

Notes

Pre-Calculus Sec. 4.5

Graphs of Sine and Cosine

Section 4.5
**GRAPHING SINE AND COSINE FUNCTIONS USING
TRANSFORMATIONS**

$$y = \pm D \pm A \cos(Bx - C) \text{ or } y = \pm D \pm A \sin(Bx - C)$$

Graphing Check List

1) Period : $\frac{2\pi}{|B|}$ (the horizontal stretch or shrink)

2) Amplitude : $|A|$ (the vertical stretch or shrink)

3) Vertical Displacement : D (the vertical shift or "new" x-axis)

$$\boxed{y=d}$$

4) *Phase Shift : opposite of $\frac{C}{B}$ (the horizontal shift) *L/R shift*
(PS)

★ *If B has been factored out then the phase shift will be equal to the opposite of C

5) Reflect about the x-axis : $-A$

6) Increments: $\frac{\text{Period}}{4}$ (distance between each x coordinate)

Same as $\frac{1}{4}$ (Period)

L.O.F

The Graph of $f(x) = \sin x$

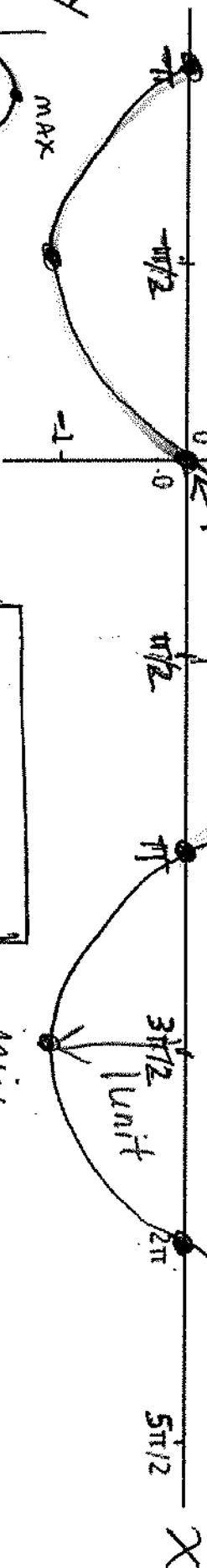
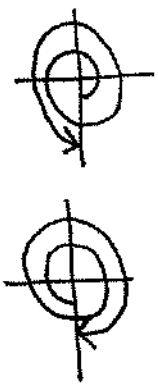
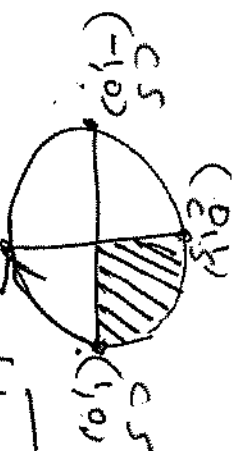
$\sin 0 = 0$
 $\sin \frac{\pi}{2} = 1$

$f(x)$

$f(x) = 1 \sin x$

Amplitude = $|1| = 1$

Period $\left[\frac{1}{4} (2\pi) \right]$



Amplitude =

$$\frac{Y_{max} - Y_{min}}{2}$$

$$= \frac{1 - (-1)}{2} = \frac{2}{2} = 1$$

Domain: $\{x | x \in \mathbb{R}\}$

Range: $\{f(x) | -1 \leq f(x) \leq 1\}$

Period:

INT: $(-\infty, \infty)$

INT: $[-1, 1]$

$\pi - (-\pi) = 2\pi$

$2\pi - 0 = 2\pi$

$X_{max} - X_{min}$
 for one complete pattern

This graph has Origin symmetry, and is an Odd function.

