Notes

Lesson 7
PreCalculus
Sec. 1.6
Transformations of Graphs

L/R Up/Down

Horizontal & Vertical Shifts

Given the graph of $f(x)$

$x+c=0$
$x=-c$

$y = f(x + c)$ moves the graph to the **left** c units.

$y = f(x - c)$ moves the graph to the **right** c units.

Horiz. changes x (opposite)

$\boldsymbol{y} = f(\boldsymbol{x}) + c$ moves the graph **upward** c units.

$\boldsymbol{y} = f(\boldsymbol{x}) - c$ moves the graph **downward** c units.

Vertical changes y

parent LOF

Stretching and Shrinking Graphs

Parent

Given the graph of $f(x)$:

$y = c\,f(x)$ **stretches** the graph vertically if $|c| > 1$ and

Vertical

Stretch

shrinks vertically if $0 < |c| < 1$

Shrink

(multiply each of its y-values by c)

$y = f(cx)$ **shrinks** the graph horizontally if $|c| > 1$ and

shrink

inside

stretches horizontally if $0 < |c| < 1$

Stretch

(divide each of its x-values by c)

$x = 1$

$9x = 1$

$x = \frac{1}{9}$

$x = 1$

$\frac{1}{3}x = 1$

$x = 3$

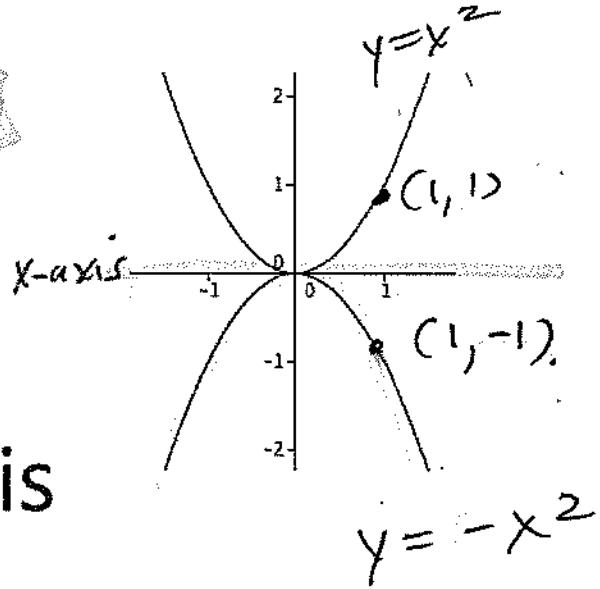
parent

Reflection Over the x-Axis

Given the graph of *f(x)*

y = **−** *f* (***x***) **reflects** the graph over the *x*-axis.

(all the y-values change their sign)



Reflection Over the y-Axis

Given the graph of *f(x)*

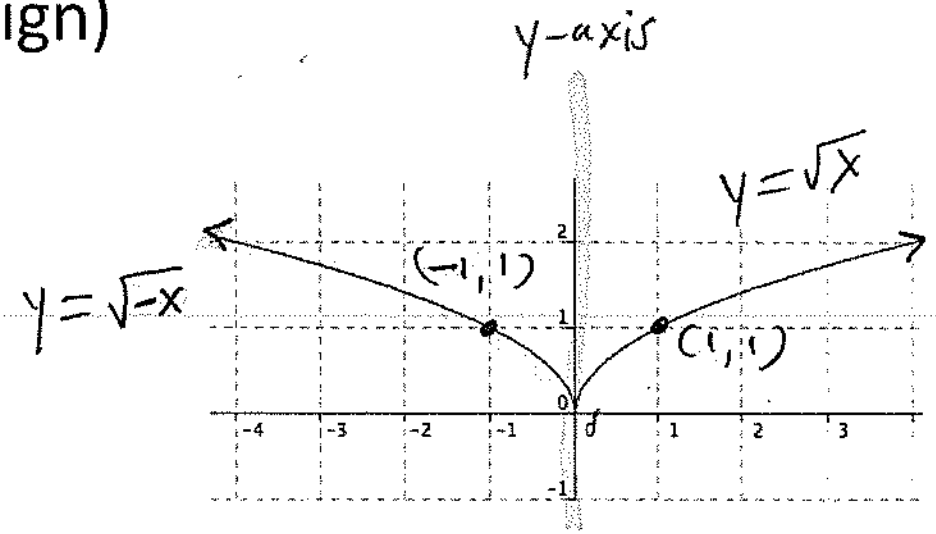
y = *f* (**−*x***) **reflects** the graph over the *y*-axis.

