Notes

Pre-Calculus
Sec. 4.4
Trigonometric Functions

Important Right Triangle
Pythagorean Triplets to Memorize:

side – side – hypotenuse

$\boxed{a^2+b^2=c^2}$

$\boxed{x^2+y^2=r^2}$

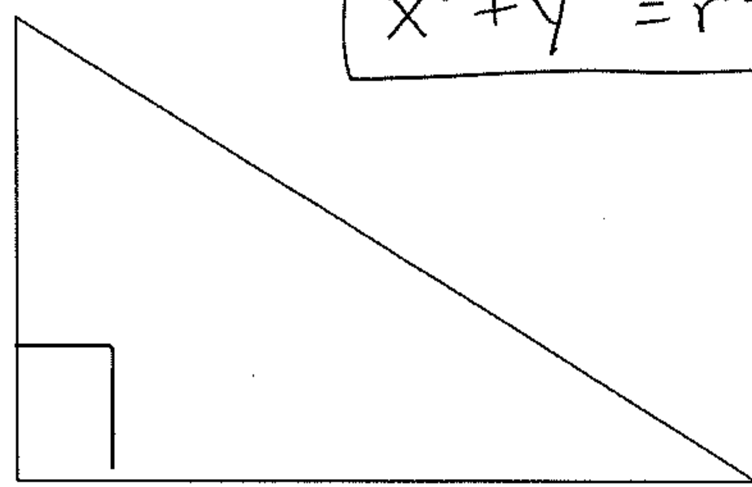
3 – 4 – 5

5 – 12 – 13

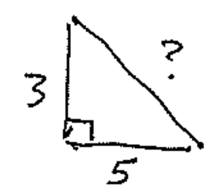
7 – 24 – 25

8 – 15 – 17

9 – 40 – 41



Given Sides $3 \,\&\, 5 \rightarrow$ ~~4~~ ~~hyp~~



not a triplet

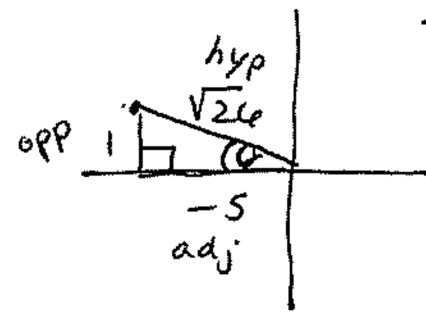
$9 + 25 = r^2$

$\sqrt{34} = r$

Ex.1) Find the value of the <u>six</u> trig. functions if the terminal side of θ passes through the point (−5, 1).

(x, y)

Quad. II



* not a unit circle b/c not given info.

$a^2+b^2=c^2$

$x^2+y^2=r^2$

$25+1=r^2$

$26=r^2$

$\sqrt{26}=r$

2 13

SOH: $\sin\theta = \frac{1}{\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}}$

$\boxed{\sin\theta = \frac{\sqrt{26}}{26}}$

CAH: $\cos\theta = \frac{-5}{\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}}$

$\boxed{\cos\theta = \frac{-5\sqrt{26}}{26}}$

TOA: $\boxed{\tan\theta = -\frac{1}{5}}$

Reciprocals:

$\boxed{\csc\theta = \sqrt{26}}$

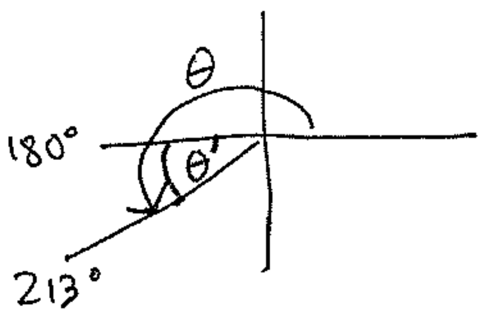
$\boxed{\sec\theta = -\frac{\sqrt{26}}{5}}$

$\boxed{\cot\theta = -5}$

S | A
T | C

Ex.2) Find the reference angle for θ. Sketch θ and θ'.

