Graphing Quadratics In Standard Form Notes

Standard Form:

$f(x) = ax^2 + bx + c$

Quadratic term — Linear term — Constant term

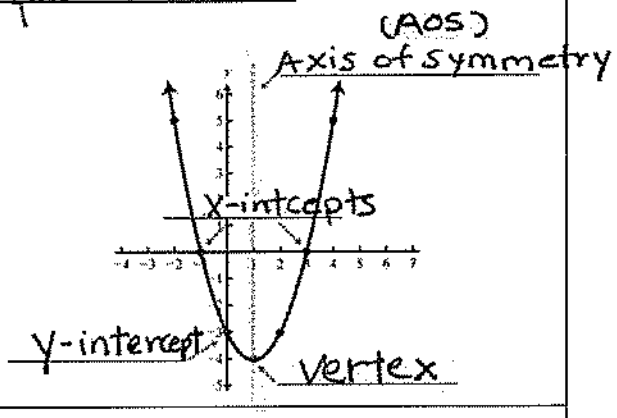
where:

a $\in \mathbb{R}$, $a \neq 0$

b $\in \mathbb{R}$

c $\in \mathbb{R}$

The graph of a quadratic is called a parabola.



To graph a quadratic function in standard form:

1. Find the vertex.

* Vertex: $x = \frac{-b}{2a}$ *

2. Plot at least two other points, one on each side of the vertex. (to four)

ex: Sketch, then state the vertex, axis of symmetry, y-intercept and domain and range in the indicated notation.

a) $y = 3x^2 - 12x + 8$

$a = 3$ positive cup

$b = -12$

$c = 8$

$x = \frac{-b}{2a} = \frac{-(-12)}{2(3)} = \frac{12}{6} = 2$

x	y
0	8
1	-1
2	-4 ✓
3	-1
4	8

(-1, +1, -1, +1)

$y = 3(0)^2 - 12(0) + 8$

$y = 8$

$y = 3(1)^2 - 12(1) + 8$

$= 3(1) - 12 + 8$

$= 3 - 12 + 8$

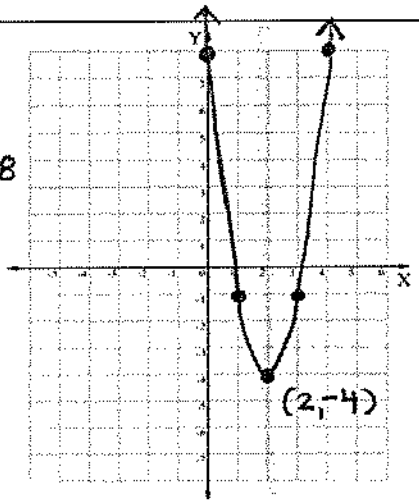
$= -9 + 8$

$= -1$

$y = 3(2)^2 - 12(2) + 8$

$= 3(4) - 24 + 8$

$= 12 - 24 + 8 = -4$



Vertex: (2, -4) AOS: $x = 2$ y-int: (0, 8)

(INT) Domain: $(-\infty, \infty)$ Range: $[-4, \infty)$

b) $y = -x^2 - 6x - 4$

$a = -1$ negative down

$b = -6$

$c = -4$

$x = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3$

x	y
-5	1
-4	4
-3	5 ✓
-2	4
-1	1

$y = -(-1)^2 - 6(-1) - 4$

$= -1 + 6 - 4$

$= 5 - 4$

$= 1$

$y = -(-2)^2 - 6(-2) - 4$

$= -4 + 12 - 4$

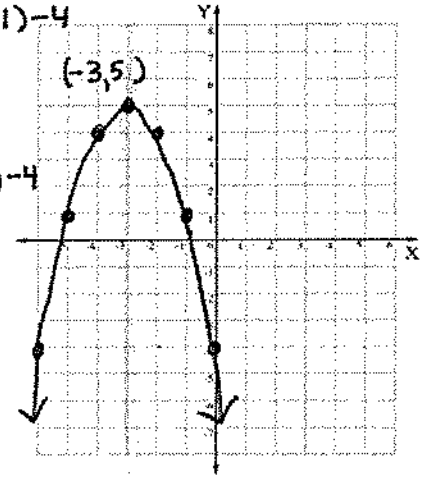
$= 8 - 4$

$= 4$

$y = -(-3)^2 - 6(-3) - 4$

$= -9 + 18 - 4$

$= 9 - 4 = 5$



Vertex: (-3, 5) AOS: $x = -3$ y-int: (0, -4)