$f(-x) = -f(x)$

$\sin(-x) = -\sin x$

$[\text{or } 360^\circ]$

LOF

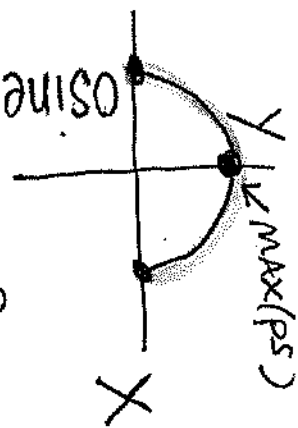
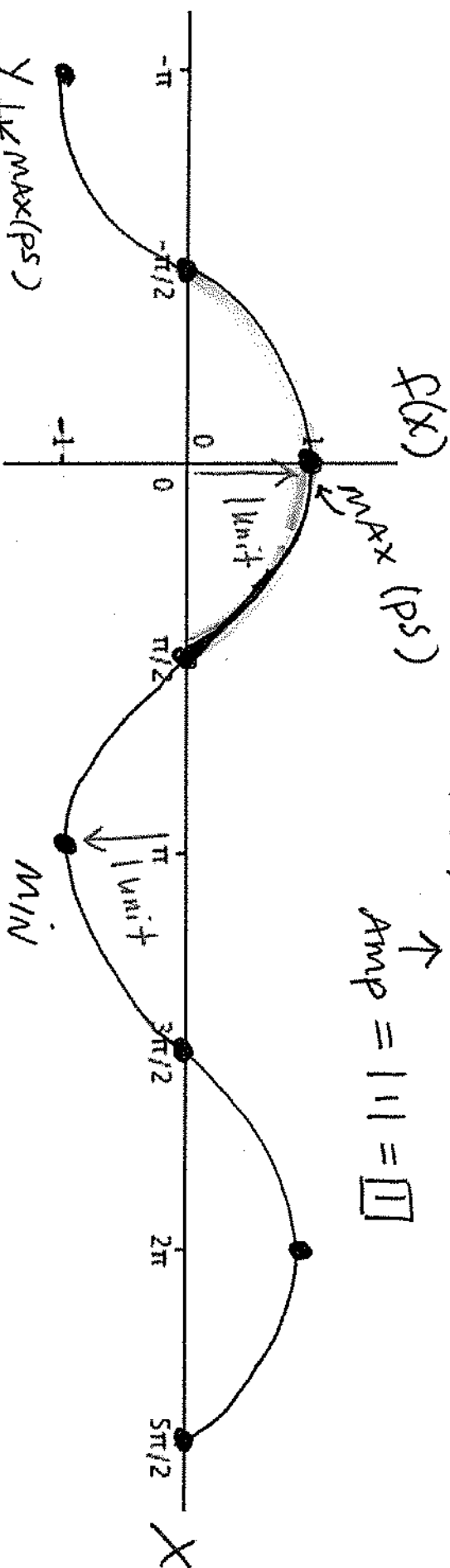
The Graph of $f(x) = \cos x$

$$\cos 0 = 1$$

$$\cos \frac{\pi}{2} = 0$$

$$f(x) = |\cos x|$$

$$\text{Amp} = |1| = 1$$



Domain: $\{x | x \in \mathbb{R}\}$

Range: $\{f(x) | -1 \leq f(x) \leq 1\}$

Period: $\frac{3\pi}{2} - (-\frac{\pi}{2})$

INT: $(-\infty, \infty)$

INT: $[-1, 1]$

$$= \frac{4\pi}{2} = 2\pi$$

$X_{\max} - X_{\min}$

or 360°

This graph has Y-axis symmetry, and is an Even function.

$$f(-x) = f(x)$$

$$\cos(-x) = \cos x$$

Recall:

- $y = c f(x)$: Vertically stretches or shrinks.
- $y = f(cx)$: Horizontally stretches or shrinks.

Because these change our "Basic" graphs, our basic **period** and **amplitude** may also change.