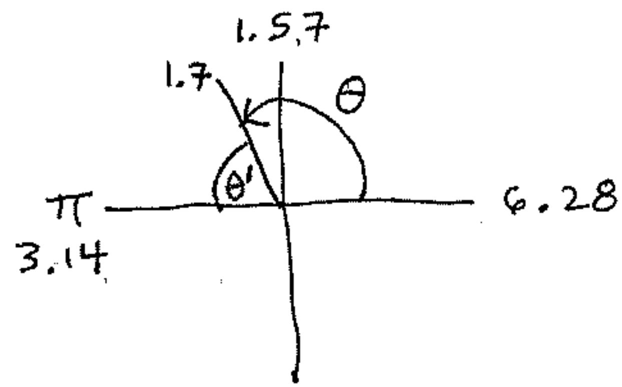
a) $\theta = 213°$



$\theta' = 213° - 180°$

$\boxed{\theta' = 33°}$

b) $\theta = 1.7$



Exact: $\boxed{\theta' = \pi - 1.7}$

(terms of π)

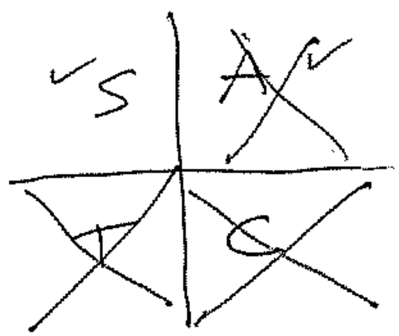
ESTIMATE: $\theta' \approx 3.14 - 1.7$

$\boxed{\theta' \approx 1.44}$

$$\begin{array}{r} 3.14 \\ -1.70 \\ \hline \end{array}$$

Ex.3: State the quadrant in which θ lies.

a) positive $\sin\theta > 0$ and $\cos\theta < 0$ negative

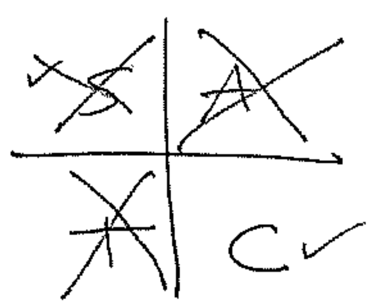


$\boxed{\text{QII}}$

b) negative $\cot\theta < 0$ and $\csc\theta < 0$ negative

also $\tan\theta < 0$

also $\sin\theta < 0$

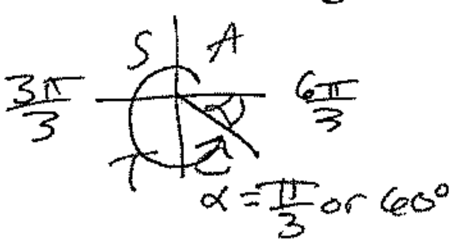


$\boxed{\text{QIV}}$

Ex.4: Evaluate:

$a)\ \sin\frac{5\pi}{3} = \ominus \sin \overset{\alpha}{60^\circ}$

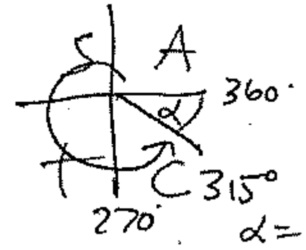
$= \boxed{-\frac{\sqrt{3}}{2}}$



$d)\ \cot 315^o = \boxed{-1}$

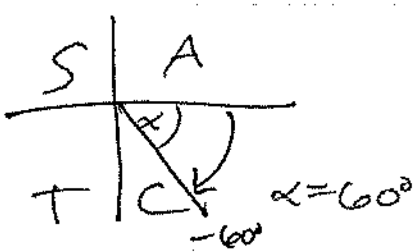
use $\tan 315^\circ = \ominus \tan 45^\circ$

