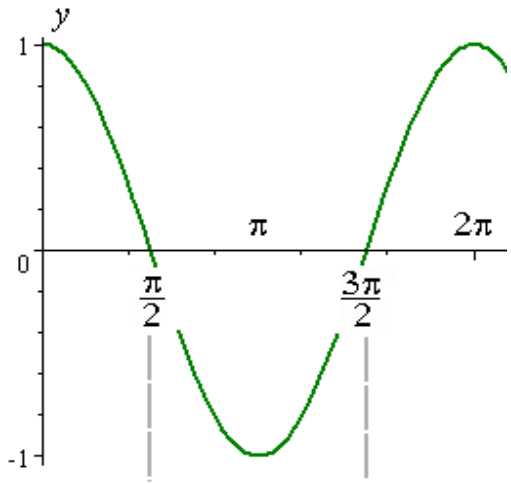


Domain:

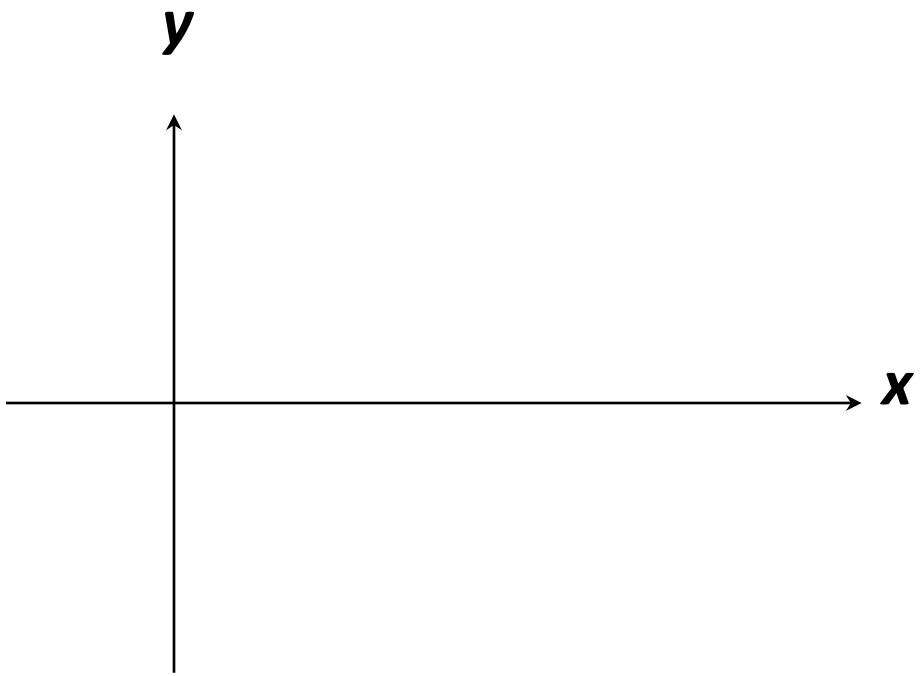
Range:

Graphing: $y = \cos^{-1} x$

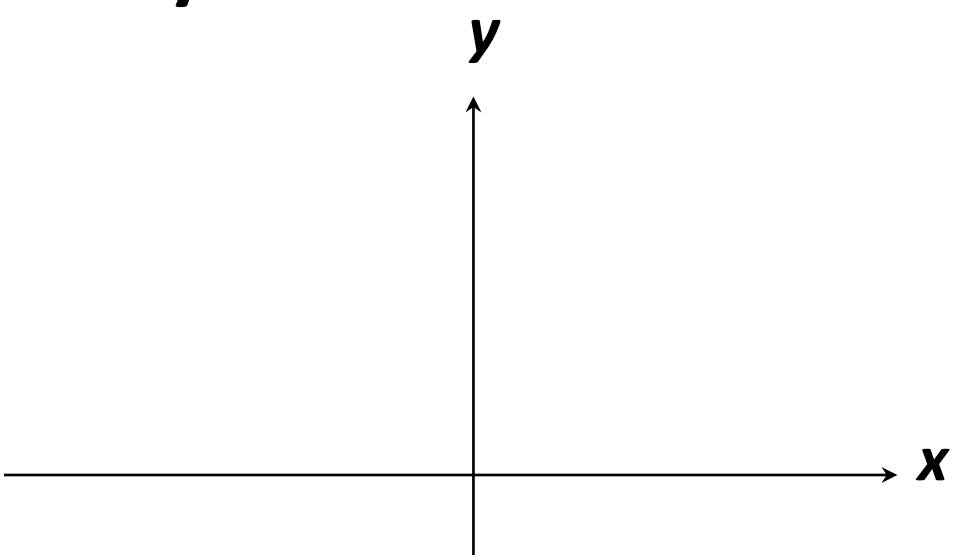
1st: graph of $y = \cos x$



2nd: Cut a Chunk!!



