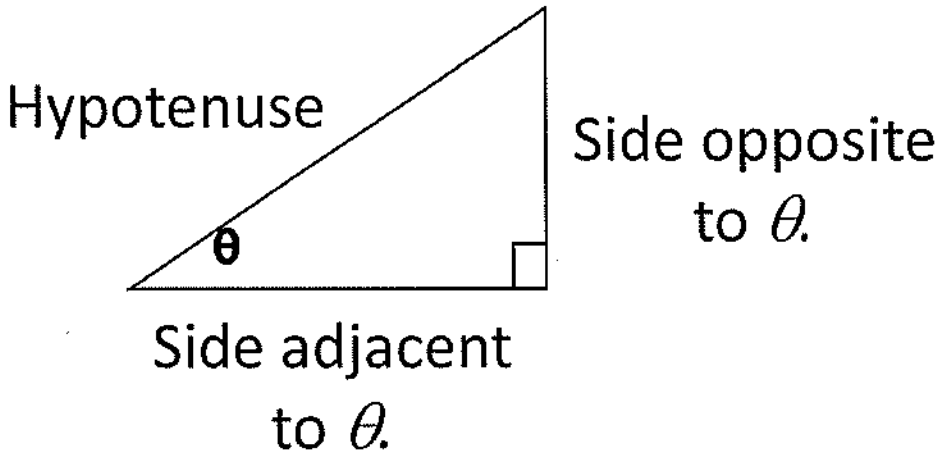
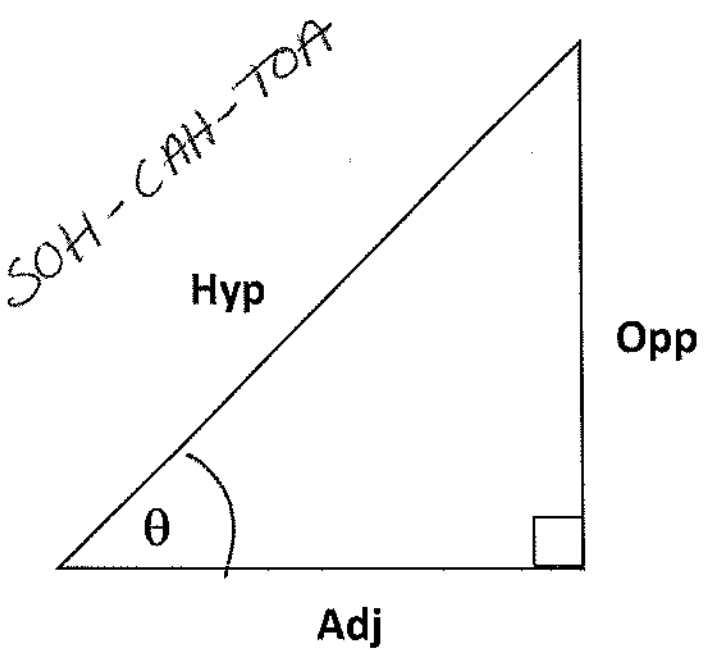
Notes

Pre-Calculus:
Sec. 4.2 Trig. Functions & The Unit Circle: Quadrant 1 and QA's

Trigonometry: measurement of triangles.



Right Triangle Definitions of Trigonometric Functions



Reciprocals

SOH

$\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$ Sine

$\csc\theta = \frac{\text{Hyp}}{\text{Opp}}$ Cosecant

CAH

$\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$ Cosine

$\sec\theta = \frac{\text{Hyp}}{\text{Adj}}$ Secant

TOA

$\tan\theta = \frac{\text{Opp}}{\text{Adj}}$ Tangent

$\cot\theta = \frac{\text{Adj}}{\text{Opp}}$ Cotangent

Opp = the length of the side *opposite* θ
Adj = the length of the side *adjacent* to θ
Hyp = the length of the *hypotenuse*

