# Pre-Calculus Sec. 1.2: Functions & Graphs

Relation: any ordered pairs. example: (x, y)

Domain: all first components or inputs (all x-values) Independent Variable

Range: all second components or outputs (all y-values) Dependent Variable

Function: Each element in the domain (x's) corresponds to **exactly one** element in the range (y's)...... So..... x-values may not repeat! There can be only one y per x-value..

Function Notation: f(x), g(x), h(x), p(x), etc.....

f(a), r(b), etc...

# Does the relation describe a function?

*Mapping*:

| D | R |
|---|---|
| $-1 \to 3$ | |
| $2 \to 3$ | |
| $2 \to 5$ | |
| $4 \to 4$ | |

(−1, 3)
(2, 3)
(2, 5)
(4, 4)

X's repeat, not a function

*Table*:

| X | Y |
|---|---|
| (2, | 3) |
| (−1, | 4) |
| 4 | 5 |

X-values do <u>not</u> repeat
∴ a function

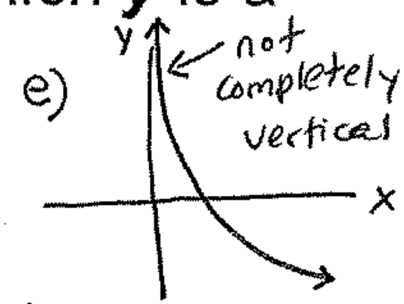
( VLT )

# The Vertical Line Test for Functions

- If any vertical line intersects a graph in more than one point, the graph does not define y as a function.

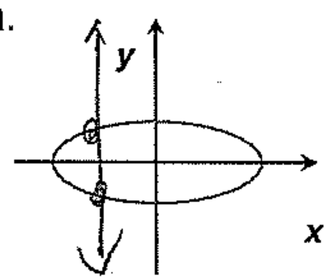
Use the vertical line test to identify graphs in which ***y*** is a function of ***x***.

e)



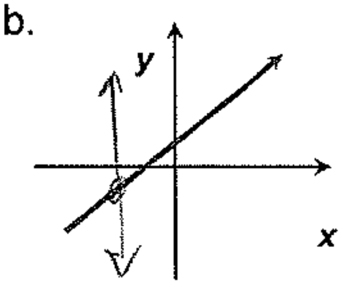

Function, passes VLT

a.



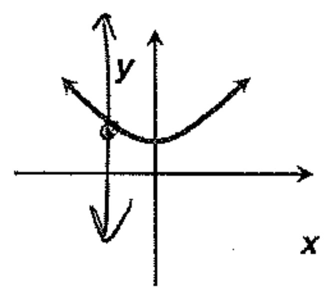

not a function, fails VLT

b.



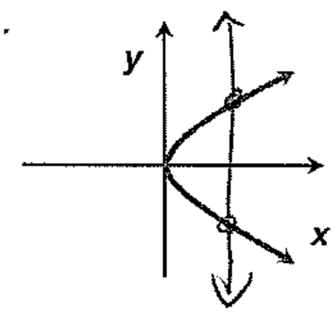

Function, passes VLT

c.




Function, passes VLT

d.




Not a function, fails VLT

# Determine whether the equation represents y as a function of x.

- Solve for y, if the equation is preceded by a $\pm$ this indicates that for a given value of x there corresponds 2 values of y. Plug in an x-value to test the number of y-values generated.

1) $x = y^2 + 1$

$y^2 + 1 = x$

$\sqrt{y^2} = \sqrt{x-1}$

$|y| = \sqrt{x-1}$

$y = \pm\sqrt{x-1}$

$\boxed{\text{not a function}}$

even power

even root

add $\pm$

Test: $x = 2$

$y = \pm\sqrt{2-1}$

$y = \pm\sqrt{1}$

$y = \pm 1$

$(\underline{\underline{2}}, 1)$ and $(\underline{\underline{2}}, -1)$

x-values repeat

2) $y = \sqrt{x+5}$

Given as a positive only!

