Pre-Calculus
Sec. 1.8
Inverse Functions

Definition of the Inverse Function

Given $f(x)$ and $g(x)$:

If $f(g(\boldsymbol{x})) = \boldsymbol{x}$ and $g(f(\boldsymbol{x})) = \boldsymbol{x}$,
then the function g is the **inverse of the function** f.

We use $f^{-1}(x)$ as inverse notation.

The **domain** of f is equal to the **range** of f^{-1}, and vice versa.

A function that has an inverse is also called a
one-to-one Function (passes both the VLT and the HLT).

To find the inverse of a set of points: switch x with y.

ie. {(1, 2), (3, 4)} ⟶ {(2, 1), (4, 3)}

One-to-One Function: a *f* (which already passes the vertical line test) must also pass the horizontal line test (HLT) (Now Y's do not repeat either).

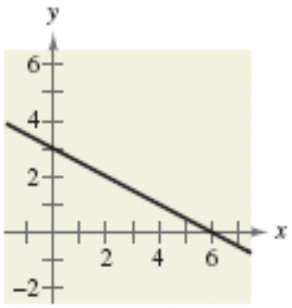
A function *f* has an inverse (f^{-1}), iff it is one-to-one (passes both the VLT and the HLT)

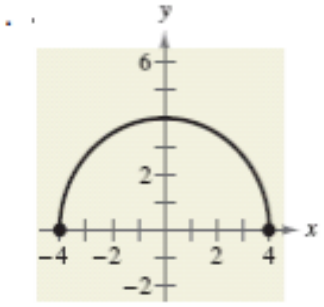
To "**find**" the inverse of an equation, that is a one-to-one function, you will switch *x* & *y*, then solve for *y*.

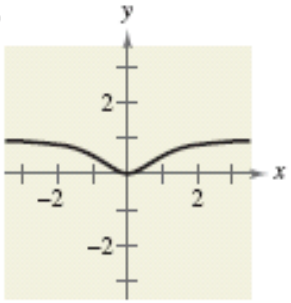
Ex.1

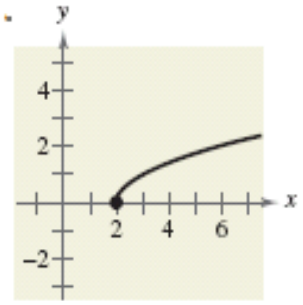
Horizontal Line Test and One-to-One Functions

Part 1 – Does this function have an inverse function? (I.E. is it One-to-One?)









Part 2 – Use a graphing calculator to graph the function and determine if it is One-To-One (I.E. it has an inverse function)

$h(x) = -2x\sqrt{16 - x^2}$

Ex.2

In Exercises 9–14, (a) show that f and g are inverse functions algebraically and (b) use a graphing utility to create a table of values for each function to numerically show that f and g are inverse functions.

10. $f(x) = \dfrac{x - 9}{4}, \quad g(x) = 4x + 9$

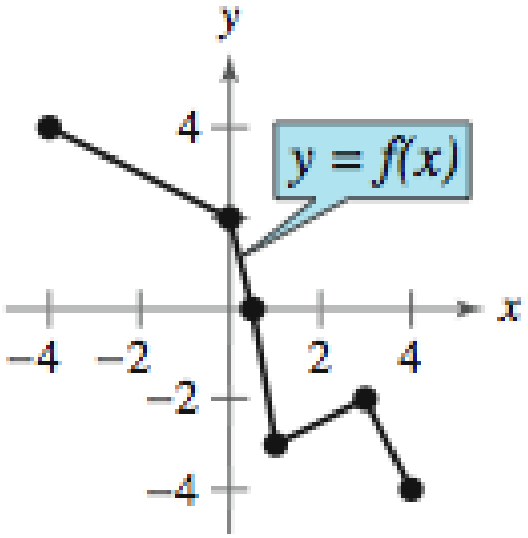
Ex.3

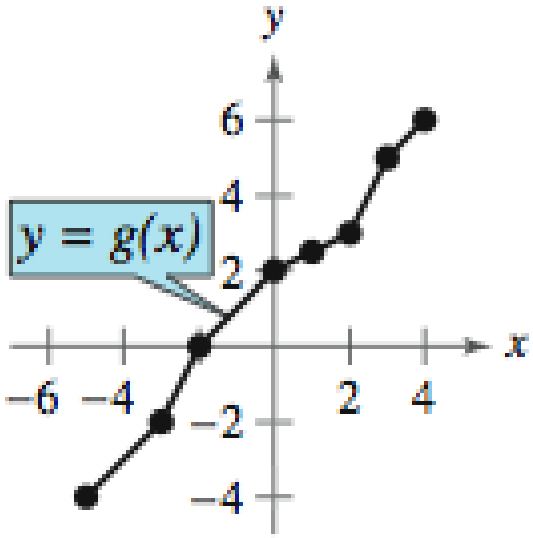
In Exercises 15–20, show that f and g are inverse functions algebraically. Use a graphing utility to graph f and g in the same viewing window. Describe the relationship between the graphs.

18. $f(x) = 9 - x^2, \quad x \geq 0; \quad g(x) = \sqrt{9 - x}$

Ex.4

In Exercises 81–88, use the graphs of $y = f(x)$ and $y = g(x)$ to evaluate the function.





84. $g(f(-4))$

87. $(g \circ f^{-1})(2)$

Ex.5 a)Find the inverse of: $f(x)=\frac{3}{2+5x}$

b) Give the domain and range of $f(x)$ and $f^{-1}(x)$ in interval notation.

Ex.6 a) Find $f^{-1}(x)$ for $f(x)=\dfrac{2x+3}{x-1}$

b) Give the domain and range of $f(x)$ and $f^{-1}(x)$ in interval notation.

Ex.7 a) Find the inverse of $f(x) = \sqrt{5 - x}$

b) Give the domain and range of $f(x)$ and $f^{-1}(x)$ in interval notation.

Ex. 8 Given:

$$f(x)=\frac{1}{8}x-3, \quad g(x)=x^3, \quad h(x)=2x+1$$

a) Find $(f \circ g)^{-1}(5)$

$$f(x)=\frac{1}{8}x-3,\quad g(x)=x^3,\quad h(x)=2x+1$$

b) Find $h(g(f(8)))$