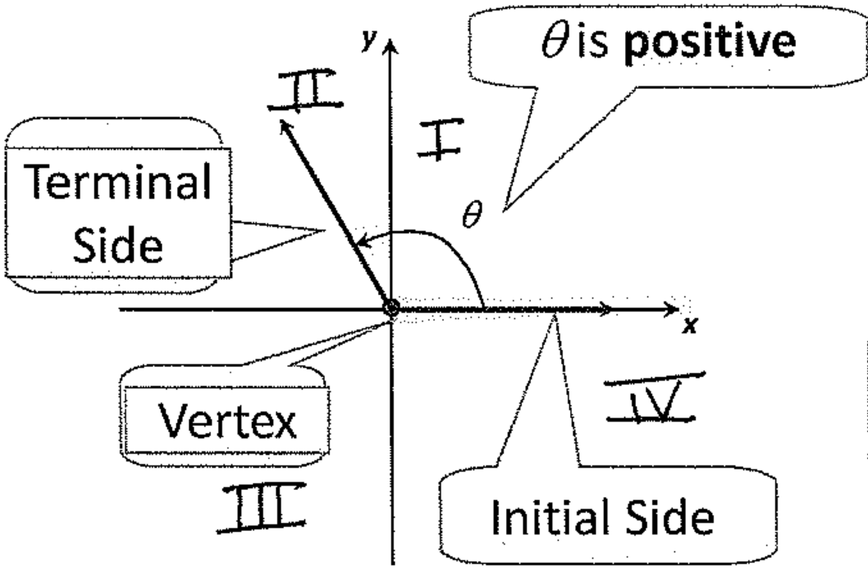
Notes

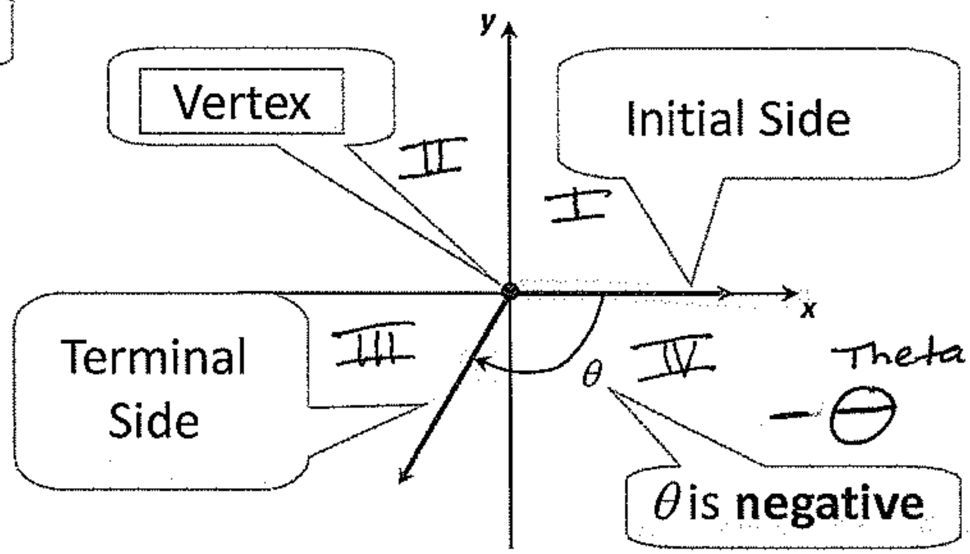
Pre-Calculus
Sec.4.1
Radian and Degree Measure

Angles are usually shown in Standard Position: with the initial (fixed) side on the positive x-axis.

Begin here.



Positive angles rotate *counterclockwise*.
(against)



Negative angles rotate *clockwise*.
(with the clock)

θ Theta

α Alpha

β Beta

Angles are often measured in Degrees, but angles can also be measured in Radians.

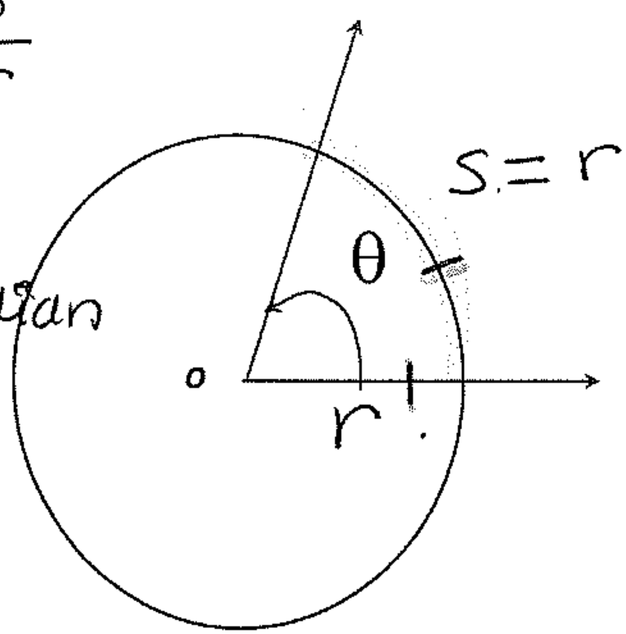
Definition of a Radian:

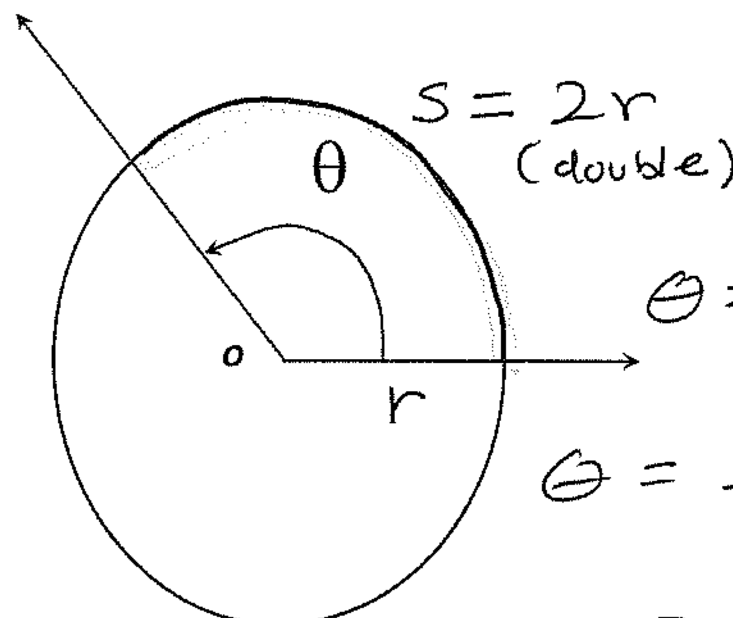
1 radian is the radian measure of the central angle (θ) that is the # of radius units in its intercepted arc length (s).

$\theta = \frac{s}{r}$

$\theta = \frac{r}{r}$

$\theta = 1$ radian





$\theta = \frac{s}{r}$

$\theta = \frac{2\cancel{r}}{\cancel{r}}$

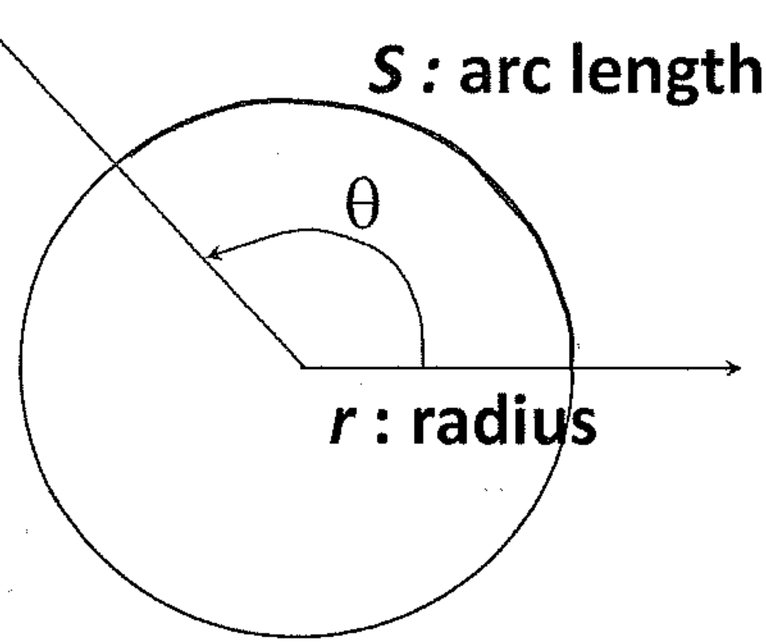
$\theta = 2$ radians

Radian Measure

Therefore, the radian measure can be computed by using: ← Definition of 1 radian

$$\theta = \frac{S}{r}$$

always in radians never in degrees; S and r must have the same units.



$$\frac{\theta}{1} \times \frac{S}{r}$$

$$S = \theta r$$

⊛ radians only

Because the circumference of a circle is $2\pi r$ units, it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of