$= -1$



$b)\ \cos(-60^\circ) = \oplus \cos 60^\circ$

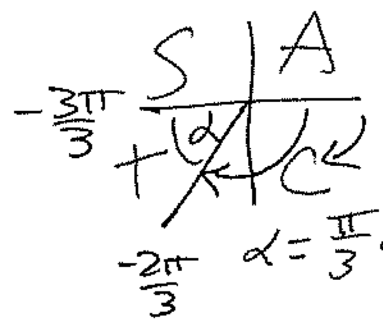
$= \boxed{\frac{1}{2}}$



$e)\ \sec\left(-\frac{2\pi}{3}\right) = \boxed{-2}$

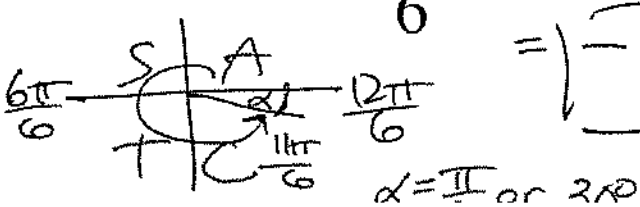
use $\ominus \cos 60^\circ$

$= -\frac{1}{2}$



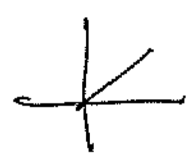
$c)\ \tan\frac{11\pi}{6} = \ominus \tan 30^\circ$

$= \boxed{-\frac{\sqrt{3}}{3}}$



$f)\ \csc\frac{9\pi}{4} = \csc\frac{\pi}{4} = \boxed{\sqrt{2}}$

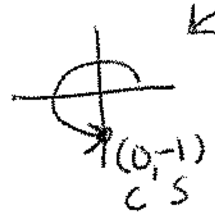
C+



use $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

Ex.5) Find the exact value of the expression.
Write the answer as a single fraction.

$$\sin\frac{3\pi}{2}\tan\left(-\frac{8\pi}{3}\right)+\cos\left(-\frac{5\pi}{6}\right)$$



$(-1)(\sqrt{3}) + \left(-\frac{\sqrt{3}}{2}\right)$

$\tan\left(-\frac{8\pi}{3}\right)$

CT

$= \tan\frac{4\pi}{3}$

$= \oplus \tan\frac{\pi}{3}$

$\frac{3\pi}{3}$ S | A, T | C

$\alpha = \frac{\pi}{3}$

even rule

$= \cos\frac{5\pi}{6}$

$= \ominus \cos\frac{\pi}{6}$

$\alpha = \frac{\pi}{6}$ S | A, $\frac{6\pi}{6}$ T | C

$= -\frac{\sqrt{3}}{2}$

$-\frac{\sqrt{3}}{1}\cdot\frac{2}{2} - \frac{\sqrt{3}}{2}$

get a common denominator

$\frac{-2\sqrt{3}}{2} - \frac{1\sqrt{3}}{2}$

$= \boxed{\frac{-3\sqrt{3}}{2}}$

Ex.6) Find two solutions of the equation, in degrees $(0° \le \theta < 360°)$ and radians $(0 \le \theta < 2\pi)$. No calculator.

