Notes

Lesson2: Pre-Calculus Sec. 1.7: Domain

2 Ways to Find the Domain of a Function

1) Graphically: Use the X-axis and the graph

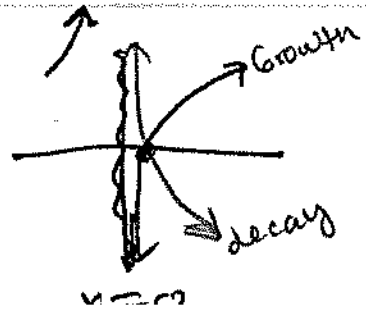
1) Algebraically: "Picky eaters"

$\frac{1}{f(x)}$, $\boxed{f(x) \neq 0}$ a) ***excluding*** any #'s that make the denominator of the equation **zero**. undefined

any even root/index

$\sqrt{f(x)}$, $\boxed{f(x) \geq 0}$ b) ***excluding*** any #'s that result in the even root equation **negative**.

$\log_b f(x)$, $\boxed{f(x) > 0}$ c) ***excluding*** any #'s that result in the log equation **zero** or **negative**.



open

(not including zero)

- asymptote there on LOF graph

Ex. 1: Find the domain of each relation. Express your answers in both *set* and *interval* notations.

{ } braces (and/or]

a) $f(x) = 4x^2 - 3x + 1$

Quadratic

SET : $\{x \mid x \in \mathbb{R}\}$

INTERVAL : $(-\infty, \infty)$

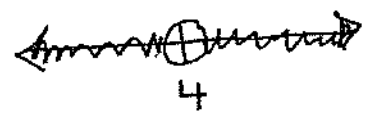
constant

b) $g(x) = \dfrac{3}{x-4}$ $\neq 0$

$x - 4 = 0$

$x = 4$

$x \neq 4$



S : $\{x \mid x \neq 4\}$

I : $(-\infty, 4) \cup (4, \infty)$

other set notation

$\{x \mid x < 4 \text{ or } x > 4\}$

$\sqrt{0} = 0$

even root

c) $g(x) = \dfrac{\sqrt{2x-3}}{1}$

$2x - 3 \geq 0$

$2x \geq 3$

$x \geq \frac{3}{2}$

$\frac{3}{2}$

$\boxed{\begin{array}{l} S: \{x \mid x \geq \frac{3}{2}\} \\ I: [\frac{3}{2}, \infty) \end{array}}$

Linear

d) $r(x) = \dfrac{2x+1}{\sqrt{x-3}}$ ✓ $\neq 0$

$x - 3 \cancel{\geq} 0$

$x - 3 > 0$

$x > 3$

3

$\boxed{\begin{array}{l} S: \{x \mid x > 3\} \\ I: (3, \infty) \end{array}}$

Recall: $\dfrac{0}{5} \to \dfrac{0}{K}$ $= 0$ | $\dfrac{5}{0} \to \dfrac{N}{0}$ way $=$ undefined

$$e)\ p(x) = \frac{\sqrt{x-2}}{x-5}$$

$x - 2 \geq 0$

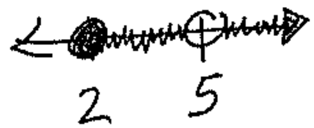
$x \geq 2$

$\neq 0$

$x - 5 = 0$

$x = 5$

$x \neq 5$



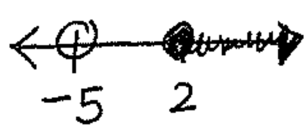
$$\boxed{\begin{array}{l} S: \{x \mid x \geq 2;\ x \neq 5\} \\ I: [2, 5) \cup (5, \infty) \end{array}}$$

other set notation

$\{x \mid 2 \leq x < 5 \text{ or } x > 5\}$

What if? $x \neq -5$ with $x \geq 2$

$S: \{x \mid x \geq 2;\ \cancel{x \neq -5}\}$



$\boxed{S: \{x \mid x \geq 2\}}$ $\boxed{I: [2, \infty)}$

$$f)\ q(x) = \log_{10}(x+2) - 5$$

open

$x + 2 > 0$

$x > -2$

$$\boxed{\begin{array}{l} S: \{x \mid x > -2\} \\ I: (-2, \infty) \end{array}}$$

What if?

$f(x) = \log x + 2$

$x > 0$

no ( ) here.

$$\boxed{\begin{array}{l} S: \{x \mid x > 0\} \\ I: (0, \infty) \end{array}}$$

$g)\ k(x) = 3^{1-x} + 2$

Exponential

$$\boxed{\begin{array}{l} S: \{x \mid x \in \mathbb{R}\} \\ I: (-\infty, \infty) \end{array}}$$

$h)\ q(x) = \dfrac{3}{x^2+4} \neq 0$

constant

$$x^2 + 4 = 0$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$|x| = 2i$$

$$x = \pm 2i$$

imaginary!

$$\boxed{\begin{array}{l} S: \{x \mid x \in \mathbb{R}\} \\ I: (-\infty, \infty) \end{array}}$$

semi circle

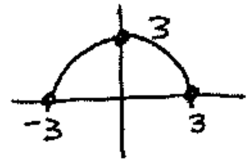
positive (top half)

i) $f(x) = \sqrt{9 - x^2}$

$r = \sqrt{9} = 3$

$f(x) = \sqrt{C - x^2}$

★ do Graphically:



• From the graph you get both D & R

<u>domain</u>:

S: $\{x \mid -3 \leq x \leq 3\}$

I: $[-3, 3]$

<u>range</u>:

S: $\{f(x) \mid 0 \leq f(x) \leq 3\}$

I: $[0, 3]$

How do I know it's a semi circle?