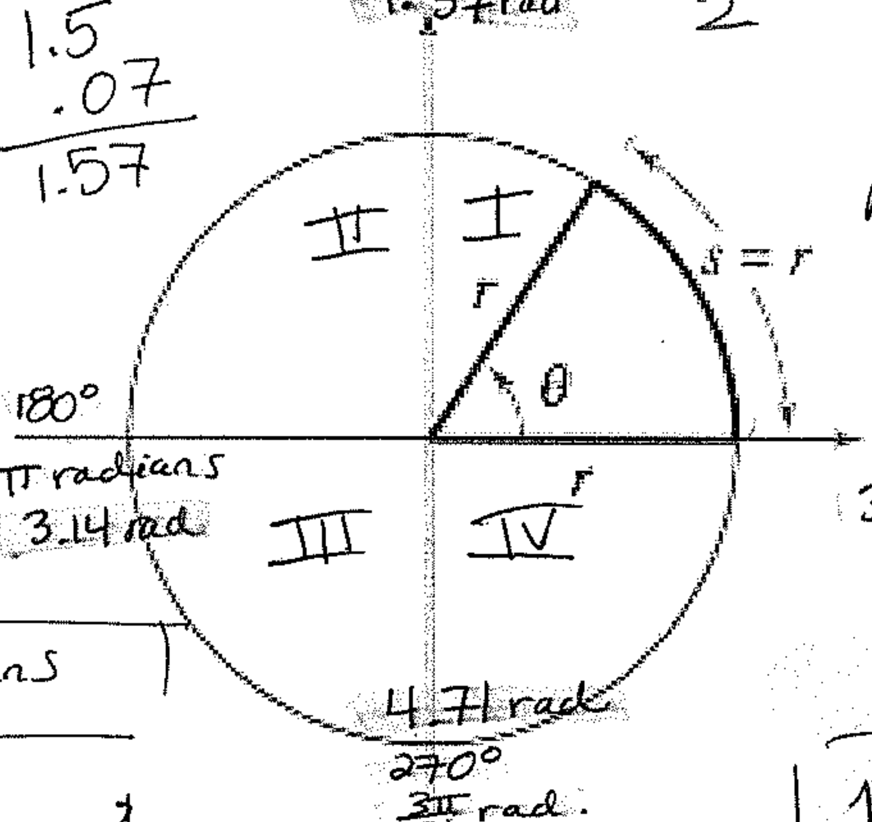
arc length $s = 2\pi r$ ← circumference

Rule: $\theta = \frac{s}{r}$

$\theta = \frac{2\pi \cancel{r}}{\cancel{r}}$

$$\begin{array}{r} 3.14 \\ +1.57 \\ \hline \end{array} \qquad \begin{array}{r} 1.5 \\ .07 \\ \hline 1.57 \end{array}$$

$\theta = 2\pi$ radians (for one revolution)



$\frac{2\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{2}$

$1\frac{1}{2}\pi$

So... $\frac{360°}{2} = \frac{2\pi \text{ rad}}{2}$.

$\boxed{180° = \pi \text{ radians}}$

$\frac{180°}{180°} = \frac{\pi \text{ rad}}{180°}$.

$\boxed{1° = \frac{\pi}{180} \text{ rad}}$

$\frac{180°}{\pi} = \frac{\pi \text{ rad}}{\pi}$

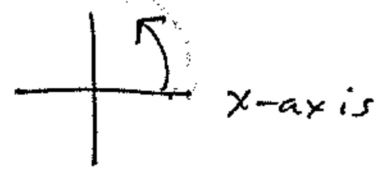
$\boxed{\left(\frac{180}{\pi}\right)° = 1 \text{ rad}}$

$\boxed{1 \text{ rad} \approx 57.3°}$

Measuring Angles Using Rotation

- One counter clockwise rotation (revolution):

  in degrees: $+360°$

  in radians: $+2\pi$

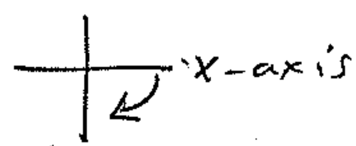
  2 positive revolutions

  Degrees: $2(360°) = \boxed{720°}$

  Radians: $2(2\pi) = \boxed{4\pi \text{ rad}}$

- One clockwise rotation (revolution):

