Notes

Review:

1. Graph: $$p(x) = \begin{cases} x, & \text{if } x < 2 \\ |x-2|, & \text{if } x = 2 \\ x+1, & \text{if } 2 < x < 5 \\ -1, & \text{if } x \geq 5 \end{cases}$$

2. Odd, even, or neither? Show your work, leave no ( ).

   $a) f(x) = -\dfrac{2x}{\sqrt{5+x^2}}$ $\qquad$ $b) g(x) = 3x^3 + x - 2$

3. Find the domain in interval notation:

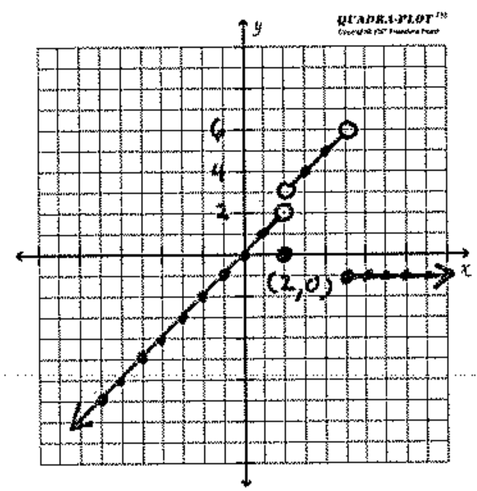
$$f(x) = \frac{3x}{\sqrt{9-x^2}}$$

Review:

1. Graph:

$$p(x)=\begin{cases} x, & \text{if } x<2 \\ |x-2|, & \text{if } x=2 \\ x+1, & \text{if } 2<x<5 \\ -1, & \text{if } x\geq 5 \end{cases}$$

one point @ (2,0)



$2<X<5$

X	X+1	
2	3	open
3	4	
4	5	
5	6	open

The graph must pass the VLT.

Review:

2. Odd, even, or neither? Show your work, leave no ( ).

$a)\ f(x) = -\dfrac{2x}{\sqrt{5+x^2}}$ $\qquad$ $b)\ g(x) = 3x^3 + x - 2$

Label

a) $f(-x) = \dfrac{-2(-x)}{\sqrt{5+(-x)^2}}$

$\boxed{f(-x) = \dfrac{2x}{\sqrt{5+x^2}}}$

same
opp
neither

insure:
$f(2) = \frac{-4}{3}$
$f(-2) = \frac{4}{3}$
opposites

$f(-x) = -f(x)$
$\therefore$ $f(x)$ is an odd function

b) $g(-x) = 3(-x)^3 + (-x) - 2$

$\boxed{g(-x) = -3x^3 - x - 2}$

same
opp
neither

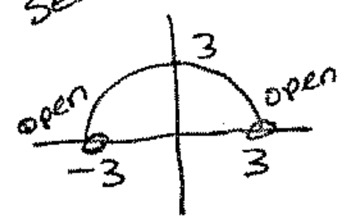
$g(-x) \neq g(x)$ or $-g(x)$
$\therefore$ $g(x)$ is neither even or odd

Review:

3. Find the domain in interval notation:

$$f(x)=\frac{3x}{\sqrt{9-x^2}}$$

semi circle r = 3



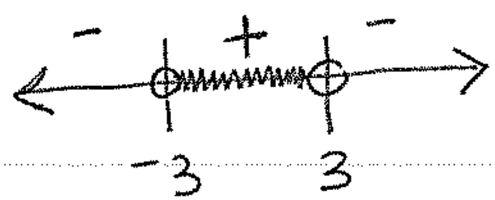
algebraically:

Test Regions here.

$9-x^2>0$ open b/c in the denominator

$(3+x)(3-x)>0$

critical values $x=-3$ $x=3$



$\boxed{(-3,3)}$

Sec. 1.7 Arithmetic Combinations of Functions

4 Basic Operations:

(Add, subtract, multiply, divide)

Given $f(x)$ and $g(x)$:

Sum: $(f+g)(x) = f(x) + g(x)$

$f+g$

Difference: $(f-g)(x) = f(x) - g(x)$

$f-g$

Product: $(fg)(x) = f(x)g(x)$

$f \bullet g$

solid dot = mult.

fg

multiply

Quotient: $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$.

$\frac{f}{g}$

Ex. 1: If $f(x) = 2x + 6$ and $g(x) = x^2 + 5x + 6$
find:

a) $(f - g)(x) = f(x) - g(x)$

$$= (2x+6) - (x^2+5x+6)$$

$$= 2x + \cancel{6} - x^2 - 5x - \cancel{6}$$

$$= \boxed{-x^2 - 3x} \quad D: (-\infty, \infty)$$

a) $(f - g)(2)$

preferred method

$f(2) - g(2)$

$[2(2)+6] - [(2)^2 + 5(2) + 6]$

$10 - 20$

$= \boxed{-10}$

OR

$(f-g)(x) = -x^2 - 3x$

$(f-g)(2) = -(2)^2 - 3(2)$

$= -4 - 6$

$= -10$

value is in the domain

$$f(x) = 2x + 6 \qquad g(x) = x^2 + 5x + 6$$

creates a hole

c) $$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x+6}{x^2+5x+6} = \frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x+2)}$$

$$\boxed{\frac{f}{g}(x) = \frac{2}{x+2}\ ;\ x \neq -3, -2}$$

d) $$\frac{f}{g}(-3) = \frac{f(-3)}{g(-3)}$$

★ prefer this method

$$= \frac{2(-3)+6}{(-3)^2+5(-3)+6} = \frac{0}{0}$$

Undefined

$$\boxed{\frac{f}{g}(-3) \text{ is undefined}}$$

OR

$$\frac{f}{g}(x) = \frac{2}{x+2}$$

$$\frac{f}{g}(\cancel{-3}) = \frac{2}{-3+2} = \frac{2}{-1} = \cancel{-2}$$

not in the domain

$x \neq -3, -2$

$$f(x) = 2x + 6 \qquad g(x) = x^2 + 5x + 6$$

e) $$\frac{f}{g}(0) = \frac{f(0)}{g(0)} = \frac{\cancel{2(0)}+6}{\cancel{(0)^2}+\cancel{5(0)}+6} = \frac{6}{6} = \boxed{1}$$