3rd: $\mathbf{y = \cos^{-1} x}$

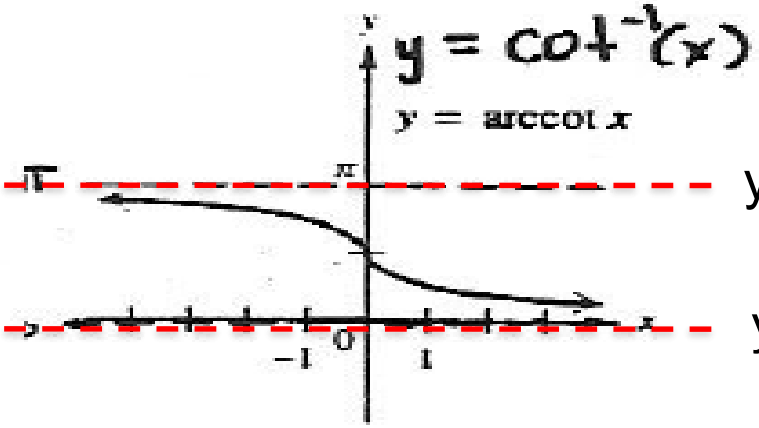


Domain:

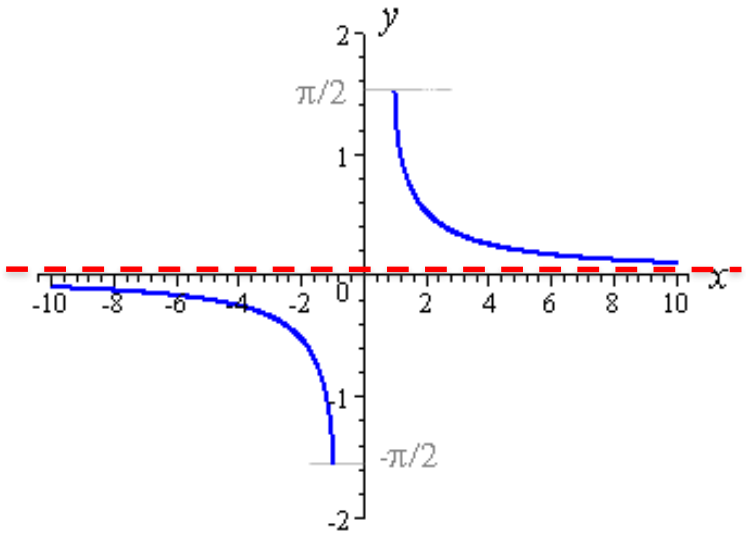
Range:

Graphs of the Other Inverse Trig. Functions

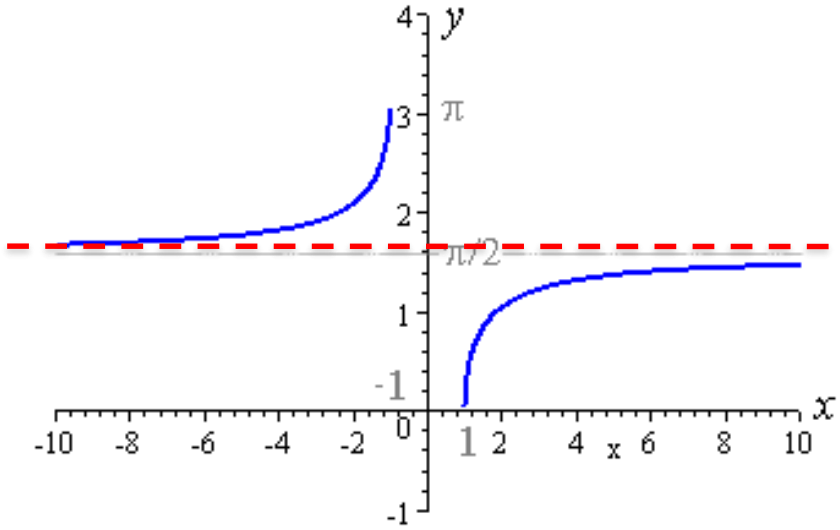
$y = \cot^{-1} x$



$y = \csc^{-1} x$



$y = \sec^{-1} x$



Since inverse trig. functions are used to find the measure of angles, let's look at the restrictions for each when expressed on the coordinate plane.