  in degrees: $-360°$

  in radians: -2π

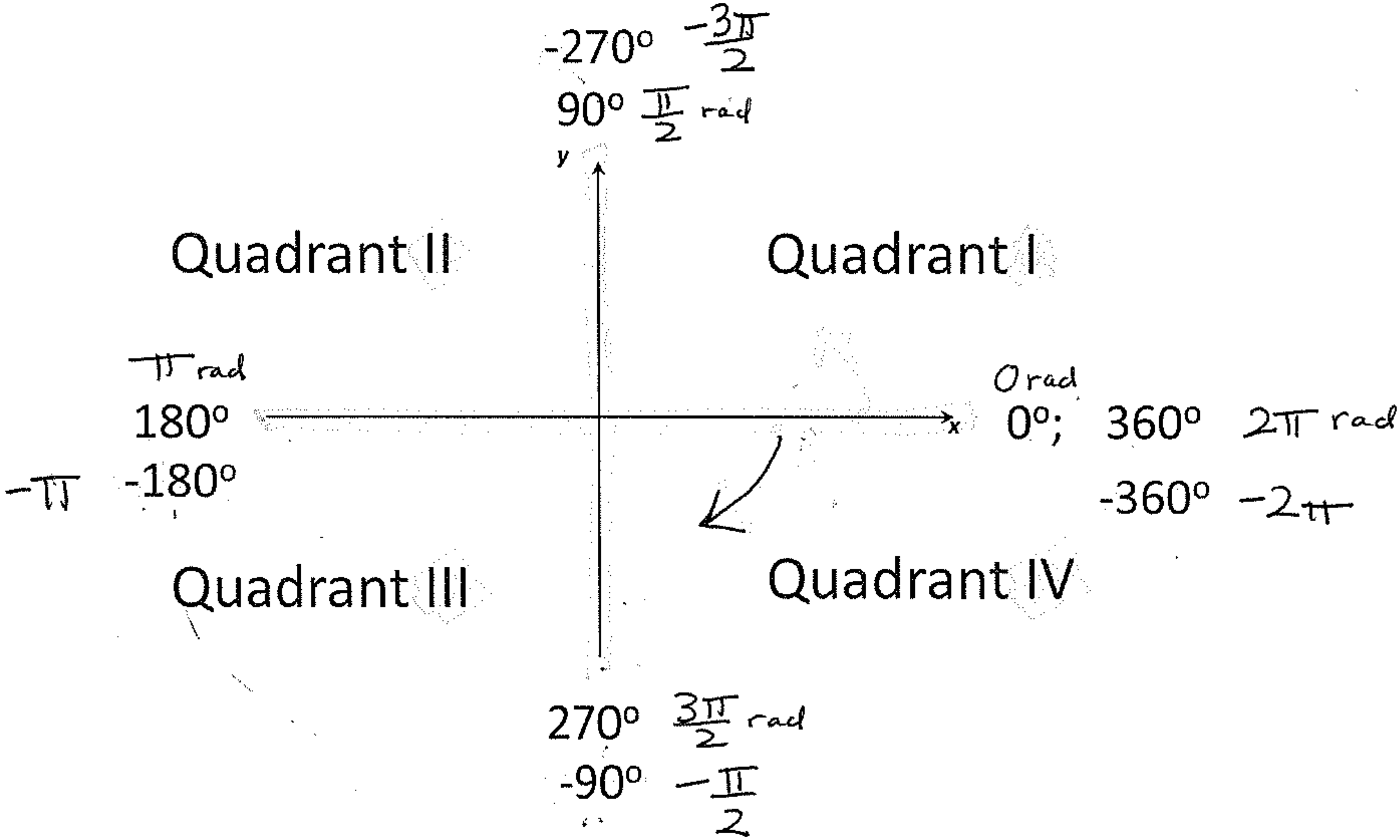
Conversion between Degrees and Radians

- Using the basic relationship π (rad.) = 180°
- To convert degrees to radians:

  multiply by $\dfrac{\pi}{180}$ ← "radians" in numerator
- To convert radians to degrees:

  multiply by $\dfrac{180}{\pi}$ ← "degrees" in numerator

Quadrants in Which the Angle Lies



Quadrantal Angles: if the terminal side lies on an axis,

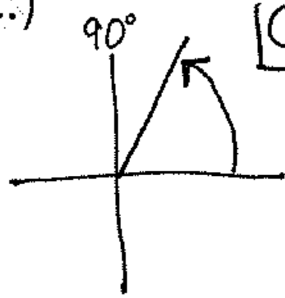
"QA"

[terminates (or ends) on the x or y axis]

Ex. 1: Determine the quadrant in which theta lies, then convert to radians: a) 55° b) 260°

(NO Calc.)