Evaluate Trigonometric functions of Special Angles

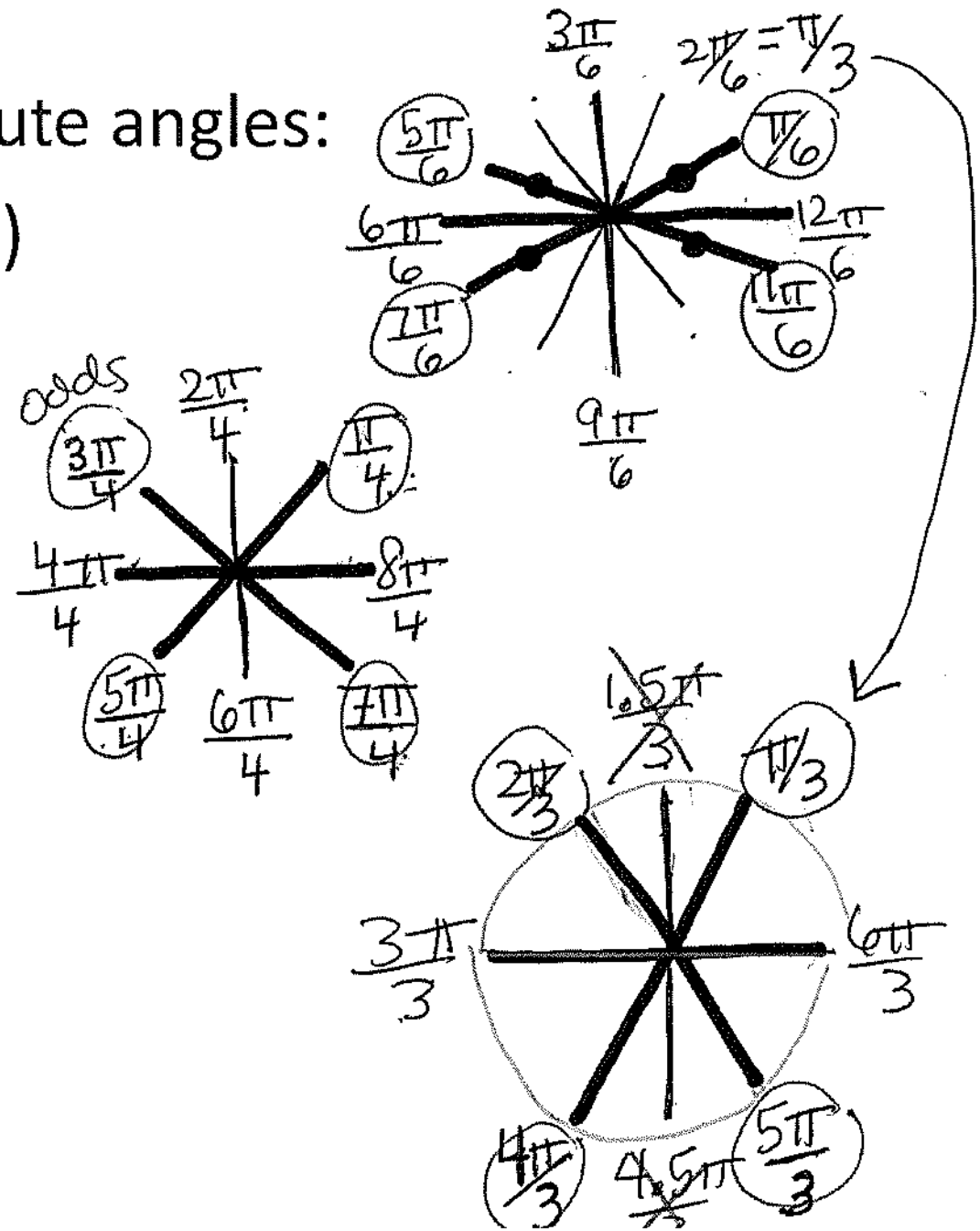
There are 3 special acute angles:

(degrees) (radians)

$30^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{6}$

$45^\circ = \frac{\pi}{4}$

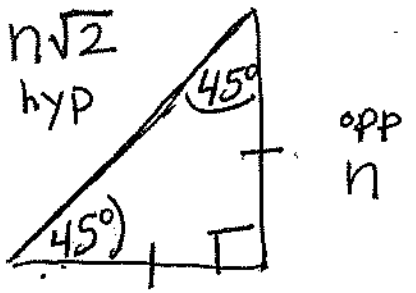
$60^\circ = \frac{\pi}{3}$



Evaluate Sine, Cosine, and Tangent of 30°, 45°, or 60°

45°-45°-90°

SOH $\sin 45^\circ = \frac{O}{H} = \frac{n}{n\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



$\sin 45^\circ = \frac{\sqrt{2}}{2}$ $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$a^2+b^2=c^2$

CAH $\cos 45^\circ = \frac{A}{H} = \frac{n}{n\sqrt{2}} = \frac{\sqrt{2}}{2}$

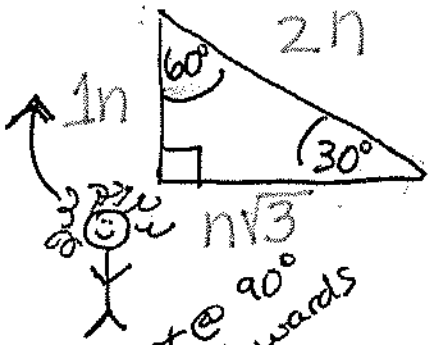
$\cos 45^\circ = \frac{\sqrt{2}}{2}$ $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