If $y = \underline{a} \sin \underline{b}x$ and $y = \underline{a} \cos \underline{b}x$; where $a \neq 0, b > 0$,

vertical *horizontal*

Then: Amplitude = $|a|$

and

Period = $\frac{2\pi}{|b|}$

Based on
the LOF
sin & cos
graphs

$$f(x) = a \sin (bx - c) + \underline{d} \quad \text{or} \quad f(x) = a \cos (\underline{b}x - c) + d$$

$$f(x) = \underline{d} + a \sin (bx - c) \quad \text{or} \quad f(x) = a \cos \underline{b}(x - \frac{c}{b}) + d$$

$$y = \sqrt{x-2} + 5$$

Right+2 Up 5

Horizontal translations (left or right): Graphs are shifted by an amount c (this # is the **phase shift**). L/R

Choose $\pm b$ \star Only use $\frac{c}{b}$ if b is not factored out.
 \star SET what is in the () = 0 to find PS.

Vertical translations (up or down): d units upward or downward, the graph oscillates about the horizontal line $y = \underline{d}$ instead of the x-axis.

\nearrow
new axis

factored out

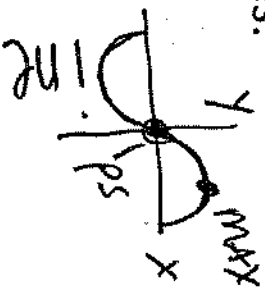
Ex.1: Graph $f(x) = 3 + 2\sin\left(x + \frac{\pi}{3}\right)$

$f(x) = 2\sin\left(x + \frac{\pi}{3}\right) + 3$

1st: Find increment ($\frac{1}{4}$ Period):

$\frac{1}{4}(2\pi) = \frac{\pi}{2}$ Add/sub. to PS

2nd: Find 5 key x-values:



- Amplitude: $|a| = |2| = 2$
- Period: $\frac{2\pi}{|b|} = \frac{2\pi}{1} = 2\pi$
- Phase Shift:
 - Set inside the () = 0
 - $x + \frac{\pi}{3} = 0$
 - $x = -\frac{\pi}{3}$