a)

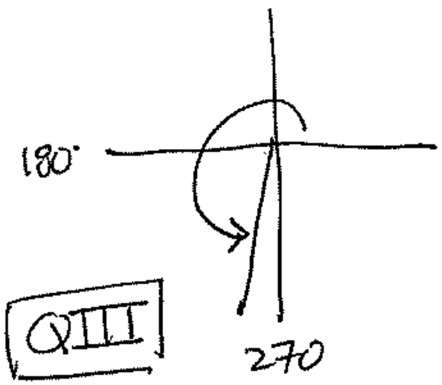


$\boxed{QI}$

$$55^\circ\left(\frac{\pi}{180^\circ}\right) = \boxed{\frac{11\pi}{36}\text{ rad}}$$

$\div 5$

b)



$\boxed{QIII}$

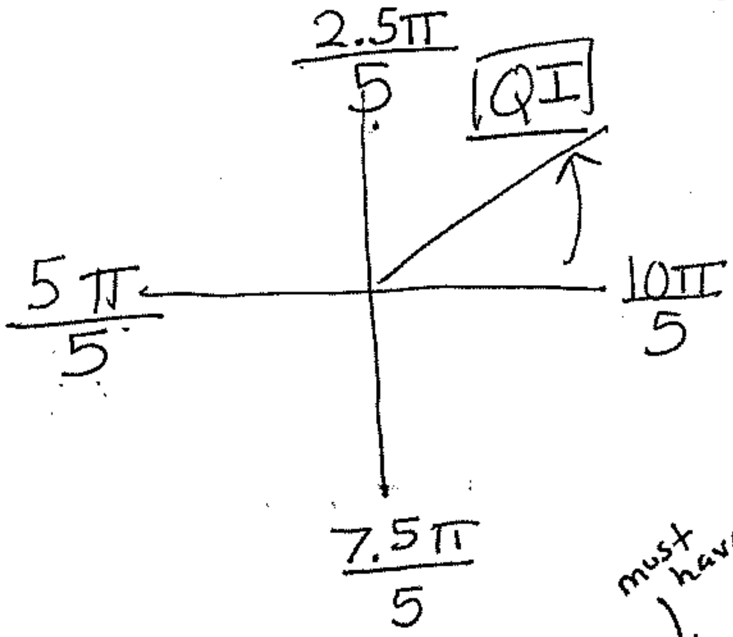
$$260^\circ\left(\frac{\pi}{180^\circ}\right) = \boxed{\frac{13\pi}{9}\text{ rad.}}$$

$\div 2$

π°

Ex. 2: Determine the quadrant in which theta lies, then convert to degrees: a) $\frac{1\pi}{5}$ b) 3 c) $-\frac{7\pi}{9}$
(NO Calc.)

a)

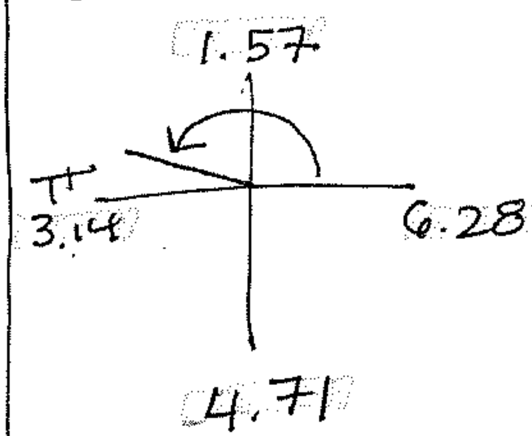


$\boxed{QI}$

$\frac{\cancel{\pi}}{5}\left(\frac{180}{\cancel{\pi}}\right)^\circ = \boxed{36^\circ}$ must have