(all the x-values change their sign)

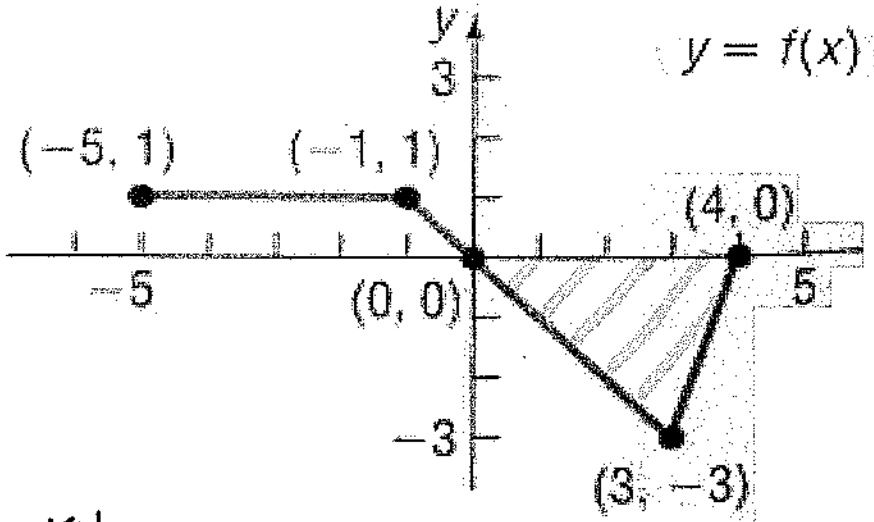
inside on the "x"

$-x = 1$

$x = -1$



Ex. 1: Sketch the following translations and reflections to the function *f(x)* shown.



x	f(x)
−5	1
−1	1
0	0
3	−3
4	0

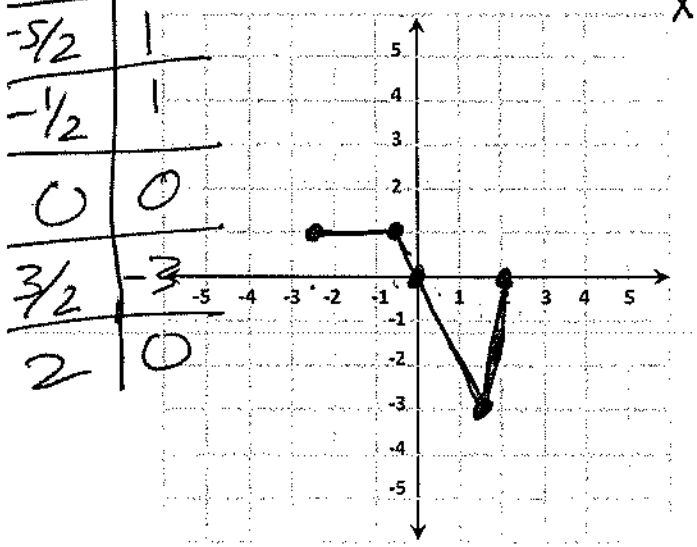
a) $f(2x)$ Horizontal Shrink/compression

X's change

$2x = 1$

$x = \frac{1}{2}$

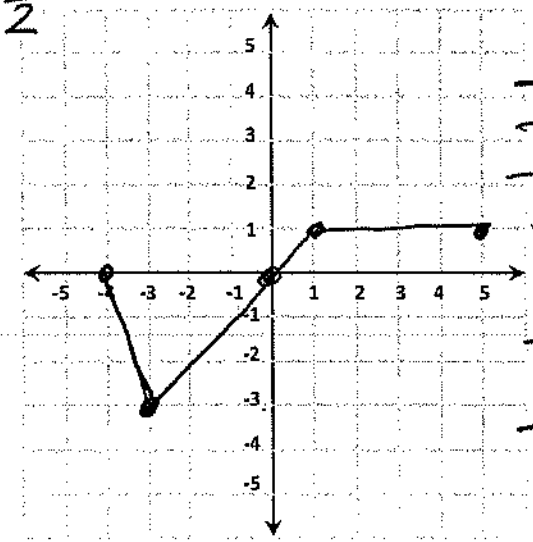
$\frac{1}{2}x$	y = f(x)
$-5/2$	1
$-1/2$	1
0	0
$3/2$	−3
2	0