Vertical Stretch

Left Shift

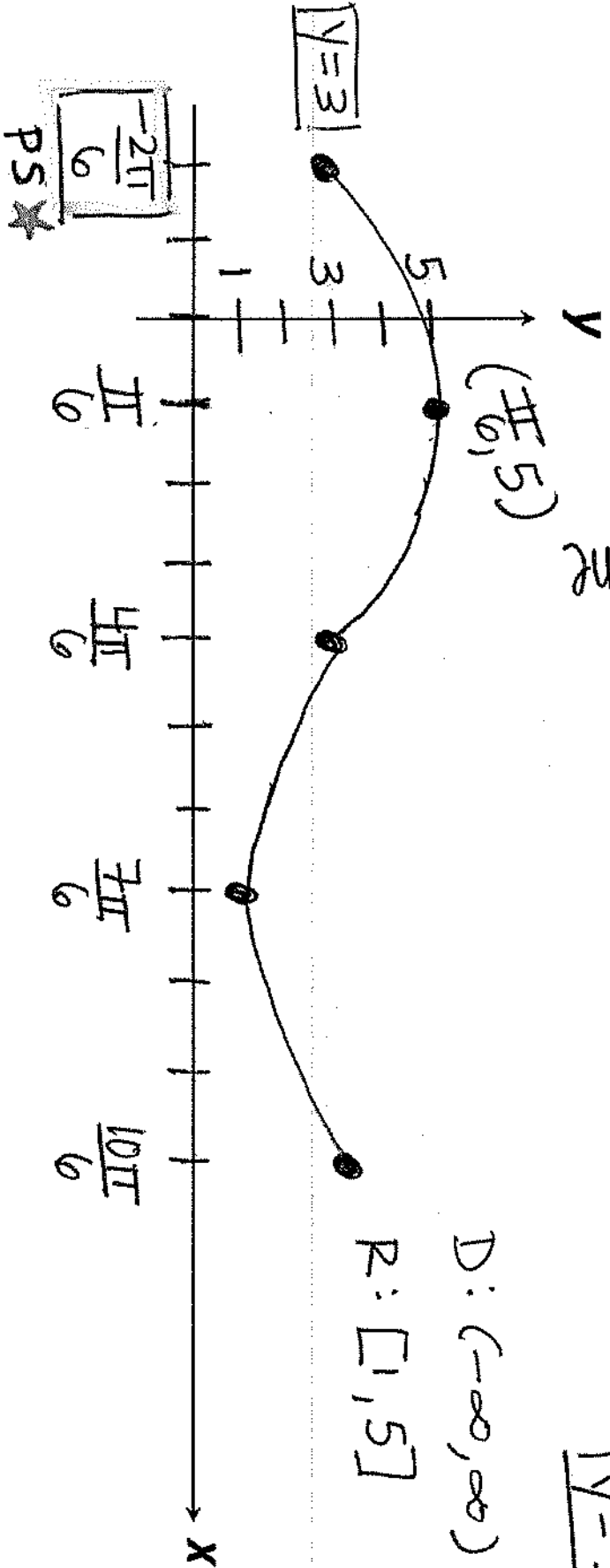
new axis: $y = d$

PS = $-\frac{\pi}{3}$

$-\frac{\pi}{3} - \frac{\pi}{2} = -\frac{2\pi}{6}$

D: $(-\infty, \infty)$

R: $[1, 5]$



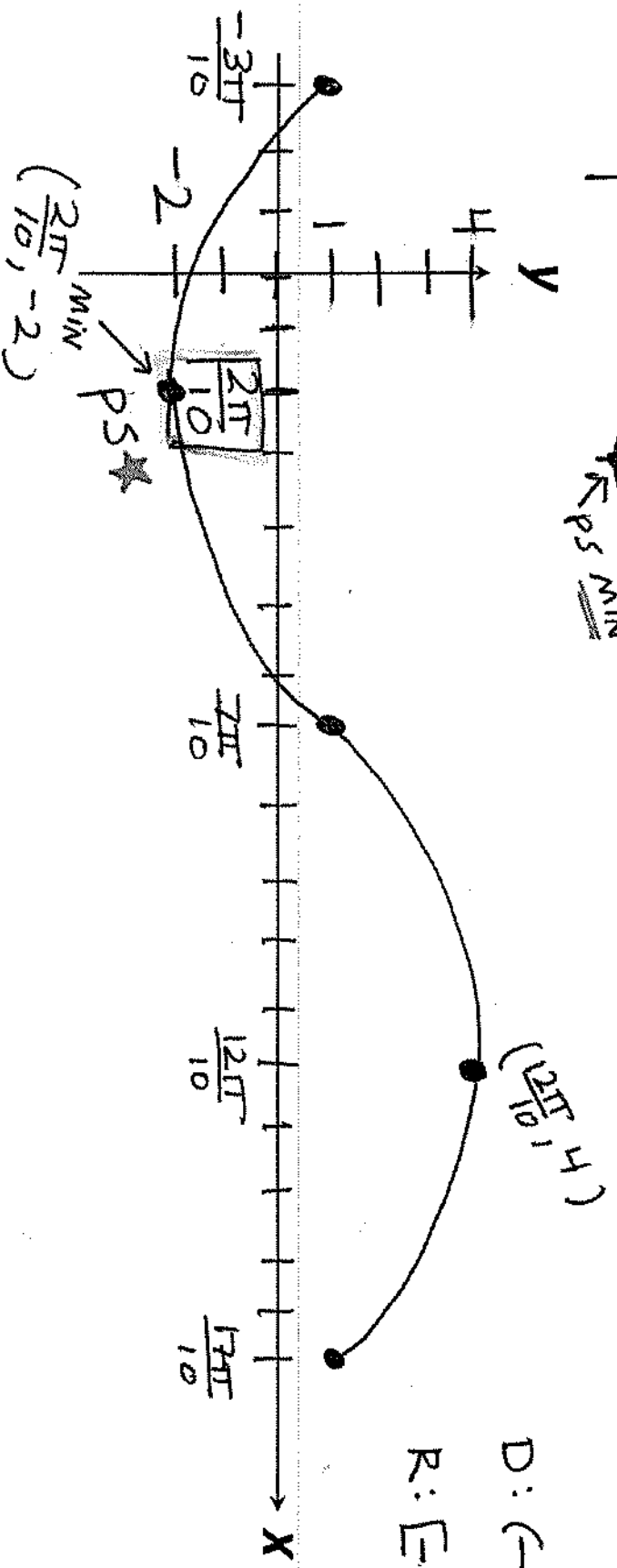
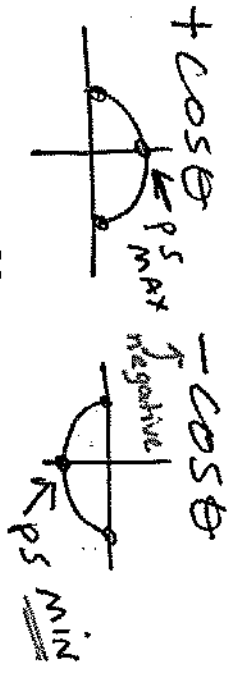
Ex. 2: Graph $f(x) = -3 \cos\left(x - \frac{\pi}{5}\right) + 1$