TOA $\tan 45^\circ = \frac{O}{A} = \frac{n}{n} = 1$

$\tan 45^\circ = 1$ $\tan \frac{\pi}{4} = 1$

* creates 6 flashcards

$30°-60°-90°$



* 12 more flashcards

SOH $\sin 30° = \frac{O}{H} = \frac{1n}{2n} = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$

$\sin \frac{\pi}{6} = \frac{1}{2}$

SOH $\sin 60° = \frac{O}{H} = \frac{n\sqrt{3}}{2n} = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

CAH $\cos 30° = \frac{A}{H} = \frac{n\sqrt{3}}{2n} = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

CAH $\cos 60° = \frac{A}{H} = \frac{1n}{2n} = \frac{1}{2}$

$\cos 60° = \frac{1}{2}$

$\cos \frac{\pi}{3} = \frac{1}{2}$

TOA $\tan 30° = \frac{O}{A} = \frac{1n}{n\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan 30° = \frac{\sqrt{3}}{3}$

$\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$

TOA $\tan 60° = \frac{O}{A} = \frac{n\sqrt{3}}{1n} = \sqrt{3}$

$\tan 60° = \sqrt{3}$

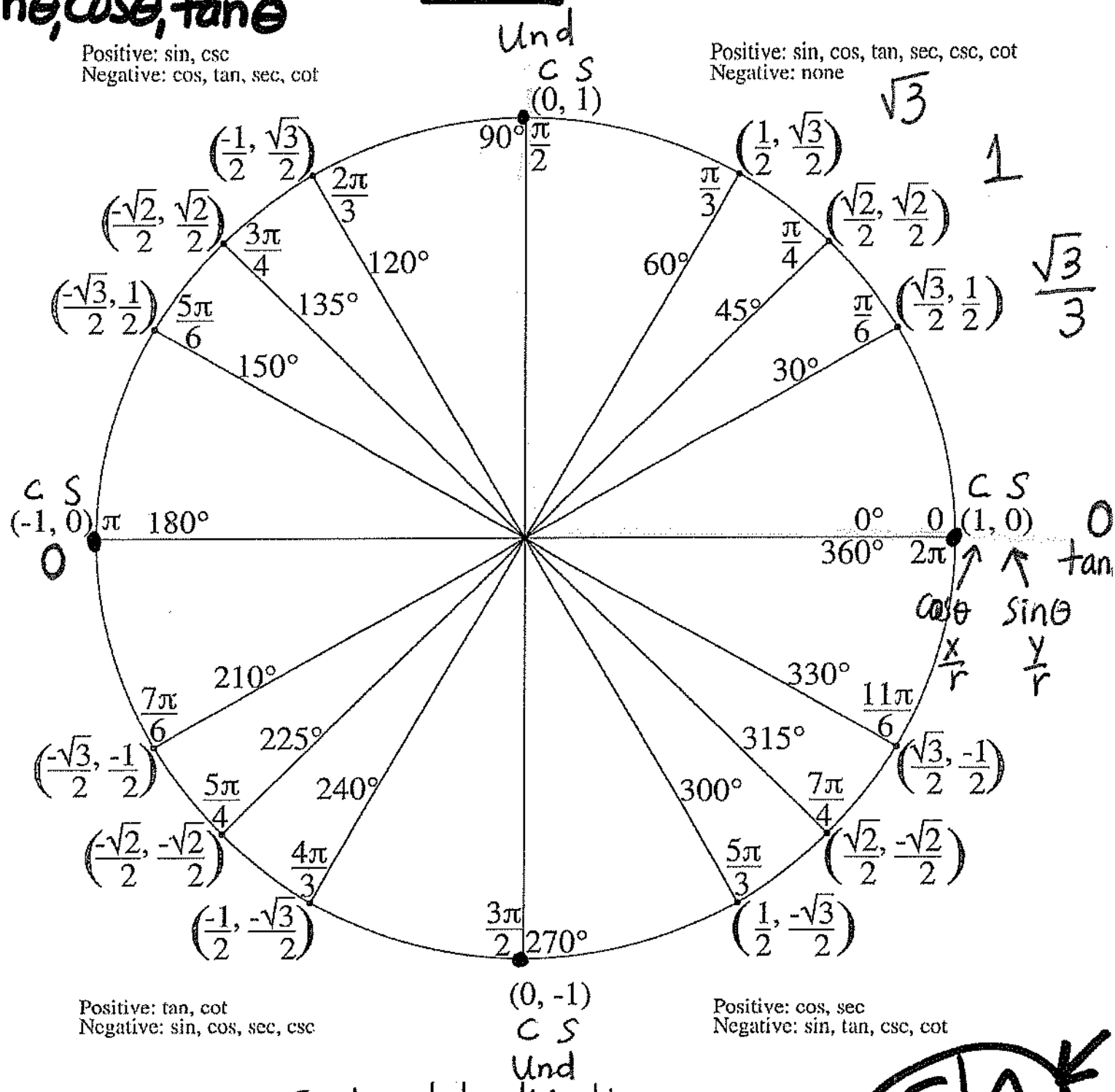
$\tan \frac{\pi}{3} = \sqrt{3}$

Need to know quad 1 & QA's for sinθ, cosθ, tanθ

r = 1

$\frac{SOH}{CAH} = TOA$

The Unit Circle



EmbeddedMath.com

Q1 and QA's

θ^o	0°	30°	45°	60°	90°	180°	270°
θ^R							
sin θ							
cos θ							
tan θ							

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

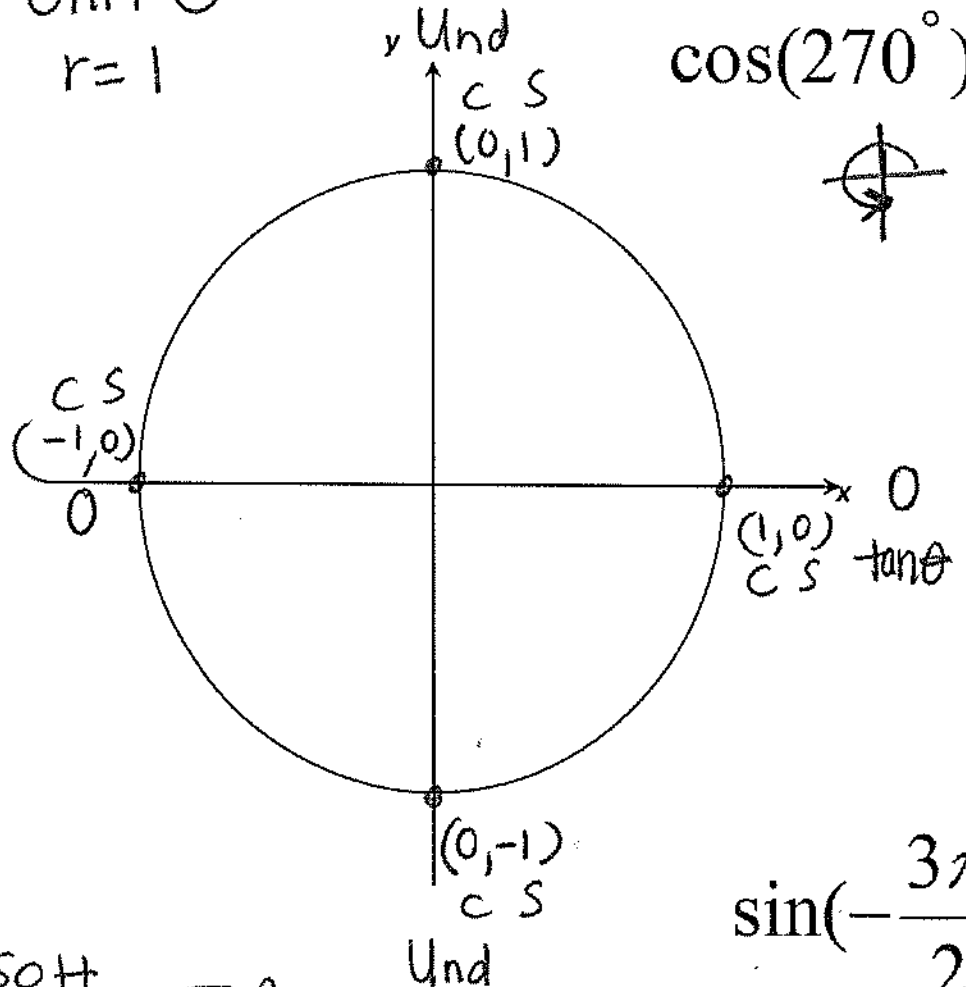
$$\frac{SOH}{CAH} = TOA$$

$$\frac{Y}{X} = \tan\theta$$

Quadrantal Angles (QA's)

• DO NOT USE reference ∡'s or rt. Δ's

Unit ⊙
r = 1



$\cos(270^\circ) = \boxed{0}$

$\cos(-180^\circ) = \boxed{-1}$

$\sin 180^\circ = \boxed{0}$

$\tan 2\pi = \overset{CT}{\tan 0_{rad}} = \frac{0}{1} = \boxed{0}$

$\sin(-\frac{3\pi}{2}) = \overset{CT}{\sin \frac{\pi}{2}} = \boxed{1}$

$+2\pi$

$\tan \frac{\pi}{2} = \frac{1}{0} = \boxed{\text{Und}}$

$\frac{SOH}{CAH} = TOA$

Signs of All Functions

- SOH
  CAH
  TOA
- Then add their reciprocal to the quadrant.