b) $f(-x)$ y-axis reflection

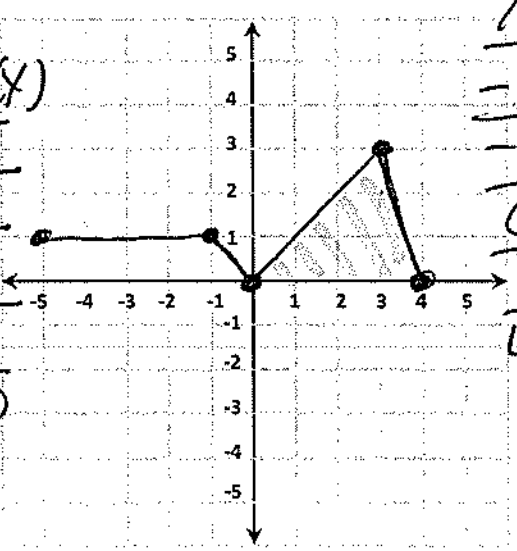
Changes X

−x	f(x)
5	1
1	1
0	0
−3	−3
−4	0



c) $+|f(x)|$ ← all y's must be positive

x	|f(x)|
−5	1
−1	1
0	0
3	+3
4	0



Ex. 2: Sketch the parent function and h(x) on the same rectangular coordinate system. Describe the transformation. State the domain and range in set notation.