$y = \sin^{-1} x$

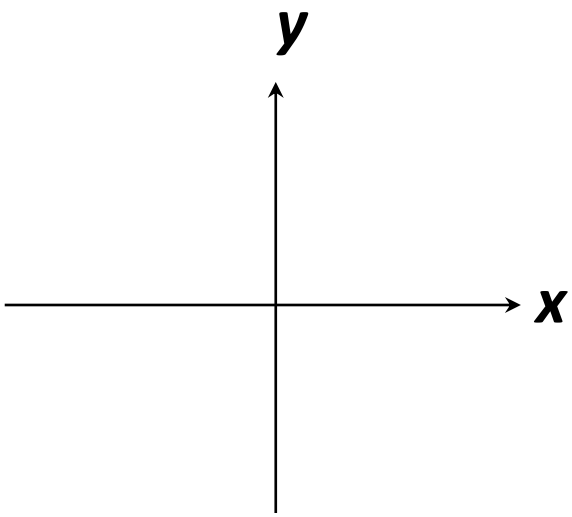
$y = \csc^{-1} x$

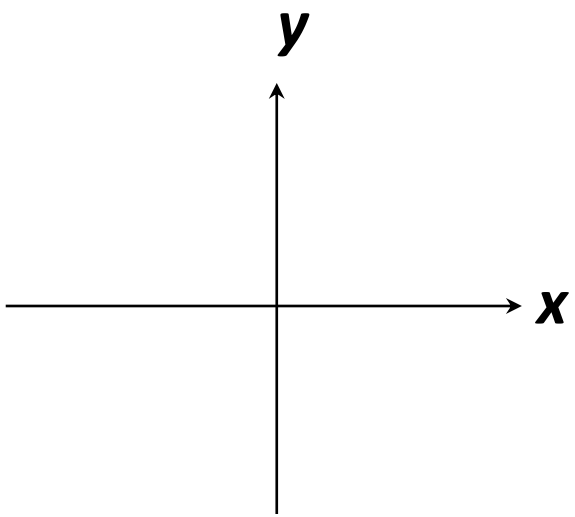
$y = \tan^{-1} x$

$y = \cos^{-1} x$

$y = \sec^{-1} x$

$y = \cot^{-1} x$





Ex. 1) Find the angle for each inverse function. (no calc.)

$\boldsymbol{a)\ \tan^{-1}\dfrac{\sqrt{3}}{3}}$

$b)\ Arc\sin\left(-\dfrac{\sqrt{3}}{2}\right)$

$\boldsymbol{c)\ \cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)}$

$\boldsymbol{d)\ Arc\tan(-1)}$

Ex. 2) Without a calculator, find:

$$\tan\left(\sin^{-1}\left(-\frac{3}{7}\right)\right)$$

Ex. 3) Evaluate. (no calc.)

a) $\mathbf{sec^{-1}(2)}$

b) $\csc^{-1}(-\frac{2\sqrt{3}}{3})$

c) $\mathbf{cot^{-1}(-\sqrt{3})}$

d) $\mathbf{sec^{-1}(-\sqrt{2})}$

Ex. 4) Without a calculator, find the exact value of:

$a)\ \cos(\csc^{-1}(-1.5))$

$b)\ \sin(\cot^{-1}(-0.5))$

Summary of the **Inverse** Trigonometric Functions

Function	Domain	Range	Quadrant of Range	No Zone
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	I and IV	
$y = \csc^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$	I and IV	
$y = \tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	I and IV	
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	I and II	
$y = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$	I and II	
$y = \cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$	I and II	

Ex. 5) *Use a calculator to find the value of the following: Express answer in degrees. Round to 3 decimal places.*

a) $\sin^{-1}(-0.852)$ b) $\sec^{-1}(-1.325)$

Ex. 6) Write the trigonometric expression as an algebraic expression.

$$\cot\left(\sin^{-1}\frac{\sqrt{x^2-9}}{x}\right)$$