$\boxed{\text{Function}}$

Test: $x = 2$

$y = \sqrt{2+5}$

$y = \sqrt{7}$

$(2, \sqrt{7})$

3) $|y| = 4 - x$

Generates two outcomes (piecewise)

$y = 4 - x$

$-y = 4 - x$

$y = -(4 - x)$

$y = \pm(4 - x)$

$\boxed{\text{not a function}}$

Test: $x = 2$

$y = \pm(4 - 2)$

$y = \pm 2$

$(\underline{\underline{2}}, 2)$ and $(\underline{\underline{2}}, -2)$

# Evaluate the functions as specified.

A) $f(x) = \sqrt{x+8} + 2$

$f(-8) = \underbrace{\sqrt{\cancel{-8+8}}}_{0} + 2 = \boxed{2}$

$f(1) = \sqrt{\underbrace{1+8}_{3}} + 2 = \boxed{5}$

$f(x-8) = \sqrt{x-\cancel{8}+\cancel{8}} + 2$

$= \boxed{\sqrt{x} + 2}$

B)

Given domain for each piece

$$g(x) = \begin{cases} 2x^2 - 1, & x \le 0 \\ 4x + 1, & x > 0 \end{cases}$$

For each piecewise function, choose the correct piece to evaluate with!

domain value

$g(-1) = 2x^2 - 1 \Rightarrow 2(-1)^2 - 1$

$2(1) - 1 = \boxed{1}$

$g(4) = 4x + 1 \Rightarrow 4(4) + 1 = \boxed{17}$

$g(0.25) = 4x + 1$

$\frac{25}{100}$

$g\left(\frac{1}{4}\right) = \cancel{4}\left(\frac{1}{\cancel{4}}\right) + 1$

$= \boxed{2}$

c) $f(x) = x^2 + 3x + 5$

$(x+3)^2 \neq x^2 + 9$

NO!

$f(x+3) = \underbrace{(x+3)^2}_{(x+3)(x+3)} + 3(x+3) + 5$

$= x^2 + 6x + 9 + 3x + 9 + 5$

$= \boxed{x^2 + 9x + 23}$

d) $g(x) = -x^2 + 2x$

$-x^2$

means

$-1 \cdot x^2$

$g(-x) = -\underbrace{(-x)^2} + 2(-x)$

$= \boxed{-x^2 - 2x}$

PreCalculus Sec. 1.3 Piecewise Functions

Definition of a Piecewise Function: A function that is defined by two (or more) equations over a specified domain is called a piecewise function.

# Graphing Piecewise Functions:

1) Find the coordinates of the endpoints for each equation with the specific domain. Make a table for each “piece”.
2) Sketch the shape of the graph for each equation by connecting its endpoints.
3) Plot a few extra points to obtain the shape if necessary.

Ex.2: Graph by hand. Make a **Table** for each piece.

$$a)\ f(x)=\begin{cases} x-1 & \text{if } x<0 \\ x^2-2x-3 & \text{if } 0\le x\le 3 \\ 0 & \text{if } x>3 \end{cases}$$

| $x$ | $x-1$ |
|---|---|
| 0 | −1 open |
| −1 | −2 |
| −2 | −3 |

linear

$x < 0$ open

| $x$ | $x^2-2x-3$ |
|---|---|
| 0 | −3 closed |
| 1 | −4 |
| 2 | −3 |
| 3 | 0 closed |