$\div 5$

b) 3 radians

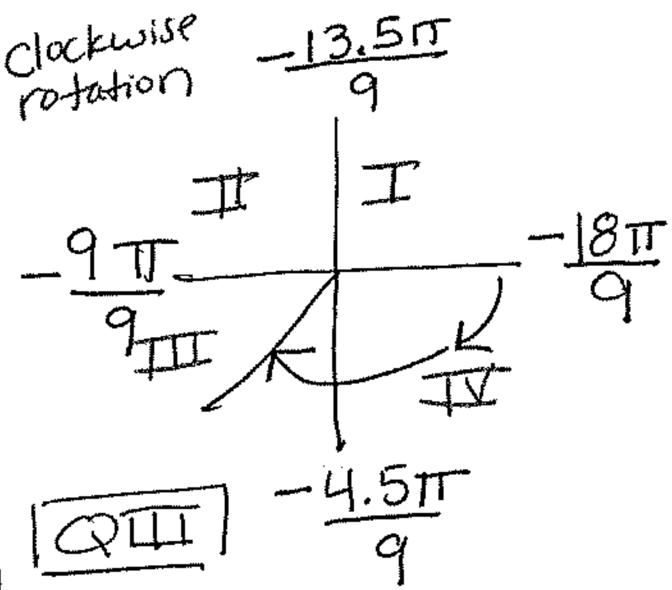


$\boxed{QII}$

$3\left(\frac{180}{\pi}\right)^\circ = \boxed{\left(\frac{540}{\pi}\right)^\circ}$ exact form

$\approx 171.9^\circ$

c) negative $-\frac{7\pi}{9}$ clockwise rotation



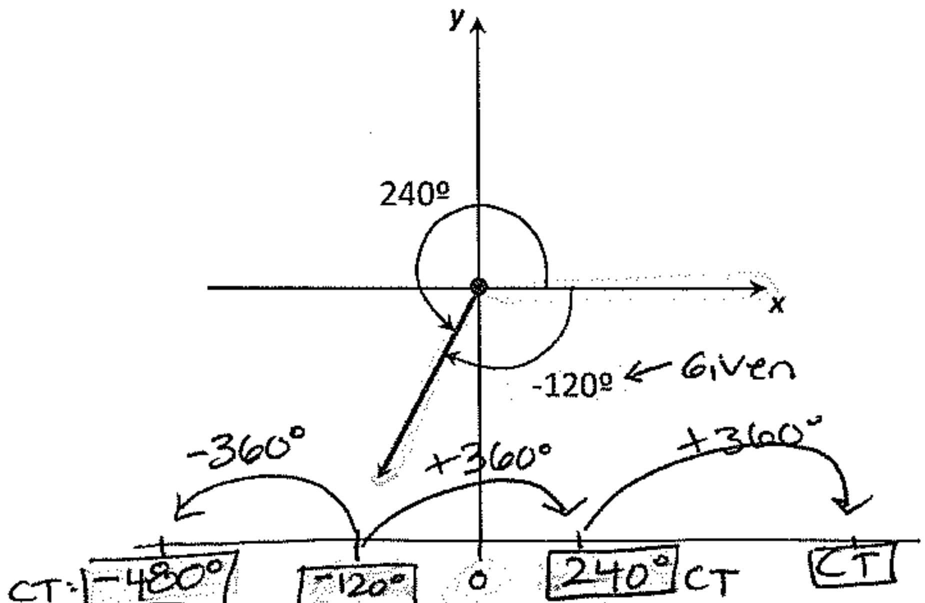
$\boxed{QIII}$

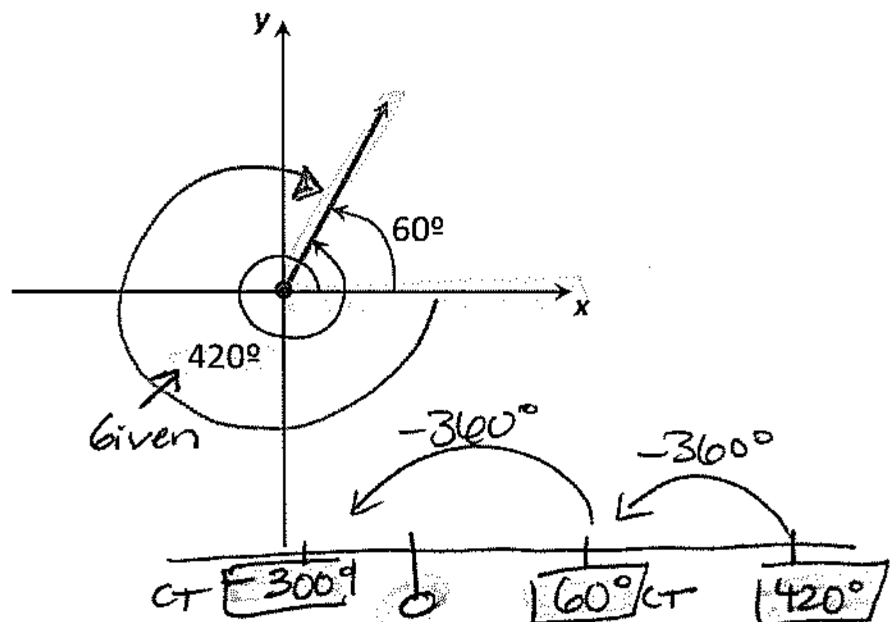
$-\frac{7\cancel{\pi}}{\cancel{9}_1}\left(\frac{\cancel{180}^{20}}{\cancel{\pi}}\right)^\circ = \boxed{-140^\circ}$

CT

Coterminal Angles

(Two angles with the same initial & terminal sides.)