Quadrant II
(-x,y)

Students

Sinθ ⊕
Cscθ ⊕

Quadrant I
(x,y)

All

All 6 trig. functions are positive here.

Quadrant III
(-x,-y)

Take

tanθ ⊕
cotθ ⊕

Quadrant IV
(x,-y)

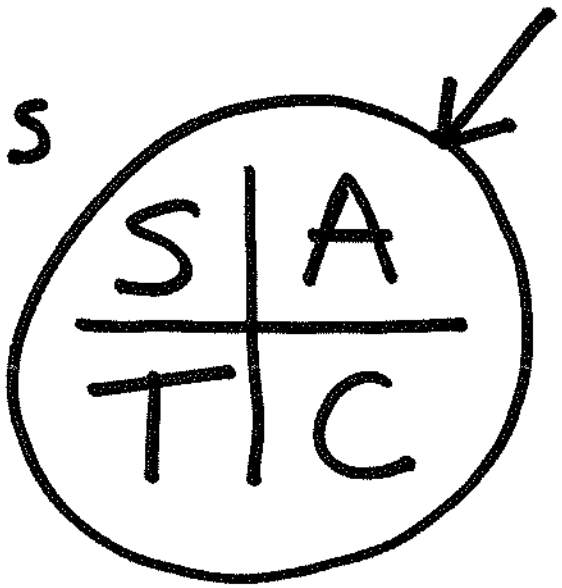
Calculus

cosθ ⊕
secθ ⊕

y

x

- Tells us which trig. functions are POSITIVE in each quadrant.



Even-Odd Identities:

negative angle

$\cos(-\theta) = \cos\theta$

$\sec(-\theta) = \sec\theta$

} Even… $f(-x) = f(x)$

opposite

$\sin(-\theta) = -\sin\theta$

$\csc(-\theta) = -\csc\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

} Odd… $f(-x) = -f(x)$ (opposite)

Ex1: The even/odd formulas can be used to rewrite a function with a negative angle as a function with a positive angle.

$$If \overset{odd}{\sin(-t)} = \frac{3}{8} \longrightarrow \underbrace{\sin(-t)}_{\text{odd rule}} = \frac{3}{8}$$

$$-\sin t = \frac{3}{8}$$

Find :

so...

$$\sin t = -\frac{3}{8}$$

$a)\ \sin t = \boxed{-\frac{3}{8}}$

same angle ↕ flip to reciprocal

$b)\ \csc t = \boxed{-\frac{8}{3}}$

Less than 90° ($\frac{\pi}{2}$)

Definition of a Reference Angle

Reference angle is a **positive acute angle** formed by the terminal side of θ and the closest side of the x-axis.

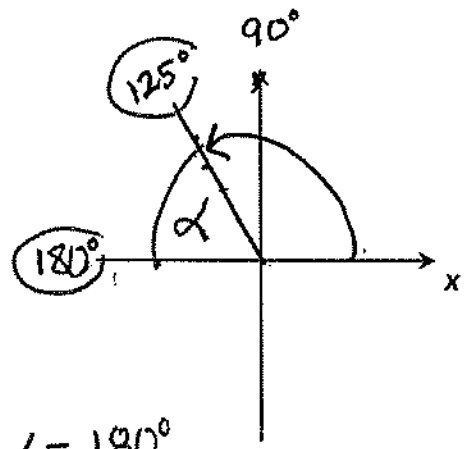
We often label the **reference angle** as α or θ'.

alpha Theta prime

Ex.2: Find the measure of its reference angle.