(INT) Domain: $(-\infty, \infty)$ Range: $(-\infty, 5]$

$y = -(0)^2 - 6(0) - 4$

$y = -4$

c) $y = x^2 + 4x - 3$ $a=1$ positive up

$b=4$

$c=-3$

$x = \frac{-(4)}{2(1)} = \frac{-4}{2} = -2$

$y = (-2)^2 + 4(-2) - 3$

$= 4 - 8 - 3$

$= -4 - 3 = -7$

x	y
-4	-3
-3	-6
-2	-7 V
-1	-6
0	-3

$y = (0)^2 + 4(0) - 3$

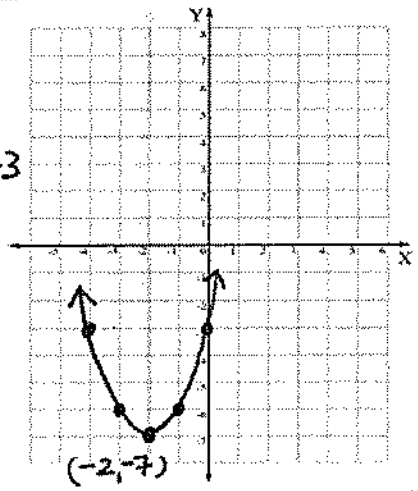
$y = -3$

$y = (-1)^2 + 4(-1) - 3$

$= 1 - 4 - 3$

$= -3 - 3$

$= -6$



Vertex: $(-2,-7)$ AOS: $x=-2$ y-int: $(0,-3)$

(INT) Domain: $(-\infty,\infty)$ Range: $[-7,\infty)$

d) $y = 5x^2 - 1$ $a=5$ positive up

$b=0$

$c=-1$

$x = \frac{-(0)}{2(5)} = \frac{0}{10} = 0$

$y = 5(0)^2 - 1$

$y = -1$

x	y
-2	19
-1	4
0	-1 V
1	4
2	19

$y = 5(1)^2 - 1$

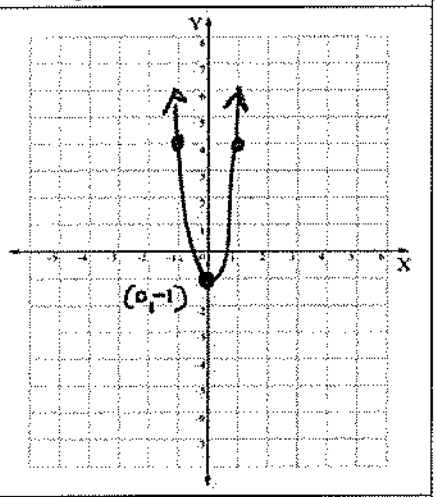
$= 5(1) - 1$

$= 5 - 1 = 4$

$y = 5(2)^2 - 1$

$= 5(4) - 1$

$= 20 - 1 = 19$



Vertex: $(0,-1)$ AOS: $x=0$ y-int: $(0,-1)$ ← same as vertex here

(INT) Domain: $(-\infty,\infty)$ Range: $[-1,\infty)$

e) $y = x^2 - x + 1$ $a=1$ positive up

$b=-1$

$c=1$

$x = \frac{-(-1)}{2(1)} = \frac{1}{2}$

$y = (\frac{1}{2})^2 - (\frac{1}{2}) + 1$

$= \frac{1}{4} - \frac{1}{2} + \frac{1}{1}$

$= \frac{1}{4} - \frac{2}{4} + \frac{4}{4} = \frac{3}{4}$

x	y
-1	3
0	1
$\frac{1}{2}$	$\frac{3}{4}$
1	1
2	3

$y = (0)^2 - (0) + 1$

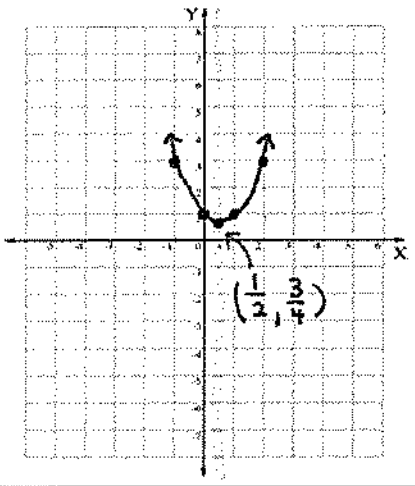
$y = 1$

$y = (2)^2 - (2) + 1$

$= 4 - 2 + 1$

$= 2 + 1$

$= 3$



Vertex: $(\frac{1}{2},\frac{3}{4})$ AOS: $x=\frac{1}{2}$ y-int: $(0,1)$

(INT) Domain: $(-\infty,\infty)$ Range: $[3/4,\infty)$

ex: Consider the graph of: $y = ax^2 + bx + c$ Standard form

a) When does the graph open up?	b) When does the graph open down?	c) What is the axis of symmetry? AOS