$y = \sqrt{9 - x^2}$

$(y)^2 = (\sqrt{9 - x^2})^2$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

w/ $r = 3$

circle centered @ $(0,0)$

$x^2 + y^2 = 9$

$\sqrt{y^2} = \sqrt{9 - x^2}$

$|y| = \sqrt{9 - x^2}$

$y = \pm\sqrt{9 - x^2}$

• Algebraically (only gives domain)

$f(x) = \sqrt{9 - x^2}$

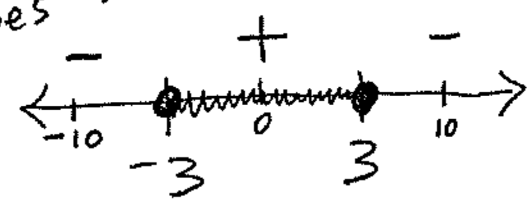
$9 - x^2 \geq 0$

$(3+x)(3-x) \geq 0$

Get Critical values

$3 + x = 0$ $\quad$ $3 - x = 0$

$x = -3$ $\quad$ $x = 3$



Test regions

$j)\ q(x) = \sqrt{x^2 - 25}$

positive (top)

what is this?

$y = \sqrt{x^2-25}$

$(y)^2 = \left(\sqrt{x^2-25}\right)^2$

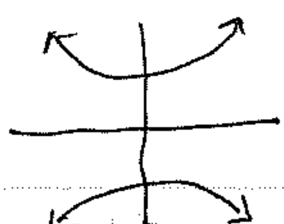
$y^2 = x^2 - 25$

$-1\left(-x^2 + y^2 = -25\right)$

$$\frac{x^2}{25} - \frac{y^2}{25} = \frac{25}{25}$$

$$\frac{x^2}{25} - \frac{y^2}{25} = 1$$

Hyperbola!



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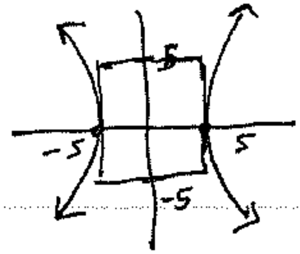
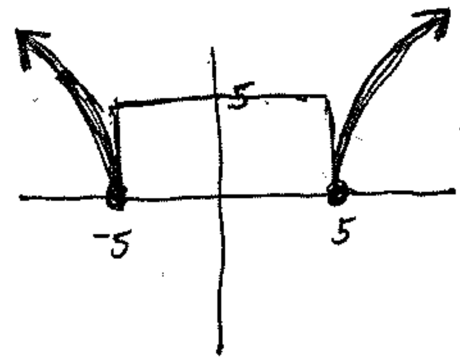


Graphs Top half only



A lot to recall for this graph, so work out the domain algebraically instead.

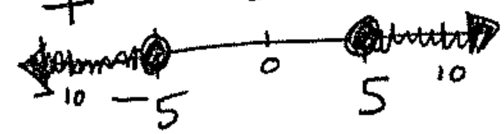
★ Better to do algebraically

$q(x) = \sqrt{x^2-25}$

$x^2 - 25 \geq 0$

$(x+5)(x-5) \geq 0$

$x = -5 \qquad x = 5$



Test regions

Domain:

S: $\{x \mid x \leq -5 \text{ or } x \geq 5\}$

I: $(-\infty, -5] \cup [5, \infty)$

even root

$k)\ g(x) = \sqrt[4]{x^4 - 8x}$

Factoring Cubes Rule
(SOAP)

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

S — same; O — opposite; AP — always positive

factor: $x^4 - 8x \geq 0$

GCF: $x(x^3 - 8) \geq 0$

factor w/ cubes: $x(x-2)(x^2+2x+4) \geq 0$

S O AP

$x^2 + 2x + 4 = 0$

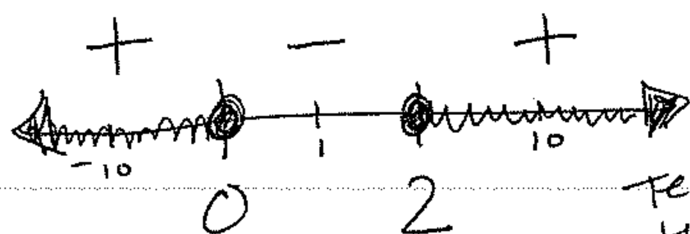
Prime → Does not factor

$D = b^2 - 4ac$

$D < 0$

= imaginary Solutions

Get critical values: $x = 0$ $\quad$ $x - 2 = 0$, $x = 2$



Test the regions

Domain:

S: $\{x \mid x \leq 0 \text{ or } x \geq 2\}$ $\quad$ I: $(-\infty, 0] \cup [2, \infty)$