a (Amplitude) b (Phase Shift) c (Phase Shift) d (Vertical Shift)
 negative cosine
 Amplitude: $|a| = |-3| = 3$
 Period: $\frac{2\pi}{|b|} = \frac{2\pi}{1} = 2\pi$
 Phase Shift: $\frac{2\pi}{5}$ Right shift
 new axis: $y = d$
 axis: $y = 1$

1st: Find increment (1/4 Period):

$$\frac{1}{4} (2\pi) = \frac{\pi}{2} \rightarrow \frac{5\pi}{10}$$

2nd: Find 5 key x-values:



D: $(-\infty, \infty)$
R: $[-2, 4]$

Ex.3: Graph $y = -2 \overset{a}{\sin} \left(4x - \frac{\pi}{3} \right) + 5$ $\overset{b}{4x} - \overset{c}{\frac{\pi}{3}} + \overset{d}{5}$

negative

Amplitude: $|a| = |-2| = 2$ Vert. Stretch

Period: $\frac{2\pi}{|b|} = \frac{2\pi}{4} = \frac{\pi}{2}$ Horizontal Shrink

Phase Shift:

1st: Find increment ($\frac{1}{4}$ Period): Add/sub to PS

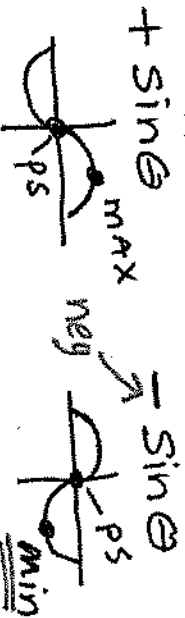
$$\frac{1}{4} \left(\frac{\pi}{2} \right) = \left| \frac{\pi}{8} \right| \Rightarrow \left| \frac{3\pi}{24} \right|$$

$$4x - \frac{\pi}{3} = 0$$

$$\frac{1}{4} \cdot 4x = \frac{\pi}{3} \cdot \frac{1}{4} \quad \star$$

$$x = \frac{\pi}{12} \quad \text{PS} = \left| \frac{+\pi}{12} \right|$$

2nd: Find 5 key x-values:

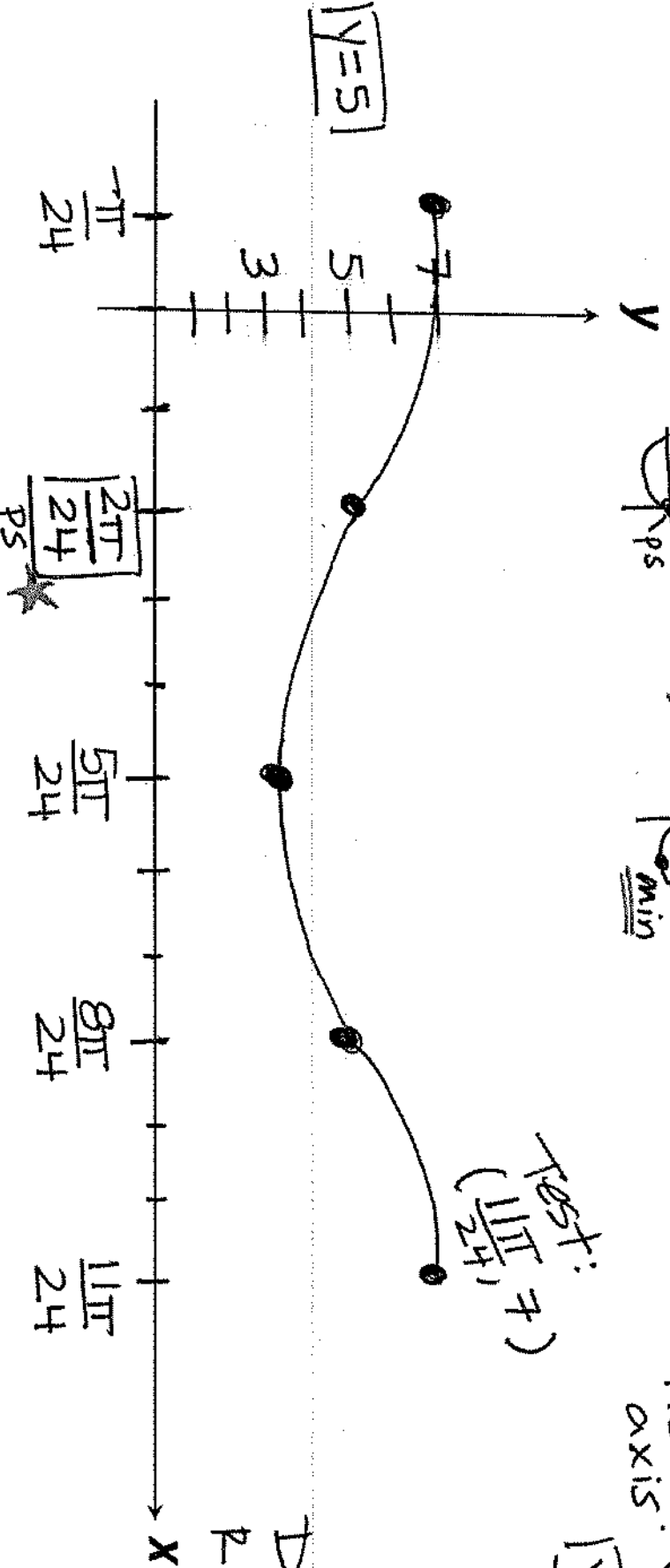


new axis: $y = d$

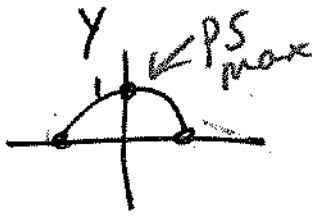
$$\left| y = 5 \right| \Rightarrow \left| \frac{2\pi}{24} \right|$$

Test: $\left(\frac{11\pi}{24}, 7 \right)$

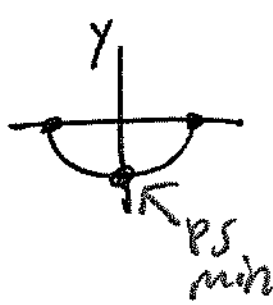
D: $(-\infty, \infty)$
R: $[3, 7]$



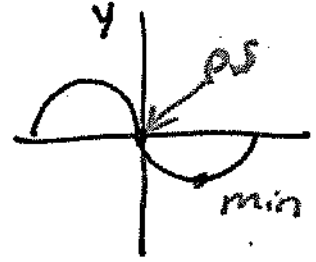
pos.
 $\oplus \cos \theta$



neg
 $\ominus \cos \theta$



neg
 $\ominus \sin \theta$



pos.
 $\oplus \sin \theta$