$a > 0$ ↑ positive	$a < 0$ ↑ negative	$x = \frac{-b}{2a}$

Maximum and Minimum Values

* The maximum or minimum of a parabola always occurs at the vertex.

*The maximum or minimum value is the y-value of the vertex

ex: State the maximum or minimum value.

a) $y = 3x^2 - 12x + 8$ (pg 1)

V: (2, -4) Up

min = -4

b) $y = -x^2 - 6x - 4$ (pg 1)

V: (-3, 5) Down

MAX = 5

c) $y = -3x^2 + 12x - 7$ $a = -3$ down, $b = 12$, $c = -7$

$x = \frac{-b}{2a} = \frac{-(12)}{2(-3)} = \frac{-12}{-6} = 2$

MAX = 5

$y = -3(2)^2 + 12(2) - 7$

$= -3(4) + 24 - 7$

$= -12 + 24 - 7$

$12 - 7 = 5$

ex: Without graphing, consider the function: $y = -\frac{1}{2}x^2 + 3$

a) What is the direction of opening? $a = -\frac{1}{2}$, $b = 0$, $c = 3$

$a < 0$

down

b) What is the axis of symmetry? $x = 0$

$x = \frac{-b}{2a} = \frac{-(0)}{2(-\frac{1}{2})} = \frac{0}{-1} \quad \frac{0}{K} = 0$

c) What is the maximum/minimum value?

vertex: (0, 3)

$y = -\frac{1}{2}(0)^2 + 3$

$y = 3$

MAX = 3

d) State the domain and range in interval notation.

(0, 3)

D: $(-\infty, \infty)$

R: $(-\infty, 3]$

ex: Write the quadratic function in standard form. $ax^2 + bx + c$

a) $y = 5(x-1)^2 + 4$

HAS NO ( )!

$y = 5[(x-1)(x-1)] + 4$ FOIL

$y = 5(x^2 - x - x + 1) + 4$

$y = 5(x^2 - 2x + 1) + 4$

$y = 5x^2 - 10x + 5 + 4$

$y = 5x^2 - 10x + 9$

b) $y = 3(x+5)(x-2)$ FOIL

$y = 3(x^2 - 2x + 5x - 10)$

$y = 3(x^2 + 3x - 10)$

$y = 3x^2 + 9x - 30$

c) $y = 2(x+7)^2 - 1$

$y = 2[(x+7)(x+7)] - 1$ FOIL

$y = 2(x^2 + 7x + 7x + 49) - 1$

$y = 2(x^2 + 14x + 49) - 1$

$y = 2x^2 + 28x + 98 - 1$

$y = 2x^2 + 28x + 97$

d) $y = -4(x-5)(x+5)$ FOIL

$y = -4(x^2 + 5x - 5x - 25)$

$y = -4(x^2 - 25)$

$y = -4x^2 + 100$

$a = -4$, $b = 0$, $c = 100$