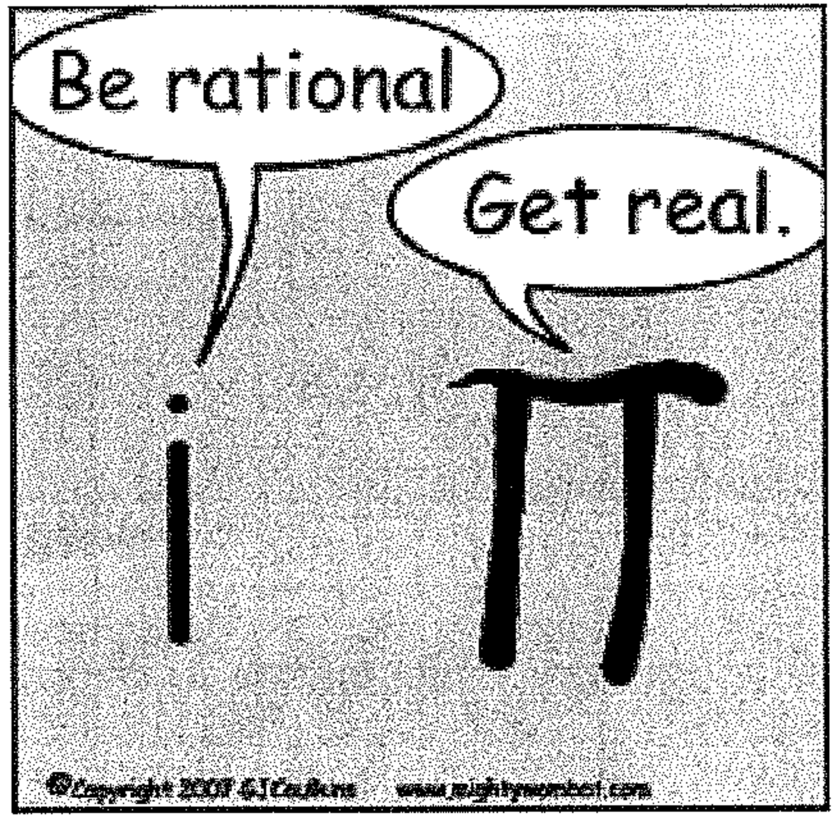
Complex Numbers

Notes

$$\sqrt{\begin{array}{l}\text{math is}\\ \text{radical!}\end{array}}$$

Imaginary Numbers

$$\sqrt{-1} = \underline{\quad i \quad}$$



ex: Simplify.

a) $\sqrt{-9} = \sqrt{-1 \cdot 9}$

$= \sqrt{-1} \cdot \sqrt{9}$

$= i \cdot 3 = \boxed{3i}$

Always take out the negative 1st

b) $\sqrt{-18} = \sqrt{-1 \cdot 18}$

$= \sqrt{-1} \cdot \sqrt{18}$

$= i \cdot 3\sqrt{2}$

$= \boxed{3i\sqrt{2}}$ $3\sqrt{2}\, i$

$2 \,|\, 18$
$3 \,|\, 9$
3

c) $\sqrt{-32}$

$$\begin{aligned} &= \sqrt{-1 \cdot 32} \\ &= i\sqrt{32} \\ &= i \cdot 4\sqrt{2} \\ &= 4i\sqrt{2} \end{aligned}$$

$$\begin{array}{r|l} 2 & 32 \\ 4 & 16 \\ & 4 \end{array}$$

d) $2\sqrt{-45}$

$$2 \cdot \sqrt{-1 \cdot 45}$$

$$2 \cdot \sqrt{-1} \cdot \sqrt{45}$$

$$\underbrace{2 \cdot i \cdot 3}\sqrt{5}$$

$$\boxed{6i\sqrt{5}}$$

$$\begin{array}{r|l} 5 & 45 \\ 3 & 9 \\ & 3 \end{array}$$

Complex Numbers

Standard Form: $\underbrace{a}_{\text{real part}} + \underbrace{bi}_{\text{imaginary part}}$

Examples of Complex Numbers:

$3 - 5i$

$517i \leftarrow \cancel{0} + 517i$

$10 \leftarrow 10 + \cancel{0i}$

$-2 + 5i$

*EVERY NUMBER CAN BE EXPRESSED AS A COMPLEX NUMBER!

in $a + bi$ form

$a+bi$

ex: Simplify. Express the answer in standard form.

add

a) $(3+6i)+(6-42i)$

*Combine like terms

$3+6i+6-42i$

$3+6+6i-42i$

$\boxed{9-36i}$ ← Real before imaginary part for standard form

b) $(16-42i)-(3-64i)$

subtraction

$16-42i-3+64i$

$16-3-42i+64i$

$\boxed{13+22i}$

c) $7(3-2i)$

multiply

$\boxed{21-14i}$

$\boxed{\sqrt{-1} = i}$ *

$(\sqrt{-1})^2 = (i)^2$

$\boxed{-1 = i^2}$ *

* d) $7i(3-2i)$

multiply

$21i - 14i^2$ ← not standard form

$21i - 14(-1)$ multiply by negative 1 (not subtraction)

$21i + 14$

reorder to standard form → $\boxed{14 + 21i}$

*e) $(1+2i)(3-5i)$ FOIL

$3-5i+6i-10i^2$

$-10(-1)$

$3-5i+6i+10 = \boxed{13+i}$

f) $(6-3i)(6+3i)$

$36+\cancel{18i}-\cancel{18i}-9i^2$

(-1)

$36+9$

$=\boxed{45}$

*g) $(1-2i)^2$ Rewrite, then FOIL!!!

