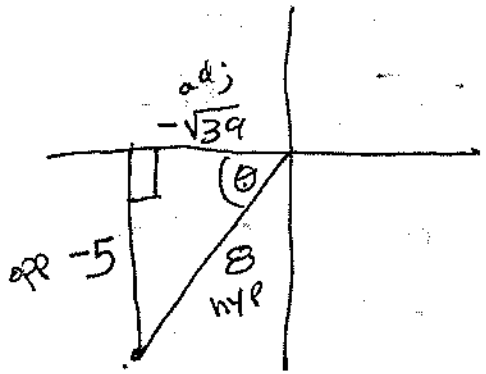
Notes

Pre-Calculus
Sec. 4.3
Right Triangle Trig.

Ex.1: Find the value of $\cos\theta$ *and* $\tan\theta$, if θ is an angle in the 3rd quadrant, and $\sin\theta = -\frac{5}{8}$.

SOH: opp / hyp



$(-x, -y)$

$(=, -5)$

$a^2 + b^2 = c^2$

$x^2 + y^2 = r^2$

$x^2 + 25 = 64$

$x^2 = 39$

$x = \pm\sqrt{39}$ ← 3rd, 13

make a choice (Based on the quadrant)

CAH

$\cos\theta = \frac{A}{H} = \frac{-\sqrt{39}}{8}$

$\boxed{\cos\theta = -\frac{\sqrt{39}}{8}}$

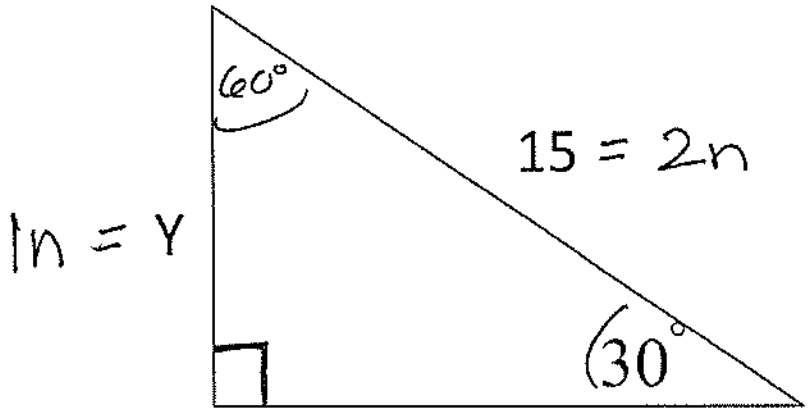
TOA

$\tan\theta = \frac{O}{A} = \frac{+5}{+\sqrt{39}} \cdot \frac{\sqrt{39}}{\sqrt{39}}$

$\boxed{\tan\theta = \frac{5\sqrt{39}}{39}}$

Ex.2: Find the exact value of the unknown variables in the right triangle.

$30°-60°-90°$



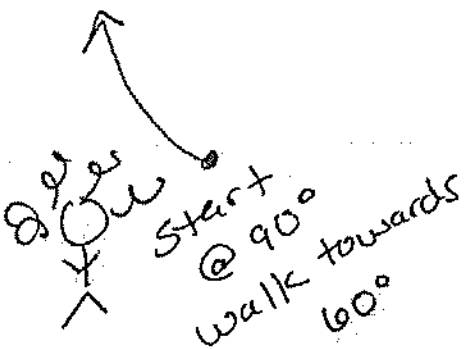
$15 = 2n$

$\frac{15}{2} = n$

$X = n\sqrt{3}$

$Y = 1n$

$\boxed{Y = \frac{15}{2}}$



$X = n\sqrt{3}$

$= \frac{15}{2} \cdot \sqrt{3}$

$\boxed{X = \frac{15\sqrt{3}}{2}}$

or use SOH CAH TOA

$\underbrace{\sin 30°}_{\downarrow} = \frac{Y}{15}$

$15 \cdot \frac{1}{2} = \frac{Y}{15} \cdot 15$

$\frac{15}{} = Y$

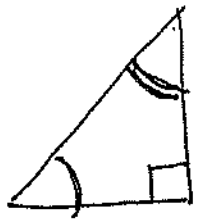
Cofunction Rules

Sine and Cosine
Secant and Cosecant
Tangent and Cotangent
} are Cofunctions

Ex. $\sin 30^\circ = \frac{1}{2} = \cos 60^\circ$ (90°)

$\sin 45^\circ = \frac{\sqrt{2}}{2} = \cos 45^\circ$ (90°)

$\tan 30^\circ = \frac{\sqrt{3}}{3} = \cot 60^\circ$



This occurs because the angles are complementary, in general, it can be shown from the right triangle definitions that ***cofunctions of complementary angles are equal***.