$a) h(x) = -\sqrt{x+2} + 3$

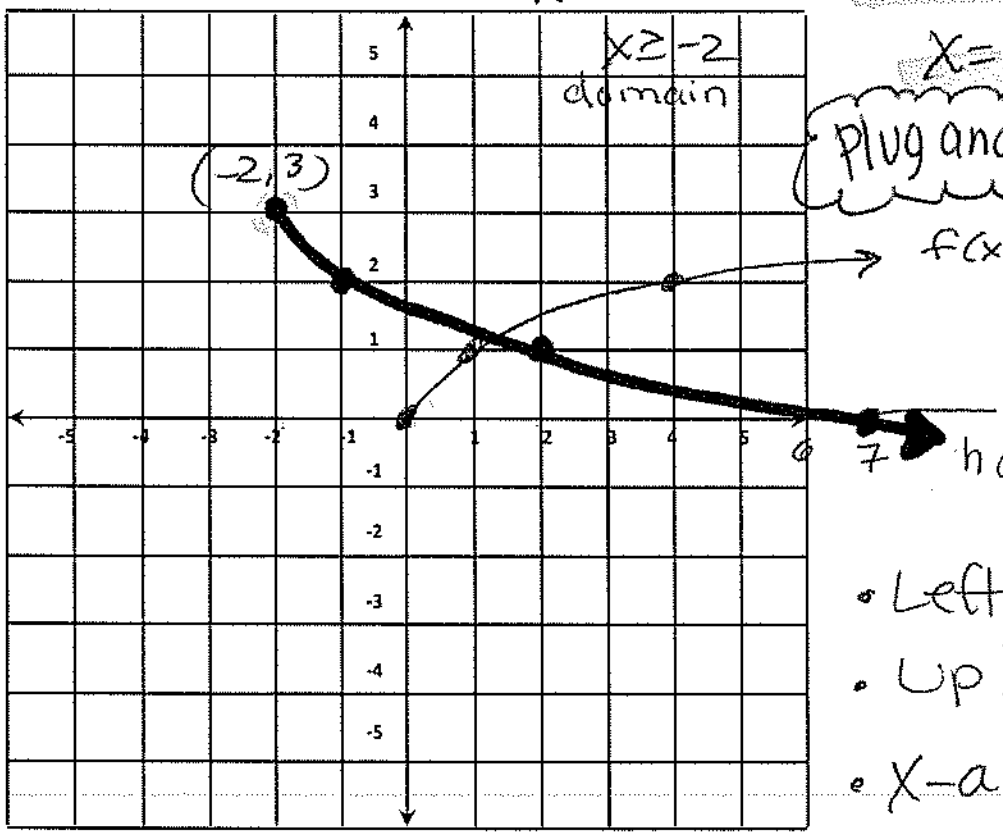
Start with $x+2=0$

$x+2 \geq 0$

$x = -2$

$x \geq -2$ domain

Parent: $f(x) = \sqrt{x}$



Plug and chug

make perfect squares

x	$h(x)$
$\sqrt{0}$ -2	3
$\sqrt{1}$ -1	2
$\sqrt{4}$ 2	1
$\sqrt{9}$ 7	0

- Left 2
- Up 3
- X-axis reflection

D: $\{x \mid x \geq -2\}$

R: $\{h(x) \mid h(x) \leq 3\}$

Integers ($\mathbb{Z}$) $\{\ldots -2, -1, 0, 1, 2, \ldots\}$

Ex. 3: Sketch the graph of the following functions. State the domain and range in set notation.

$$a) f(x) = \left[\frac{1}{4}x\right]$$

w/greatest integer function use the rules for the table

Parent: $y = [\![x]\!]$ greatest Integer function

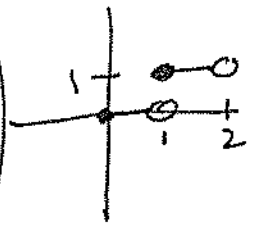
$f(5) = [\![\frac{1}{4}(5)]\!]$
$= [\![\frac{5}{4}]\!]$
$= 1$

Brackets

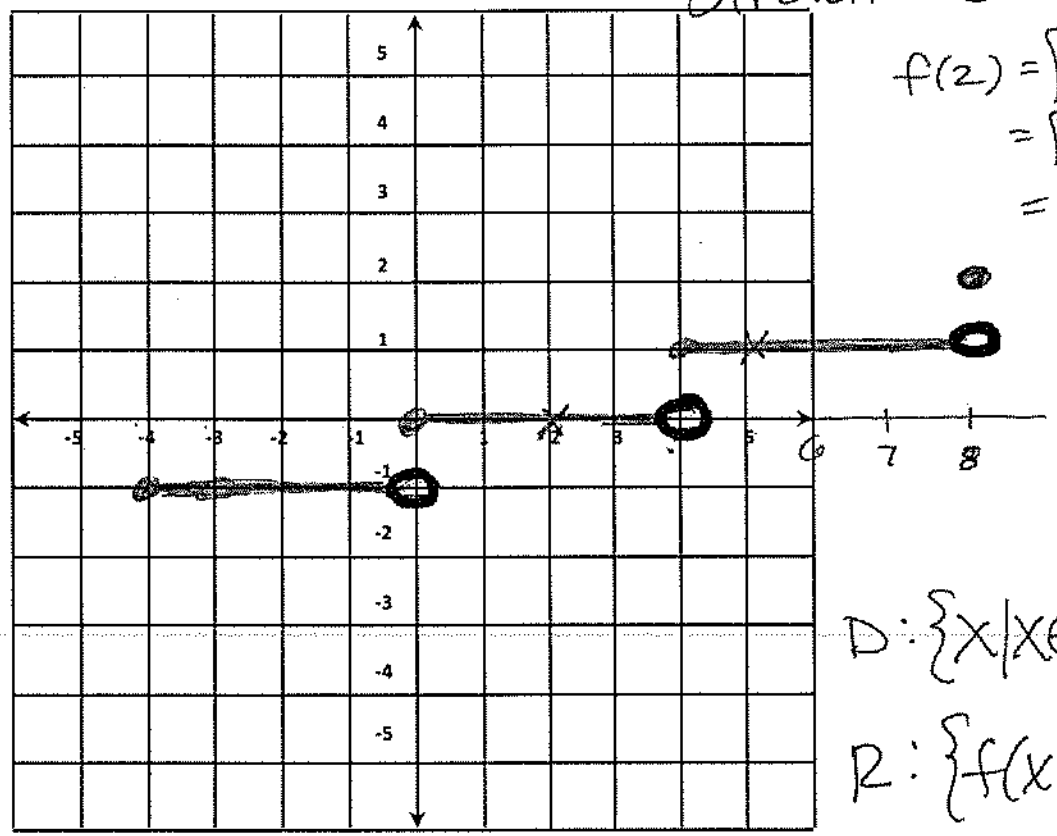
Horizontal stretch (by 4)