$a)\ \cos\theta = -\boxed{\frac{\sqrt{2}}{2}}$

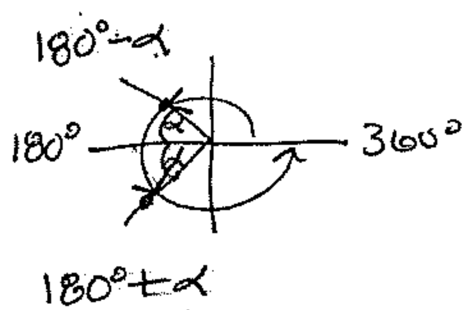
angle

neg

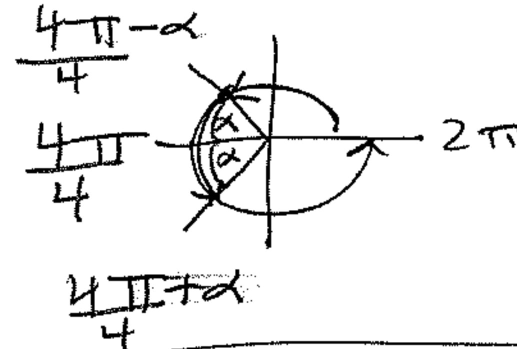
S A
T C

not QA

$\alpha = 45°$ or $\frac{\pi}{4}$



$\boxed{\theta = 135°, 225°}$



$\boxed{\theta = \frac{3\pi}{4}, \frac{5\pi}{4}}$

$b)\ \csc\theta = -\frac{2\sqrt{3}}{3}$

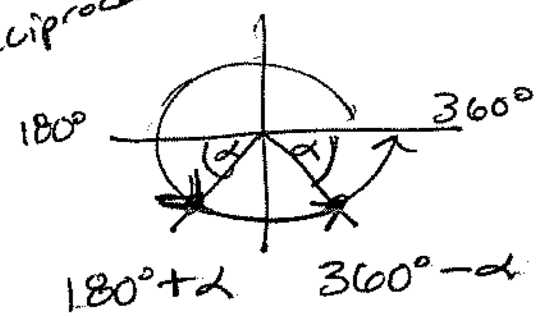
reciprocal

use $\sin\theta = -\boxed{\frac{\sqrt{3}}{2}}$

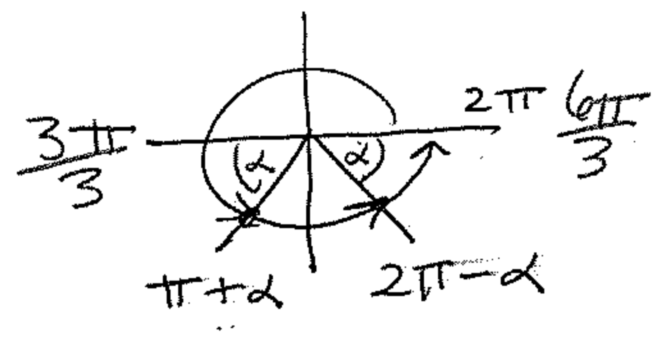
neg

S A
T C

$\alpha = 60°$ or $\frac{\pi}{3}$



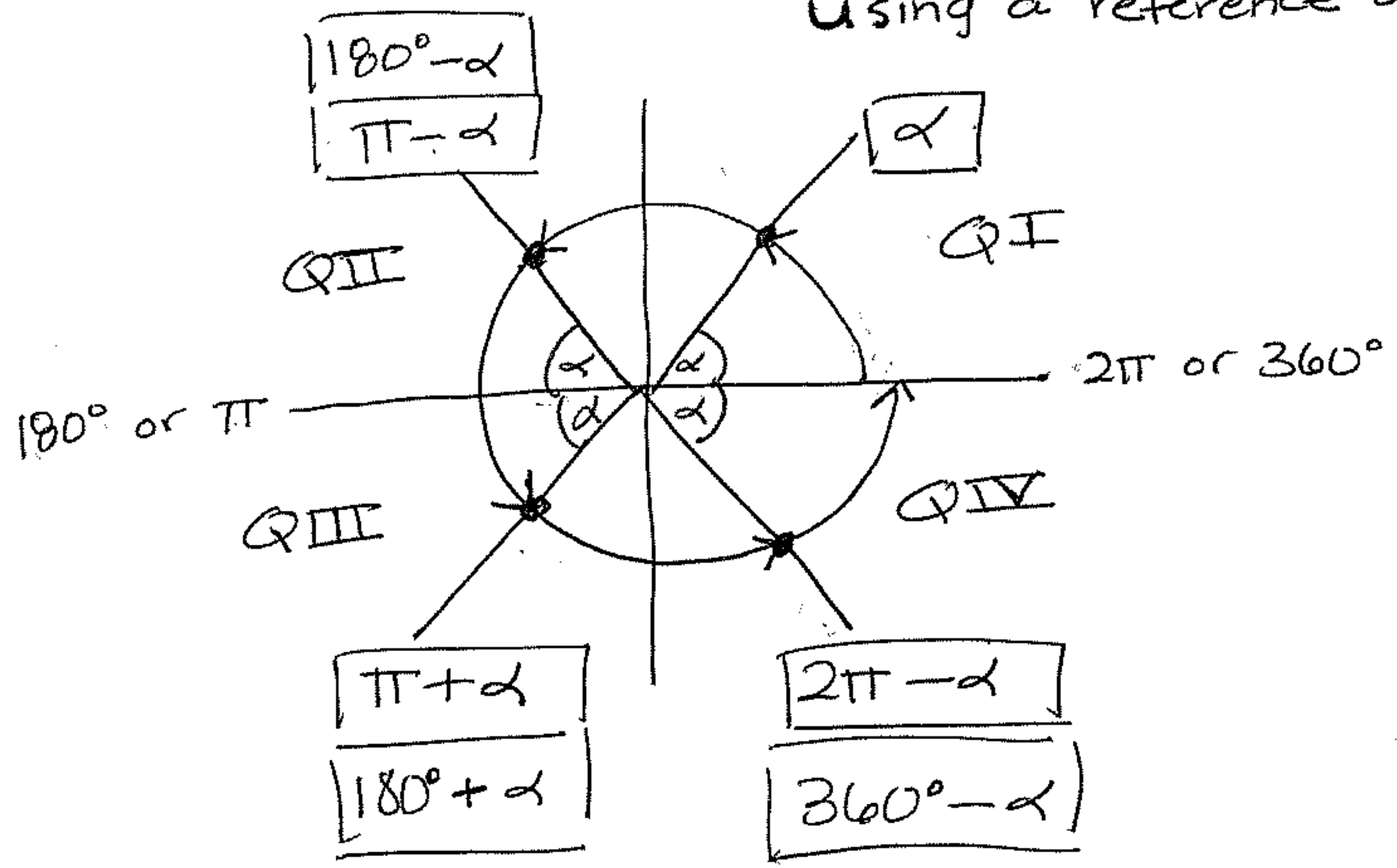
$\boxed{\theta = 240°, 300°}$



$\boxed{\theta = \frac{4\pi}{3}, \frac{5\pi}{3}}$

STUDY THIS! KNOW THIS!

To find angles btwn $[0, 2\pi)$ or $[0, 360°)$ using a reference angle.



Ex. 7: If $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$, find the exact value of the following (no calculator) if $\theta = 225°$:

a) $f(\theta) + g(\theta)$

$\sin\theta + \cos\theta$

$\sin 225° + \cos 225°$

$\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)$

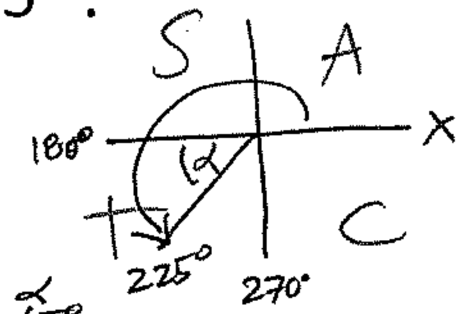
$-\frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \boxed{-\sqrt{2}}$

$\sin 225° = \pm \sin \underset{}{\overset{\alpha}{45°}}$

$= -\frac{\sqrt{2}}{2}$

$\cos 225° = \pm \cos \overset{\alpha}{45°}$

$= -\frac{\sqrt{2}}{2}$



$\alpha = 225° - 180°$

$\alpha = 45°$

b) $[g(\theta)]^2$

$[\cos\theta]^2$

$[\cos 225°]^2$

$\left(-\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \boxed{\frac{1}{2}}$

a) $2f(\theta)$

$2\sin\theta$

$2\sin 225°$

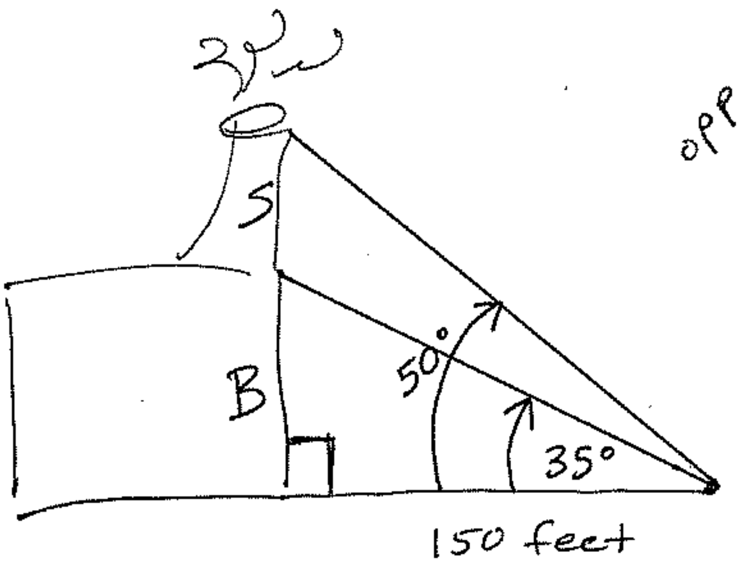
$\cancel{2}\left(-\frac{\sqrt{2}}{\cancel{2}}\right) = \boxed{-\sqrt{2}}$