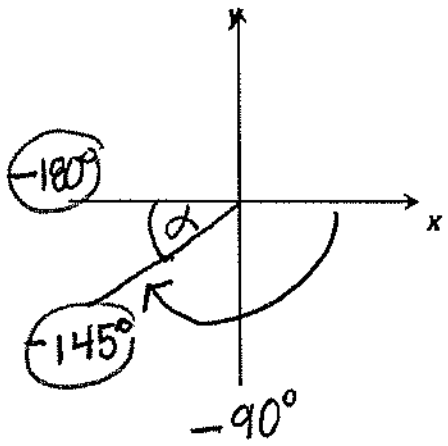
a. 125° b. -145° c. $\frac{8\pi}{5}$ d. 30°

negative



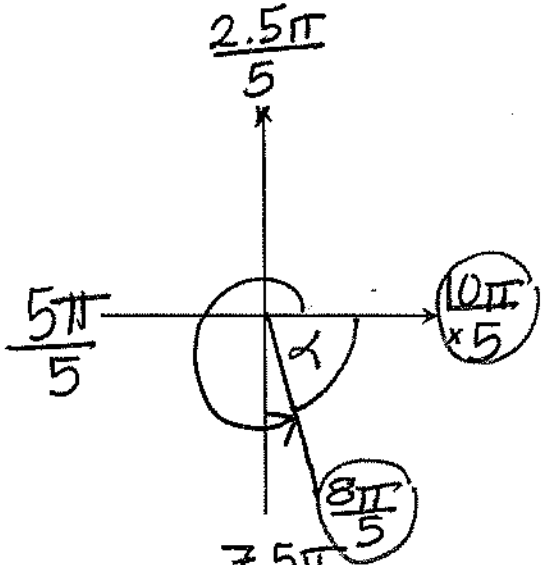
$\alpha = 180° - 125°$

$\alpha = 55°$

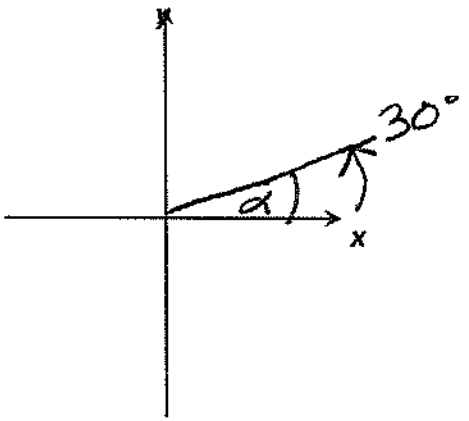


$\alpha = 35°$

positive



$\alpha = \frac{2\pi}{5}$



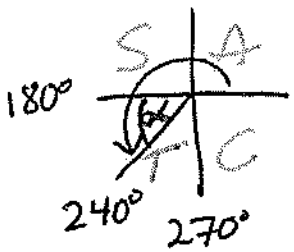
$\alpha = 30°$

Ex. 3: Evaluate:

a) $\sin 240^\circ = \pm \sin\alpha$

not QA

$= \pm \sin 60^\circ$

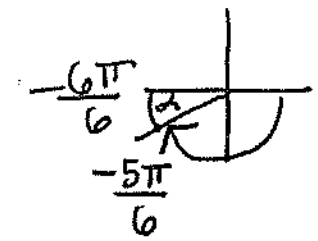


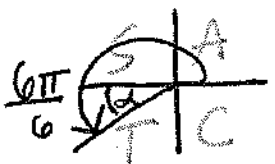
$\alpha = 60^\circ$

$= \boxed{-\frac{\sqrt{3}}{2}}$

b) $\cos\left(-\frac{5\pi}{6}\right) \overset{CT}{=} \cos\frac{7\pi}{6} = \cos\frac{5\pi}{6}$ (EVEN)

can do this one 3 ways



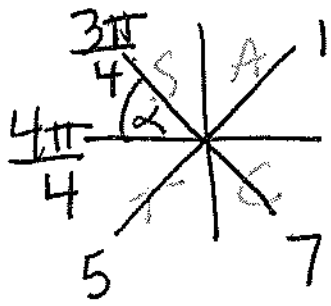


$\pm\cos\alpha = \pm\cos\frac{\pi}{6} = \pm\cos\frac{\pi}{6}$

$= \boxed{-\frac{\sqrt{3}}{2}}$

c) $\sin\frac{3\pi}{4} = \pm\sin\alpha$

$= \oplus \sin\frac{\pi}{4}$



$= \boxed{\frac{\sqrt{2}}{2}}$

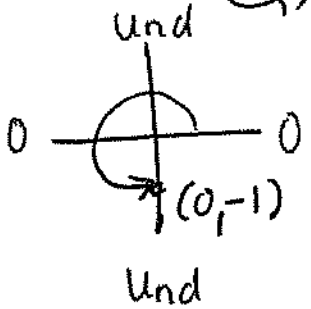
$\alpha = \frac{\pi}{4}$

d) $\tan 630^\circ$

-360°

$\overset{CT}{=} \tan 270^\circ = \frac{-1}{0} = \boxed{\text{Und}}$

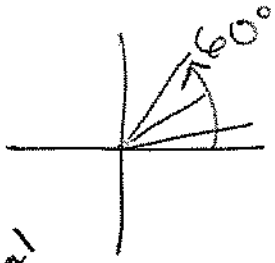
QA



Ex. 4: Evaluate:

a) Sec 60° = 2

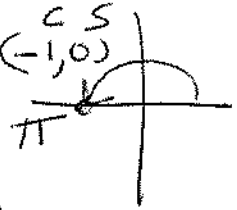
Use $\cos 60° = \frac{1}{2}$ → flip to reciprocal



b) $\csc \pi$ = Undefined

QA

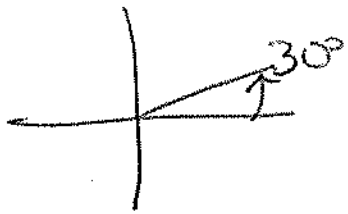
Use $\sin \pi = \frac{0}{1}$ → flip to reciprocal



c) $\cot \frac{\pi}{6}$ = $\sqrt{3}$

Use $\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$ → flip to reciprocal

$\frac{\pi}{6} \rightarrow 30°$



Reciprocals

$1 \longleftrightarrow 1$

$-1 \longleftrightarrow -1$

$0 \longleftrightarrow$ Und

$\frac{1}{2} \longleftrightarrow 2$

$\frac{\sqrt{2}}{2} \longleftrightarrow \sqrt{2}$

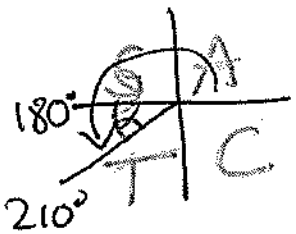
$\frac{\sqrt{3}}{3} \longleftrightarrow \sqrt{3}$

$\frac{\sqrt{3}}{2} \longleftrightarrow \frac{2\sqrt{3}}{3}$

Evaluate.

d) $\sec 210^\circ = \boxed{\frac{-2\sqrt{3}}{3}}$

not QA



$\alpha = 30^\circ$

use $\cos 210^\circ = \pm\cos\alpha$

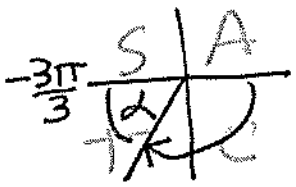
$= \pm\cos 30^\circ$

$= -\frac{\sqrt{3}}{2}$

flip to the reciprocal

e) $\csc\left(-\frac{2\pi}{3}\right) = \boxed{\frac{-2\sqrt{3}}{3}}$

use $\sin\left(-\frac{2\pi}{3}\right)$



$= \pm\sin\alpha$

$= \pm\sin\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}$

f) $\cot\frac{17\pi}{4} = \cot\frac{\pi}{4} = \boxed{1}$ (CT)

use CT angles

$\frac{2\pi}{1} \rightarrow \frac{8\pi}{4}$

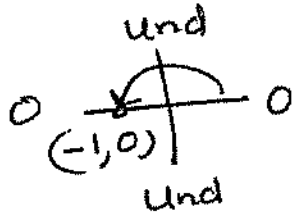
use $\tan\frac{\pi}{4} = 1$

g) $\cot 7\pi = \cot\pi = \boxed{\text{Und}}$

use CT

use $\tan\pi = \frac{0}{-1} = 0$

QA



w/ even & odd rules

h) $\cos\left(-\frac{\pi}{3}\right) = +\cos\frac{\pi}{3}$ ($60°$)

even

$= \boxed{\frac{1}{2}}$

i) $\sin\left(-\frac{2\pi}{3}\right) = -\sin\frac{2\pi}{3}$ (opposite)

odd

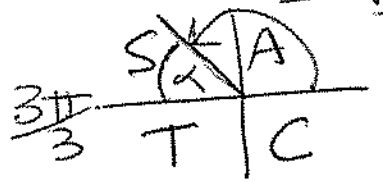
$= -\left(\frac{\sqrt{3}}{2}\right)$

$= \boxed{-\frac{\sqrt{3}}{2}}$

Find:

$\sin\frac{2\pi}{3} = \pm\sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

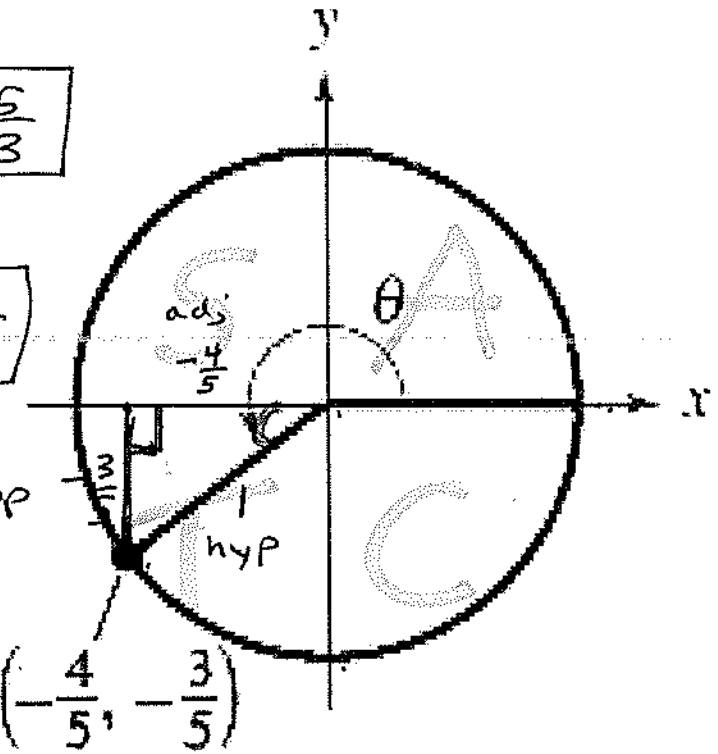


$\alpha = \frac{\pi}{3}$

Ex. **5** Determine the exact values of the six trigonometric functions of the angle θ on the unit circle.

$r = 1$

4.



SOH

$\sin\theta = \frac{O}{H} = \frac{-\frac{3}{5}}{1}$

$\sin\theta = -\frac{3}{5}$

CAH $\cos\theta = -\frac{4}{5}$

TOA $\tan\theta = \frac{O}{A} = \frac{+\frac{3}{5}}{+\frac{4}{5}}$

$\tan\theta = \frac{3}{4}$

Reciprocals

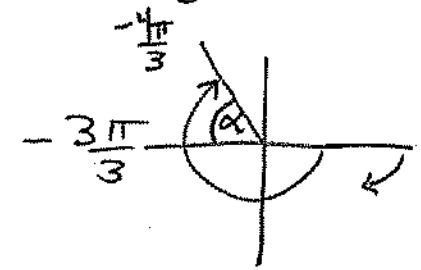
$\csc\theta = -\frac{5}{3}$

$\sec\theta = -\frac{5}{4}$

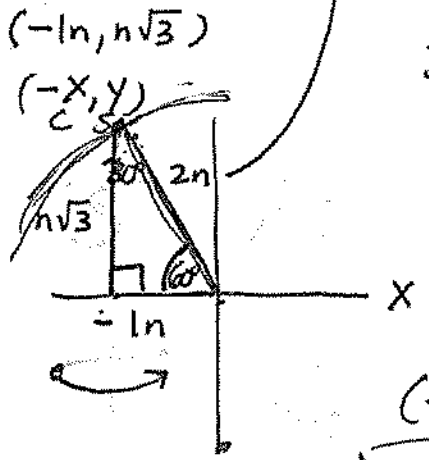
$\cot\theta = \frac{4}{3}$

Ex.**6**: Find the point (x,y) on the unit circle that corresponds to the real number *t*.

a) $t = -\frac{4\pi}{3}$



$\alpha = \frac{\pi}{3}$ or $60°$



$30°-60°-90°$

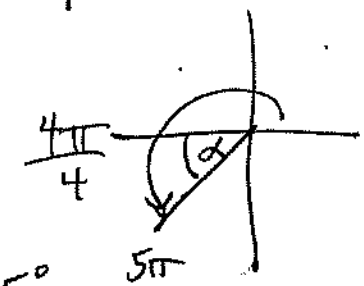
$2n = 1$ unit ⊙ radius

$n = \frac{1}{2}$

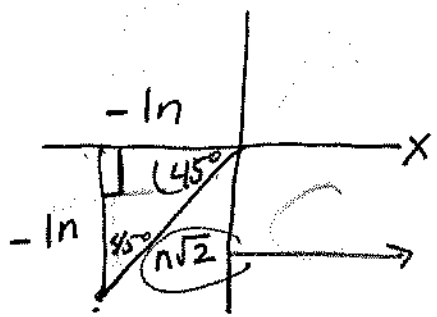
$(-1n, n\sqrt{3})$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

b) $t = \frac{5\pi}{4}$



$\alpha = \frac{\pi}{4}$ or $45°$



$45°-45°-90°$

$n\sqrt{2} = 1$ unit ⊙ radius

$n = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$(-1n, -1n)$

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$