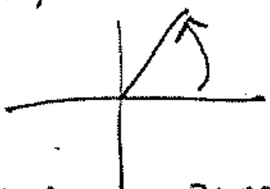


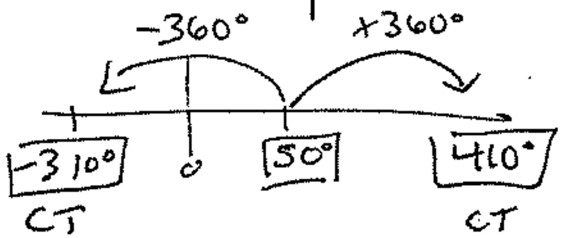
To find coterminal angles just keep adding or subtracting 360° (or 2π) to the given angle. There are an infinite amount of coterminal angles that can be found for each given angle.

- We will usually ask for one positive and one negative (closest to zero) NOT including the given angle.
- stay in the given units. $\begin{cases} \text{Degrees} \to \text{Degrees} \\ \text{Radians} \to \text{Radians} \end{cases}$

Ex. 3: Find a positive and negative coterminal angle (as close to zero as possible) to the following given angles. Give answers in the same form as the question. (NO Calc.)

a. 50°

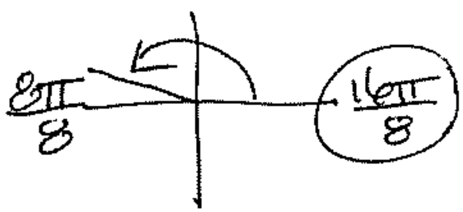


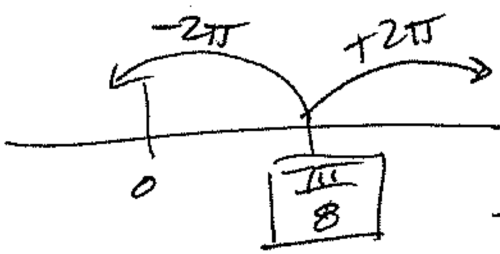


✓ b. -120°

• on previous page

c. $\frac{7\pi}{8}$





$\frac{2\pi}{1} \rightarrow \frac{16\pi}{8}$

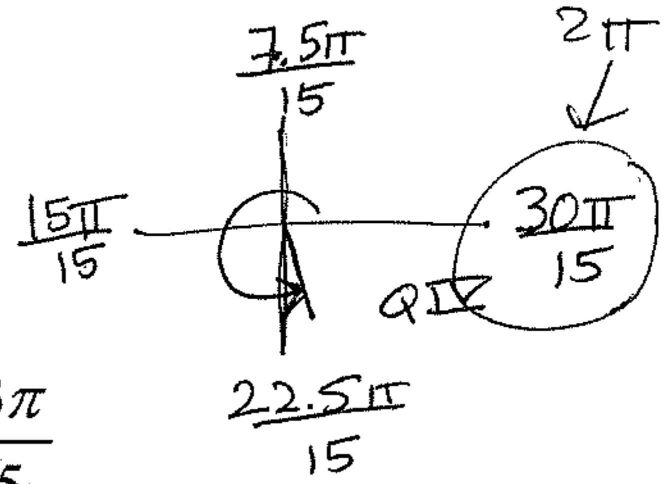
CT

$\frac{7\pi}{8} + \frac{16\pi}{8} = \boxed{\frac{23\pi}{8}}$

$\frac{7\pi}{8} - \frac{16\pi}{8} = \boxed{-\frac{9\pi}{8}}$

CT

d. $\frac{23\pi}{15}$



CT

$\frac{23\pi}{15} + \frac{30\pi}{15} = \boxed{\frac{53\pi}{15}}$

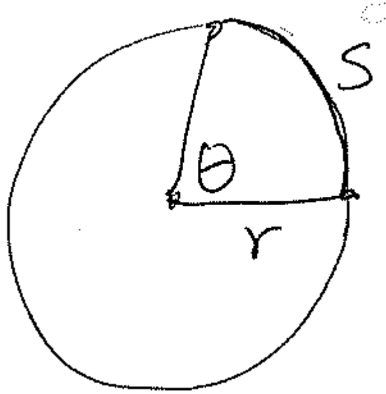
$\frac{23\pi}{15} - \frac{30\pi}{15} = \boxed{-\frac{7\pi}{15}}$

CT

Recall: $\theta = \dfrac{s}{r}$ always in radians, never in degrees

$s = \theta r$

Ex.4: Find the radian measure of a central angle that intercepts an arc of 10 cm on a circle with a radius of 15 cm.