EX. 8)

Right Triangles: At a point 150 feet from the base of a building with a smokestack on top of it, the angle of elevation to the ***bottom*** of the smokestack is 35°, and the angle of elevation to the ***top*** is 50°. Find the height of the smokestack. Round to the nearest thousandth.

B

S

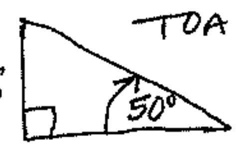


• Find the height of the bldg (B) from the small Δ.

TOA $\frac{\tan 35°}{1} = \frac{B}{150}$

$B = 150 \tan 35°$

• Now use the larger Δ.



TOA $\frac{\tan 50°}{1} = \frac{B+S}{150}$

$B + S = 150 \tan 50°$

$S = 150 \tan 50° - B$

exact $\boxed{S = 150 \tan 50° - 150 \tan 35° \text{ feet}}$

Mode → Degrees w/calc. $S \approx 73.732$ feet

Ex.9) Review: LINEAR SPEED

A satellite in a circular orbit 1250 km above Earth makes one complete revolution every 110 minutes. Assume that Earth is a sphere of radius 6400 km.

A) What is its angular speed in radians per minute (in terms of π)?

$$A.S. = \frac{\theta}{t} = \frac{1\ \cancel{rev}}{110\,\text{min}} \cdot \frac{2\pi\ \text{rad.}}{1\ \cancel{rev}} = \frac{2\pi}{110}\ \text{rad/min}$$

$$= \boxed{\frac{\pi}{55}\ \text{rad/min}}$$

angular speed.

B) What is its linear speed in km per hour?

Give the answer in terms of π (exact form). Then use a calculator to give the answer rounded to 3 decimal places.

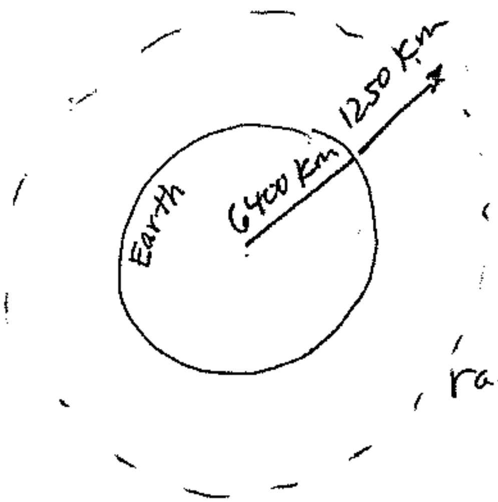
$$L.S. = \frac{s}{t} = \frac{\theta r}{t} = (A.S.) \cdot r$$

$$L.S. = \frac{\pi}{\cancel{55}_{11}\ \cancel{\text{min}}} \cdot \frac{7650\ \text{km}}{1} \cdot \frac{\cancel{60}^{12}\ \cancel{\text{min}}}{1\ \text{hr}}$$

$$= \boxed{\frac{91800\pi}{11}\ \text{km/hr}}$$ exact

$$\approx \boxed{26{,}218.019\ \text{km/hr}}$$

$$\begin{array}{r} 7650 \\ \times\ 12 \\ \hline 15300 \\ 76500 \\ \hline \end{array}$$



$$\begin{array}{r} \text{radius} = 6400 \\ +\ 1250 \\ \hline 7650\ \text{km} \end{array}$$