int(X)

X	Y
-1	-1
0	0
1	1
2	2



$f(2) = [\![\frac{1}{4}(2)]\!]$
$= [\![\frac{1}{2}]\!]$
$= 0$



4·X	f(X)
-4	-1
0	0
4	1
8	2

Test a point

$f(8) = [\![\frac{1}{4}(8)]\!]$
$= [\![2]\!]$
$= 2$

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{f(x) \mid f(x) \in \mathbb{Z}\}$

Integers

Can't write range in interval notation

b) $g(x) = 2[x] + 1$

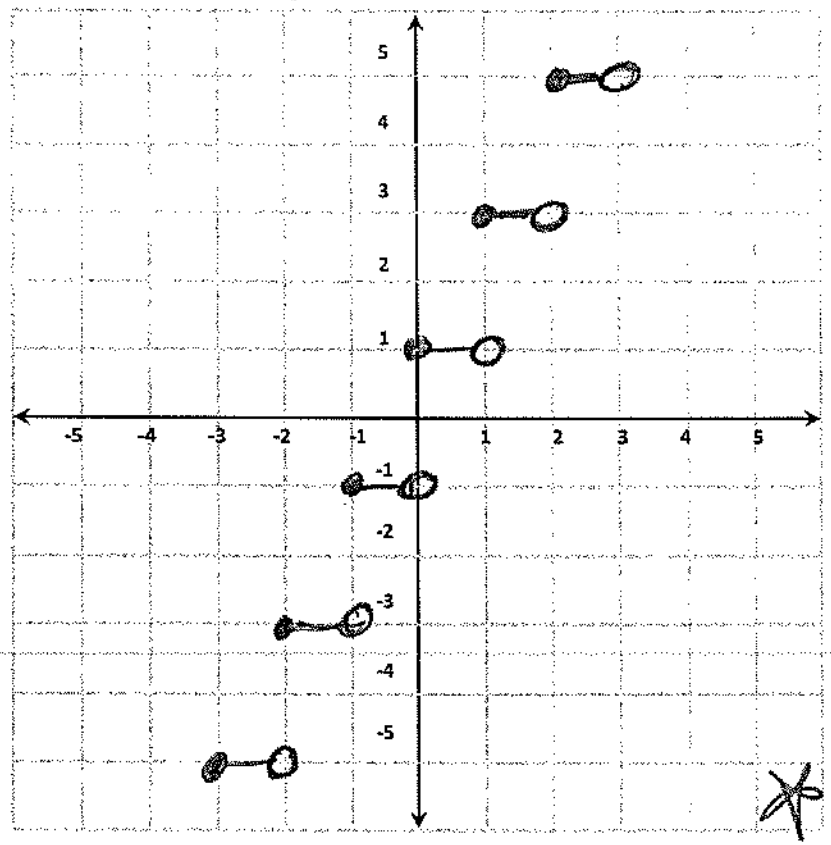
Vertical stretch by 2 (y's)

• Up one (y's)

$f(x) = [\![x]\!]$

x	f(x)
−1	−1
0	0
1	1
2	2

x	2·y + 1
−1	−1
0	1
1	3
2	5



$g(2) = 2[\![2]\!] + 1$

$= 2(2) + 1$

$= 5$ ✓

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{g(x) \mid g(x) = 2n + 1;\ n \in \mathbb{Z}\}$

