Notes

PreCalculus
Sec. 1.3
Difference Quotient

Ex1) *Given* : $f(x) = 2x^2 - x + 3$

Find the Difference Quotient (simplified) : $\dfrac{[f(x+h)] - [f(x)]}{h}; h \neq 0$

$$DQ = \frac{\left[2\underbrace{(x+h)^2}_{\text{foil}} - (x+h) + 3\right] - \left[2x^2 - x + 3\right]}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - x - h + 3 - 2x^2 + x - 3}{h}$$

$$= \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{- x} - h \cancel{+ 3} \cancel{- 2x^2} \cancel{+ x} \cancel{- 3}}{h}$$

$$= \frac{4xh + 2h^2 - h}{h}$$

$$= \frac{\cancel{h}(4x + 2h - 1)}{\cancel{h}} = \boxed{4x + 2h - 1;\ h \neq 0}$$

Ex2) $Given: f(x)=\sqrt{x}$ Given as a positive only

Find the Difference Quotient (simplified) : $\frac{[f(4+h)]-[f(4)]}{h}; h \neq 0$ value

$$DQ = \frac{\left[\sqrt{4+h}\right]-\left[\sqrt{4}\right]}{h}$$

multiply by the conjugate

$$= \left(\frac{\sqrt{4+h}-2}{h}\right) \cdot \frac{\left(\sqrt{4+h}+2\right)}{\left(\sqrt{4+h}+2\right)}$$

$$= \frac{(\cancel{4}+h)+\cancel{2\sqrt{4+h}}-\cancel{2\sqrt{4+h}}-\cancel{4}}{h\left(\sqrt{4+h}+2\right)}$$

$$= \frac{\cancel{h}1}{\cancel{h}\left(\sqrt{4+h}+2\right)}$$

$$= \boxed{\frac{1}{\sqrt{4+h}+2}; h \neq 0}$$

$\sqrt{4}=2$

$\sqrt{4} \neq \pm 2$

$x^2=4$

$\sqrt{x^2}=\sqrt{4}$

$|x|=2$

$x=\pm 2$

$\sqrt{4+h} \neq 2\sqrt{h}$

$\sqrt{4h}=2\sqrt{h}$

See what if $\rightarrow$

From Ex2

What if?

conj.

$$(2-\sqrt{4+h})(2+\sqrt{4+h})$$

$$= 4 + \cancel{2\sqrt{4+h}} - \cancel{2\sqrt{4+h}} - (4+h)$$

$$= \cancel{4} - \cancel{4} - h$$

$$= -h$$

Ex3) *Given*: $f(x) = x^3 - 8x$

Find the Difference Quotient (simplified): $\dfrac{[f(x+h)] - [f(x)]}{h}; h \neq 0$

$$DQ = \frac{[(x+h)^3 - 8(x+h)] - [x^3 - 8x]}{h}$$

$$= \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{8x} - 8h - \cancel{x^3} + \cancel{8x}}{h}$$

$$= \frac{3x^2h + 3xh^2 + h^3 - 8h}{h}$$

$$= \frac{\cancel{h}(3x^2 + 3xh + h^2 - 8)}{\cancel{h}}$$

$$= \boxed{3x^2 + 3xh + h^2 - 8; \quad h \neq 0}$$

$(x+h)^3$

$= (x+h)\underbrace{(x+h)(x+h)}$

$= (x+h)(x^2 + 2xh + h^2)$

$= x^3 + 2x^2h + xh^2$
$\quad + x^2h + 2xh^2 + h^3$

$x^3 + 3x^2h + 3xh^2 + h^3$

PreCalculus

Sec. 1.3

Even or Odd Functions

To Determine Even or Odd Functions

- Always find $f(-x)$ first!!! Then…

If $f(-x) = f(x)$:

(Same as Original)

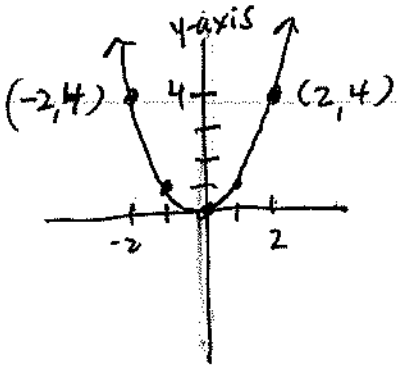
Then the function is **Even**

The graph of Even function has y-axis symmetry.

(both $(-x, y)$ and (x, y) are on the graph)

LOF $y = x^2$

Quadratic



If $f(-x) = -f(x)$:

(Opposite of the Original)

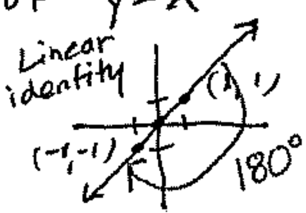
Then the function is **Odd**

The graph of Odd function has origin symmetry.