(The angles add up to 90° or π/2.)

$\sec 20^\circ = \csc 70^\circ$ (90°)

Cofunction Identities

$\frac{\pi}{2}$

$\sin(90° - \theta) = \cos\theta$ $\quad\quad$ $\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$ $\quad\quad$ $\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \csc\theta$ $\quad\quad$ $\csc(90° - \theta) = \sec\theta$

↑ never distribute the abbreviation of a trig. function → $\sec(90° - \theta) \neq \sec 90° - \sec\theta$ (NO) NEVER!!!

For radians $\pi/2$ will be substituted for 90°.

Note: These equations move from both left to right and right to left. Expand and Condense!!!!

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

condenses

often used in proofs (verify)

Ex3) Find a cofunction with the same value as the given expression.

a) $sin35° =$ $\cos 55°$

$90° - 35° = 55°$

b) $\tan \frac{2\pi}{11} =$ $\cot\left(\frac{7\pi}{22}\right)$ (radians)

$\frac{11}{11} \cdot \frac{\pi}{2} - \frac{2\pi}{11} \cdot \frac{2}{2}$

$\frac{11\pi}{22} - \frac{4\pi}{22} = \frac{7\pi}{22}$

c) $sec17° =$ $\csc 73°$

$90° - 17° = 73°$

Mode Key

Radians ⟷ Degrees

Ex4: Evaluate with a calculator (Round to 4 decimal places.)

a) $\tan 35° \approx$ .7002

↑ degrees

b) $\cot 35°$

↑ degrees

$= \frac{1}{\tan 35°}$ or $\tan 35°$ x^{-1}

↑ used to take the reciprocal

≈ 1.4281

c) $\csc 5$ ← Radians (no degree symbol) (change mode)

$= \frac{1}{\sin 5}$ or $\sin 5$ x^{-1}

≈ -1.0428

d) $\cot 7\pi/5$

radians

$= \frac{1}{\tan \frac{7\pi}{5}}$ or $\tan \frac{7\pi}{5}$ x^{-1}

≈ 0.3249

* e) $\tan(7/5\pi) \approx .4777$

need ( ) because of order of operations

$\tan(7/(5\pi))$

mode?

Ex5) Use a calculator to find the value of the acute angle θ in radians, round to 3 decimal places.

a) $sin\theta = 0.9499$

inverse

$\theta = \sin^{-1}(0.9499)$ ← write this on your paper

$\theta \approx 1.253$ radians

mode must be set for the units you want → radians

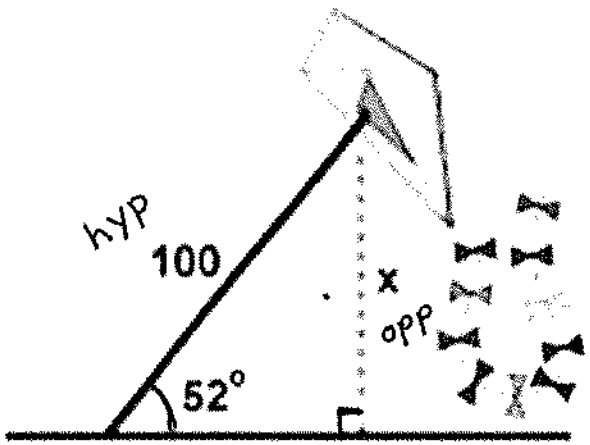
b) $tan\theta = 0.5117$

inverse

$\theta = \tan^{-1}(0.5117)$

$\theta \approx 0.473$ radians

Ex.6) A girl flies a kite with a 100 foot string. The angle of elevation of the string is 52°. How high off the ground is the kite? Round answer to 3 decimal places.