Standard form ax^2+bx+c

Quadratic

c) $p(x) = x^2 + 6x + 4$

Up $a>0$ $a=1$ $b=6$ $c=4$

Vertex

Use $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

X Y

y-int: $y=4$ or $(0,4)$

$x = \frac{-b}{2a} = \frac{-(6)}{2(1)} = -3$

$y = p(-3) = (-3)^2 + 6(-3) + 4$

$= 9 - 18 + 4$

$= -5$

vertex: $(-3, -5)$

AOS $x=-3$

$p(-2) = -4$

$p(-1) = -1$

$y = x^2 + 6x + 4$

$y - 4 = 1x^2 + 6x$

$\frac{+b}{2} = (+3)^2 = +9$

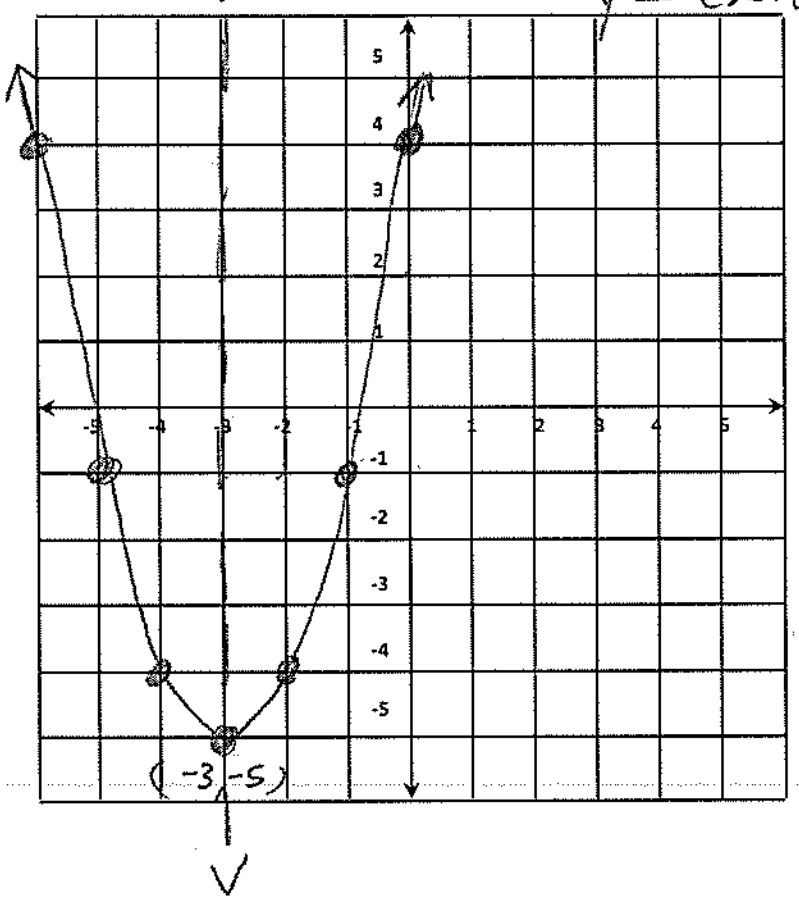
$y - 4 \underline{+9} = x^2 + 6x \underline{+9}$

$y + 5 = (x+3)^2$

$y = (x+3)^2 - 5$

vertex form

V: $(-3, -5)$



- Left 3
- Down 5

X	p(x)
-6	4
-4	-1
-3	-5 V
-2	-1
0	4

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{p(x) \mid p(x) \geq -5\}$

- also can change to vertex form →

$p(x) = x^2 + 6x + 4$ Change to vertex form.