* prefer this method

OR $$\frac{f}{g}(x) = \frac{2}{x+2} \; ; \; x \neq -3, -2$$

so $$\frac{f}{g}(0) = \frac{2}{0+2} = \frac{2}{2} = \boxed{1}$$

value in the domain

yes ✓

Sec. 1.7 Composite Functions

open circle

$(f \circ g)(\mathbf{x}) = \underbrace{f}_{\text{Outside}}\,(\underbrace{g(\mathbf{x})}_{\text{Inside}})$: Read as *"f of g(x)"*

Means: all of the ***x's*** in the *f* function are replaced by the equation of *g(x)*

Vice versa for $(g \circ f)(\mathbf{x})$. $= \underbrace{g}_{\text{Outside}}\,(\underbrace{f(x)}_{\text{Inside}})$

$h(x)$ $p(x)$ $d(x)$ etc...

$h \circ g(x)$ $p \circ h(x)$

$d \circ g(x)$

Finding the Domain

vs

For Operations: +, −, ×, ÷	For Composition: f o g & g o f
The domain comes from ***all*** the functions involved and the result.	The overall domain includes both the **result's** and the **"inside"** function's domain restrictions. NOT THE OUTSIDE FUNCTION!

Ex.1: If $f(x)=\dfrac{2}{x+3}$ and $g(x)=\dfrac{1}{x}$

Find $f \circ g$ and its domain.

$$= f(g(x)) = f\left(\frac{1}{x}\right) = \frac{\frac{2}{1}\cdot x}{\cancel{x}\cdot\frac{1}{\cancel{x}}+\frac{3}{1}\cdot x} = \boxed{\frac{2x}{1+3x}}$$

$f \circ g(x) =$

never leave a complex fraction

Result

$1+3x=0$

$3x=-1$

$x \neq -\frac{1}{3}$

Inside:

$g(x)=\frac{1}{x}$

$x \neq 0$

Domain of $f \circ g(x)$:

Set: $\{x \mid x \neq -\frac{1}{3}, 0\}$

INT: $(-\infty,-\frac{1}{3})\cup(-\frac{1}{3},0)\cup(0,\infty)$

$-\frac{1}{3}$ 0

Ex. 2: If $f(x) = \sqrt{x}$ and $g(x) = x^2 + 3$,

a) Find $g \circ f$ and its domain.

$$= g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 + 3$$

$$\boxed{g \circ f(x) = x + 3}$$

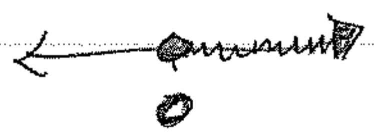
Result

Linear → all $\mathbb{R}$'s

Inside:

$f(x) = \sqrt{x}$

$x \geq 0$



Domain of $g \circ f(x)$

SET: $\{x \mid x \geq 0\}$

INT: $[0, \infty)$

b) Find $g \circ f(4) = g(f(4))$

value

★ prefer this method

$= g(2)$

$= (2)^2 + 3$

$= 4 + 3$

$= \boxed{7}$

Label:

$f(4) = \sqrt{4}$

$= 2$

OR

$g \circ f(x) = x + 3;\ x \geq 0$

$g \circ f(4) = 4 + 3$

$= \boxed{7}$

in the domain

c) Find $g \circ f(-1)$

★ prefer this method

$= g(f(-1))$

$= \boxed{\text{Undefined}}$

b/c −1 is not in the domain

$f(-1) = \sqrt{-1}$

$= i$

imaginary

OR

$g \circ f(x) = x + 3;\ x \geq 0$

$g \circ f(-1) = -1 + 3$

$= \cancel{\boxed{2}}$

not in the domain $x \geq 0$

This is a wrong answer.

Ex. 3: If $f(x)=\dfrac{1}{x}$ and $g(x)=\dfrac{1}{2x+1}$,

Find $f \circ g$ and its domain.

$$f(g(x)) = f\left(\frac{1}{2x+1}\right) = \frac{1}{\frac{1}{2x+1}}$$

$$\boxed{(f\circ g)(x) = 2x+1}$$

Result
Linear all $\mathbb{R}$'s

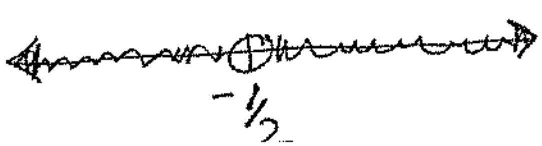
Inside:

$g(x)=\dfrac{1}{2x+1}$

$2x+1=0$

$2x=-1$

$x \neq -\frac{1}{2}$



Domain of $f\circ g(x)$:

SET: $\{x \mid x \neq -\frac{1}{2}\}$

INT: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$