(both (x, y) and $(-x, -y)$ are on the graph)

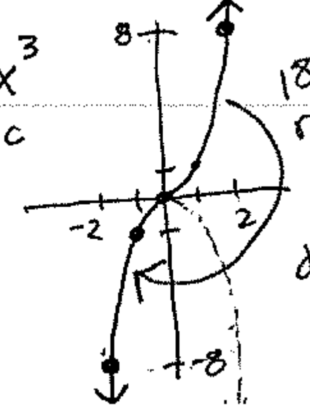
LOF $y = x^1$

Linear identity



$y = x^3$

Cubic



180° rotation or double flip

Find the coordinates of a 2^{nd} point on the graph:

Given:	Even Function change the x-values	Odd Function change both x & y values
(-2, 3)	(2, 3)	(2, -3)
(6, -1)	(-6, -1)	(-6, 1)
(-5,-2)	(5, -2)	(5, 2)

• work must be shown as taught

Ex. 1: Algebraically determine whether each of the following functions are even, odd, or neither.

a) $h(x) = \frac{-x}{\sqrt{7+x^2}}$

Find $h(-x) = \frac{-(-x)}{\sqrt{7+(-x)^2}}$ (Label)

$\boxed{h(-x) = \frac{x}{\sqrt{7+x^2}}}$

same
opp
neither

must write conclusion

$h(-x) = -h(x)$

$\therefore$ h(x) is an odd function

unsure?

Plug into the original:

$h(2) = -\frac{2}{\sqrt{11}}$

$h(-2) = +\frac{2}{\sqrt{11}}$

opposites

b) $f(x) = \dfrac{x-1}{x-2x^2}$

Find $f(-x) = \dfrac{(-x)-1}{(-x)-2(-x)^2}$

$$\boxed{f(-x) = \frac{-x-1}{-x-2x^2}}$$

$$= \frac{\cancel{-}(x+1)}{\cancel{-}(x+2x^2)}$$

or

$$\boxed{f(-x) = \frac{x+1}{x+2x^2}}$$

same

opp

neither

unsure?

$f(2) = -\frac{1}{6}$

$f(-2) = \frac{3}{10}$

neither

$$\boxed{\begin{array}{l} f(-x) \neq f(x) \text{ or } -f(x) \\ \therefore f(x) \text{ is neither even or odd} \end{array}}$$

c) $g(x) = 2x\left|x^2 - 3\right|$

<u>unsure</u> :

$g(2) = 4$

$g(-2) = -4$

opposites

Find $g(-x) = 2(-x)\left|(-x)^2 - 3\right|$

$\boxed{g(-x) = -2x\left|x^2 - 3\right|}$

same

opp

neither

$g(-x) = -g(x)$

$\therefore$ $g(x)$ is an odd function

d) $f(x) = -2x^3 - x + 3$

$f(-x) = -2(-x)^3 - (-x) + 3$

$-2(-x^3)$

$\boxed{f(-x) = 2x^3 + x + 3}$

same

opp

neither

$f(-x) \neq f(x)$ or $-f(x)$

$\therefore f(x)$ is neither even or odd

e) $r(x) = \dfrac{3x^2 - 7}{|x| + 5}$

unsure?

$r(2) = \frac{5}{7}$

$r(-2) = \frac{5}{7}$

Same

Find $r(-x) = \dfrac{3(-x)^2 - 7}{|(-x)| + 5}$

$\boxed{r(-x) = \dfrac{3x^2 - 7}{|x| + 5}}$

same
opp
neither

$r(-x) = r(x)$

$\therefore$ $r(x)$ is an even function

Review: Find the domain for the following functions.
Express your answers in *set* and *interval* notation.

SET INT.

a) $f(x)=\sqrt{49-x^2}$ $\{x \mid -7 \le x \le 7\}$ $[-7,7]$

b) $g(x)=\dfrac{2x}{\sqrt{x-3}}$ $\{x \mid x>3\}$ $(3,\infty)$

c) $h(x)=\dfrac{3x}{\sqrt{x^2+2}}$ $\{x \mid x\in\mathbb{R}\}$ $(-\infty,\infty)$

d) $k(x)=2+\log_5(x-1)$ $\{x \mid x>1\}$ $(1,\infty)$

e) $f(x)=\dfrac{-7x}{\log_3(x-5)}$ $\{x \mid x>5;\ x\neq 6\}$ $(5,6)\cup(6,\infty)$

or

$\{x \mid 5<x<6 \text{ or } x>6\}$