Rt. Δ use:

SOH CAH TOA

SOH

$\sin\theta = \frac{O}{H}$

$\frac{\sin 52^\circ}{1} \times \frac{X}{100}$

Show this. $X = 100 \sin 52^\circ$

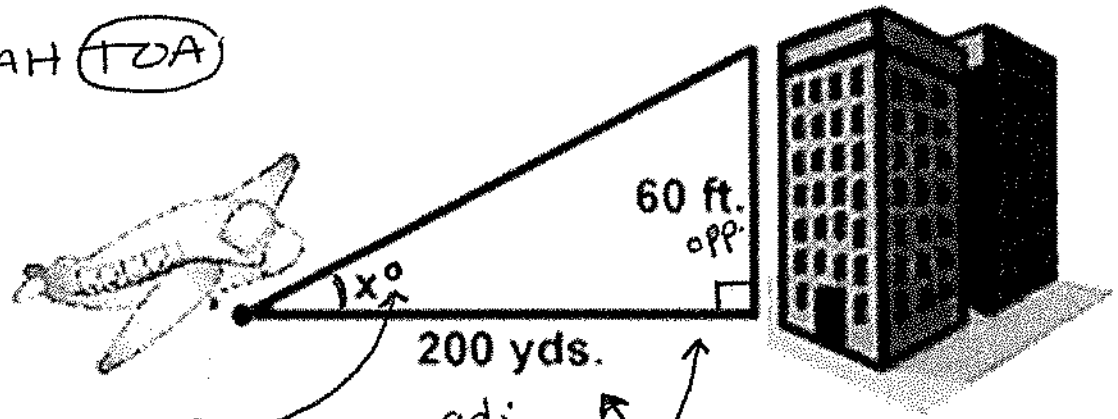
must be in degree mode

$X \approx 78.801$ feet

Ex. 7) An airplane takes off 200 yards in front of a 60 foot building. At what angle of elevation must the plane take off in order to avoid crashing into the building? Assume that the airplane flies in a straight line and the angle of elevation remains constant until the airplane flies over the building. Round answer to 3 decimal places.

RT. Δ

SOH CAH (TOA)



Degree mode

need same units

TOA

$\tan\theta = \frac{O}{A}$

$\tan X = \frac{60}{600}$

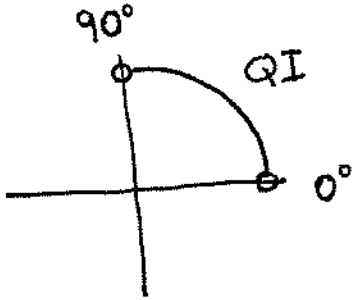
$X = \tan^{-1}\left(\frac{60}{600}\right)$

$X \approx 5.711°$

3 feet = 1 yard

$\frac{200\text{yds}}{1} \cdot \frac{3\text{ft}}{1\text{yd}} = 600\text{ft}$

Ex.8) Find each value of θ in degrees $(0° < \theta < 90°)$ and radians $(0 < \theta < \frac{\pi}{2})$ without a calc.



$a) \cot\theta = \frac{\sqrt{3}}{3}$

same angle where

reciprocal

$\tan\theta = \sqrt{3}$

$\theta = 60°$ or $\frac{\pi}{3}$

- you can take the reciprocal of a value, but the angle stays the same.

$b) \sec\theta = \sqrt{2}$

same angle where

reciprocal

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = 45°$ or $\frac{\pi}{4}$

Ex.9) Evaluate:

Use reciprocal

$\sin\frac{\pi}{6} = \frac{1}{2}$

a) $\tan\frac{\pi}{4} + \csc\frac{\pi}{6}$

$(\ 1\ ) + (\ 2\ )$

$= \boxed{3}$

Use reciprocal

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

b) $6\tan\frac{3\pi}{4} + \sin\frac{\pi}{3}\sec\frac{\pi}{6}$

$6(-1) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{3}}\right)$

PEMDAS

$\alpha = \frac{\pi}{4}$

$= \pm\tan\alpha$

$= -\tan\frac{\pi}{4}$

$= -1$

$-6 + 1$

$= \boxed{-5}$