$y = x^2 + 6x + 4$

$y - 4 = x^2 + 6x$ now complete the square

$y - 4 + \boxed{9} = x^2 + 6x + \boxed{9}$ $c = \left(\frac{b}{2}\right)^2$

$y + 5 = (x+3)^2$ $c = \left(\frac{+6}{2}\right)^2 = (+3)^2 = 9$

$y = (x+3)^2 - 5$ Vertex form

vertex : $(-3, 5)$

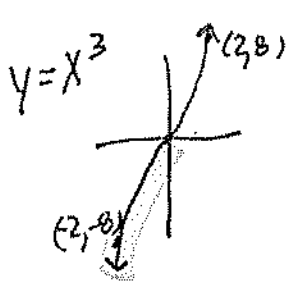
d) $f(x) = 3 - |x - 4|$

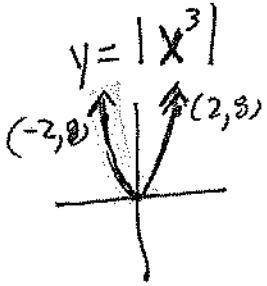
$f(x) = -|x-4| + 3$

- $-$: x-axis reflection
- $x-4$: Right 4
- $+3$: Up 3

Parent: $y = |x|$ LOF

$y = \overset{+}{|} x^3 |$



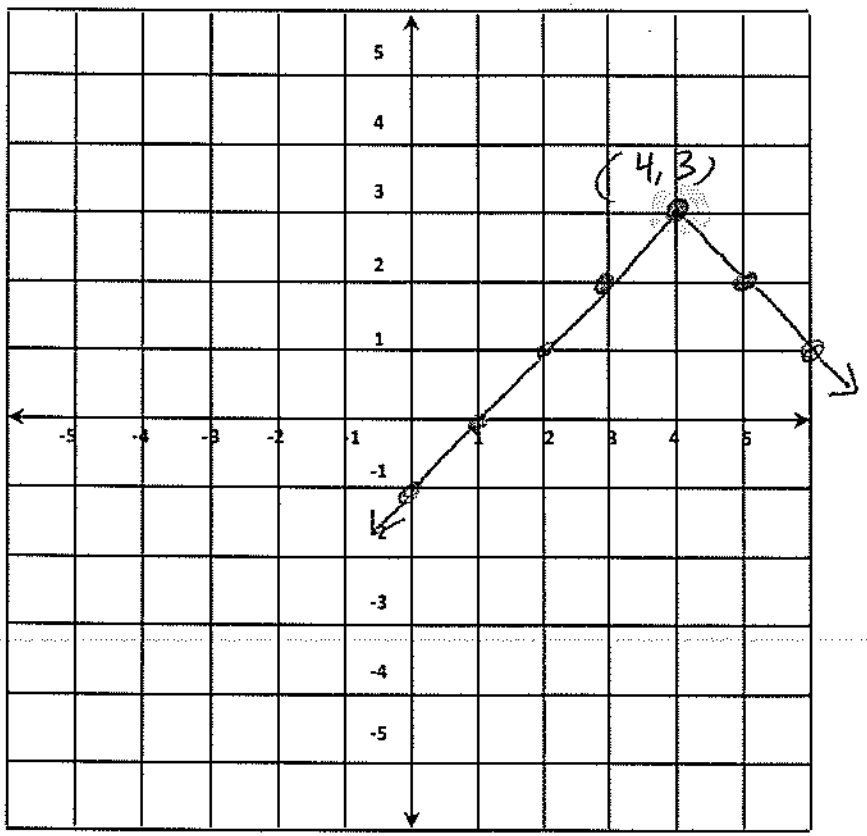


Plug and chug

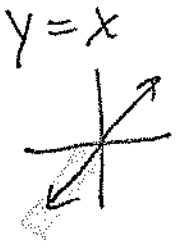
$x - 4 = 0$

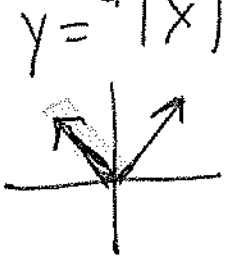
$x = 4$

$y = |x|$



x	f(x)
2	1
3	2
4	3
5	2
6	1





D: $\{x \mid x \in \mathbb{R}\}$

R: $\{f(x) \mid f(x) \leq 3\}$

Ex. 4: Use function notation to write g(x) in terms of the parent function f(x).

$$a)\ g(x) = \frac{1}{2}|x-2| - 3$$

Parent : $f(x) = |x|$

$g(x) = \frac{1}{2} f(x-2) - 3$

$$b)\ g(x) = -\sqrt{x+1} - 6$$

Parent: $f(x) = \sqrt{x}$

$g(x) = -f(x+1) - 6$