$(1-2i)(1-2i)$

$1-2i-2i+4\underset{(-1)}{i^2}$

$1-2i-2i-4 = \boxed{-3-4i}$

h) $\frac{6}{i}\cdot\frac{i}{i}$ ← Rationalize the denominator

$= \frac{6i}{i^2}$

$= \frac{6i}{-1} = \boxed{-6i}$

i) $\frac{2}{3i} \cdot \frac{i}{i} = \frac{2i}{3i^2} = \frac{2i}{3(-1)} = \frac{2i}{-3} = \boxed{-\frac{2i}{3}}$ *

or

$\boxed{-\frac{2}{3}i}$

* j) $\frac{5}{(2+i)} \frac{(2-i)}{(2-i)}$

Rationalize the denominator w/ 2 terms → need a conjugate!!!

FOIL

add ( )

$= \frac{10-5i}{4-\cancel{2i}+\cancel{2i}-\underset{-(-1)}{i^2}}$

$= \frac{10-5i}{4+1} = \frac{10-5i}{5}$

$\frac{10}{5} - \frac{\cancel{5}i}{\cancel{5}}$

$= \boxed{2-i}$

$a + bi$

k) $$\frac{2i}{(3-i)}\frac{(3+i)}{(3+i)} = \frac{6i + 2\overset{(-1)}{i^2}}{9 + \cancel{3i} - \cancel{3i} - \underset{(-1)}{i^2}}$$

add

Foil

$$= \frac{6i - 2}{9 + 1}$$

$$= \frac{-2 + 6i}{10}$$ Reorder w/ real part in front.

$$= \frac{-2}{10} + \frac{6i}{10}$$

$$= \boxed{-\frac{1}{5} + \frac{3i}{5}}$$ Standard form

$$a + bi$$

FOIL

1) $$\frac{(5+2i)}{(3-2i)} \cdot \frac{(3+2i)}{(3+2i)} = \frac{15+10i+6i+4\overset{(-1)}{i^2}}{9+\cancel{6i}-\cancel{6i}-4\underset{(-1)}{i^2}}$$

FOIL

add

Conjugate

$$= \frac{15+10i+6i-4}{9+4}$$

$$= \frac{11+16i}{13}$$

$$= \boxed{\frac{11}{13} + \frac{16}{13}i}$$

$a + bi$

Standard form of a complex #

$\boxed{\text{Powers of } i}$

$$\boxed{\begin{array}{c} \sqrt{-1} = i \\ \hline -1 = i^2 \end{array}}$$

$$i^0 = \boxed{1}$$

$$i^1 = \boxed{i}$$

$$i^2 = \boxed{-1}$$

$$i^3 = i^2 \cdot i^1 = (-1)\, i = \boxed{-i}$$

$$i^4 = i^2 \cdot i^2 = (-1)(-1) = \boxed{1}$$

$$i^{10} = (i^2)^5 = (-1)^5 = \boxed{-1}$$

$10 \leftarrow$ even

$5 \leftarrow$ odd

$$i^9 = i^8 \cdot i^1 = (i^2)^4 \cdot i = (-1)^4 \cdot i = 1 \cdot i = \boxed{i}$$

$$i^{3281} \leftarrow \text{odd}$$

$$\begin{aligned} i^{3281} &= i^{3280} \cdot i \\ &= (i^2)^{\overset{\text{even}}{1640}} \cdot i \\ &= (-1)^{1640} \cdot i \\ &= 1 \cdot i \\ &= \boxed{i} \end{aligned}$$

$$\begin{array}{r} 1500 \\ 100 \\ 40 \\ \hline 1640 \end{array}$$

$$\begin{aligned} i^{726} &= (i^2)^{363}_{\text{odd}} \\ &= (-1)^{363} \\ &= \boxed{-1} \end{aligned}$$

$$\begin{array}{r} 350 \\ 10 \\ +\ 3 \\ \hline 363 \end{array}$$

Review

ex: Find the roots.

"Solve"

$$8x^2 - 14x = 15$$

gcf? no

$$8x^2 - 14x - 15 = 0$$

$$(4x+3)(2x-5) = 0$$

$4x+3=0$ | $2x-5=0$

$4x=-3$ | $2x=5$

$\boxed{x=\frac{-3}{4}}$ | $\boxed{x=\frac{5}{2}}$

-120

$\frac{8x}{+6}$ $\frac{8x}{-20}$

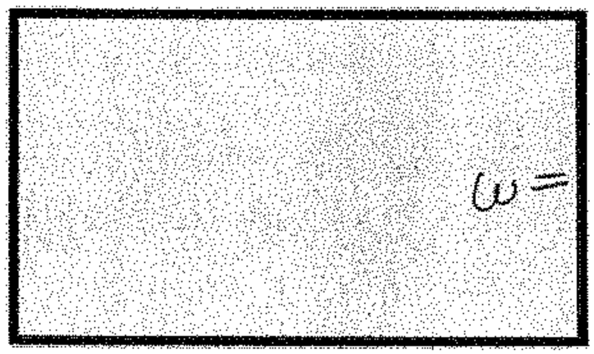
$\div 2$ $\div 4$

$\frac{4x}{+3}$ $\frac{2x}{-5}$

Review

ex: Find x.

Area of rectangle = 84 A



$w = x + 2$

$l = x + 7$

$A = l \cdot w$

$84 = (x+7)(x+2)$

Foil

$84 = x^2 + 2x + 7x + 14$

$84 = x^2 + 9x + 14$

$-84 \qquad -84$

$0 = x^2 + 9x - 70$

$0 = (x-5)(x+14)$

-70	
1	70
2	35
-5	+14

$x - 5 = 0$

$\boxed{x = 5}$

$x + 14 = 0$

$\cancel{x = -14}$

extraneous