$s = 10cm$ — same units ✓ — $r = 15cm$

$s = \theta r$

$10cm = \theta \cdot 15cm$

$\dfrac{10\cancel{cm}}{15\cancel{cm}} = \theta$

$\boxed{\theta = \frac{2}{3} \text{ radians}}$

s

Ex. 5: Find the length of the arc on a circle of radius r intercepted by a central angle θ. Given $r = 9$ feet; $\theta = 60^\circ$

$$\theta = 6\cancel{0}^\circ\left(\frac{\pi}{18\cancel{0}}\right) = \frac{\pi}{3} \text{ rad}$$

Radians only $\rightarrow$ $s = \theta r$

$$s = \frac{\pi}{\cancel{3}} \cdot \overset{3}{\cancel{9}} \text{ ft}$$

$$\boxed{s = 3\pi \text{ feet}}$$

(exact form ... in terms of π)

$\boxed{S = \theta r}$ radians only

Linear Speed measures how fast a particle moves, and Angular Speed measures how fast the angle changes.

Angular speed = $\dfrac{\text{Central angle}}{\text{time}} = \dfrac{\theta}{t}$ Linear speed = $\dfrac{\text{arc length}}{\text{time}} = \dfrac{s}{t}$

$\boxed{A.S. = \dfrac{\theta}{t}}$ $\boxed{L.S. = \dfrac{s}{t} = \dfrac{\theta r}{t}}$

Ex. 6: The circular blade on a saw has a diameter of 9 inches and rotates at 3600 revolutions per minute.

1 revolution = 2π radians

A)Find the angular speed of the blade in radians per second. (in terms of π)

$$A.S. = \frac{\theta}{t} = \frac{\overset{60}{36\cancel{00}}\text{ rev}}{1\text{ min}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} \cdot \frac{1\text{ min}}{\underset{1}{\cancel{60}}\text{ sec}} = \boxed{120\pi \text{ rad/sec}}$$

B)Find the linear speed of the saw teeth (in feet per second) as they contact the wood being cut. Give the answer in terms of π (exact form). Then use a calculator to give the answer rounded to 3 decimal places.

$$L.S. = \frac{s}{t} = \frac{\theta r}{t} = \frac{\theta}{t} \cdot r = \frac{\overset{10}{\cancel{120}}\pi}{1\text{ sec}} \cdot \frac{4.5\text{ inches}}{1} \cdot \frac{1\text{ ft}}{\underset{1}{\cancel{12}}\text{ inches}}$$

A.S.

$$= \boxed{45\pi \text{ ft/sec}} \approx \boxed{141.372 \text{ ft/sec}}$$

exact

Diam = 9 inches

Rad = 4.5 inches

Ex. 7: A 15 inch diameter tire on a car makes 9.3 revolutions per second.

A)Find the angular speed in radians per second. (in terms of π)

$$A.S. = \frac{\theta}{t} = \frac{9.3\,\cancel{rev}}{1\,sec} \cdot \frac{2\pi\,rad}{1\,\cancel{rev}} = \boxed{18.6\pi \text{ rad/sec}}$$

B)Find the linear speed in feet per minute. Give the answer in terms of π (exact form). Then use a calculator to give the answer rounded to 3 decimal places.

$$L.S. = \frac{s}{t} = \frac{\theta r}{t} = (A.S.)\cdot \text{radius}$$

$$= \frac{18.6\pi}{1\,\cancel{sec}} \cdot \frac{7.5\,\cancel{inches}}{1} \cdot \frac{1\,ft}{\cancel{12}\,\cancel{in}} \cdot \frac{\cancel{60}^{5}\,\cancel{sec}}{1\,min}$$

Diam = 15 in.

Rad. = 7.5 in.

$$\begin{array}{r} 18.6 \\ \times\ 5 \\ \hline 93.0 \end{array} \qquad \begin{array}{r} 93 \\ \times 7.5 \\ \hline 465 \\ 6510 \\ \hline 697.5 \end{array}$$

$$= \boxed{697.5\pi \text{ ft/min}}$$

exact

$$\approx \boxed{2191.261 \text{ ft/min}}$$