Quadratic

$0 \le x \le 3$ closed

| $x$ | 0 |
|---|---|
| 3 | 0 open |
| 4 | 0 |
| 5 | 0 |
| 6 | 0 |

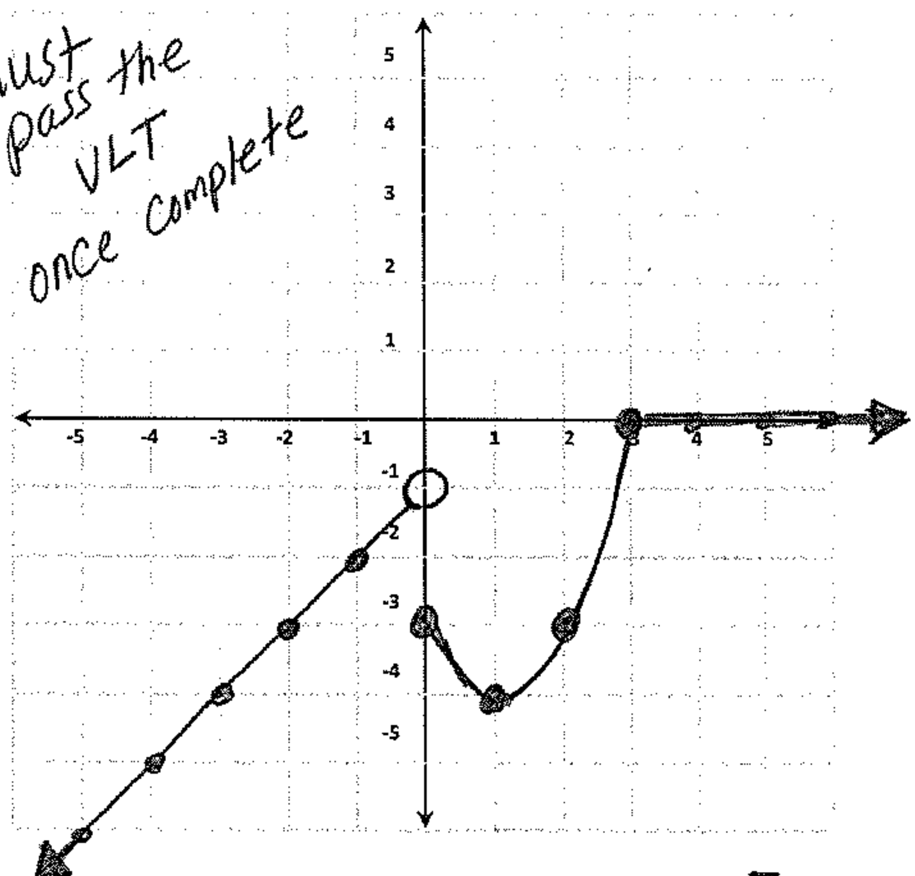
constant

Horiz. line

$f(x) = 0$

$x > 3$ open

must pass the VLT once complete




D: $(-\infty, \infty)$  R: $(-\infty, 0]$

b) $f(x) = \begin{cases} x^2 & \text{for } x < 1 \\ 5 & \text{for } x = 1 \\ 1 - x & \text{for } x > 1 \end{cases}$

| $x$ | $x^2$ |
|---|---|
| 1 | 1 open |
| 0 | 0 |
| −1 | 1 |
| −2 | 4 |
| ↓ | |

LOF Quadratic

$x < 1$ open

| $x$ | 5 |
|---|---|
| 1 | 5 |
| | ~~5~~ |
| | ~~5~~ |

constant
Horiz. line

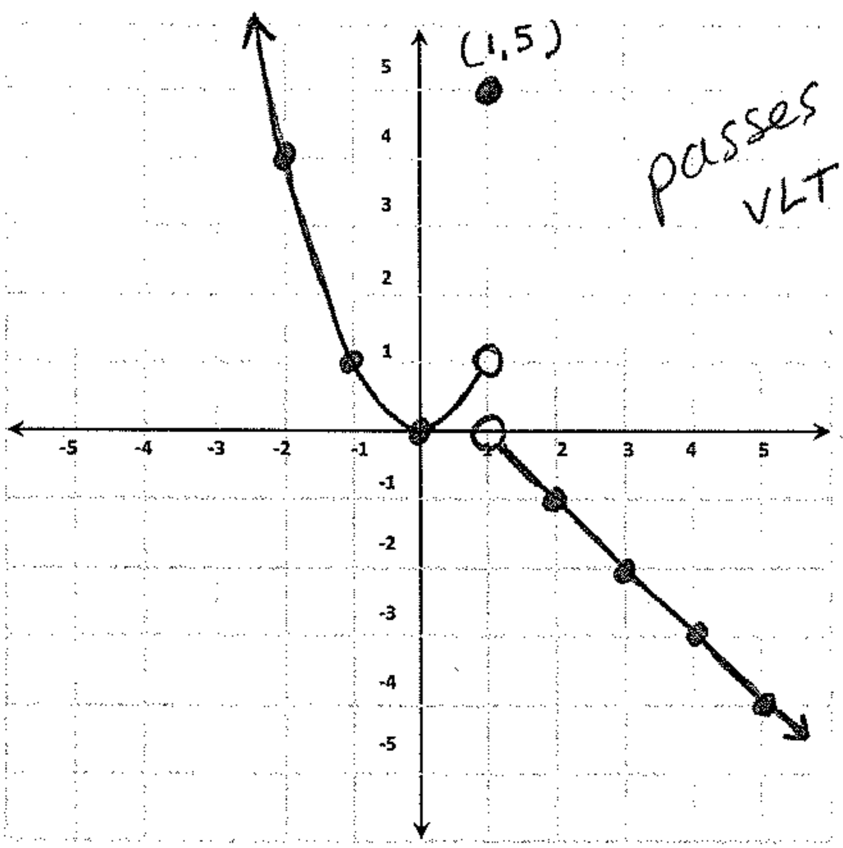
$x = 1$

one point only!

| $x$ | $1-x$ |
|---|---|
| 1 | 0 open |
| 2 | −1 |
| 3 | −2 |
| ↓ | |

linear

$x > 1$ open




passes VLT!

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$