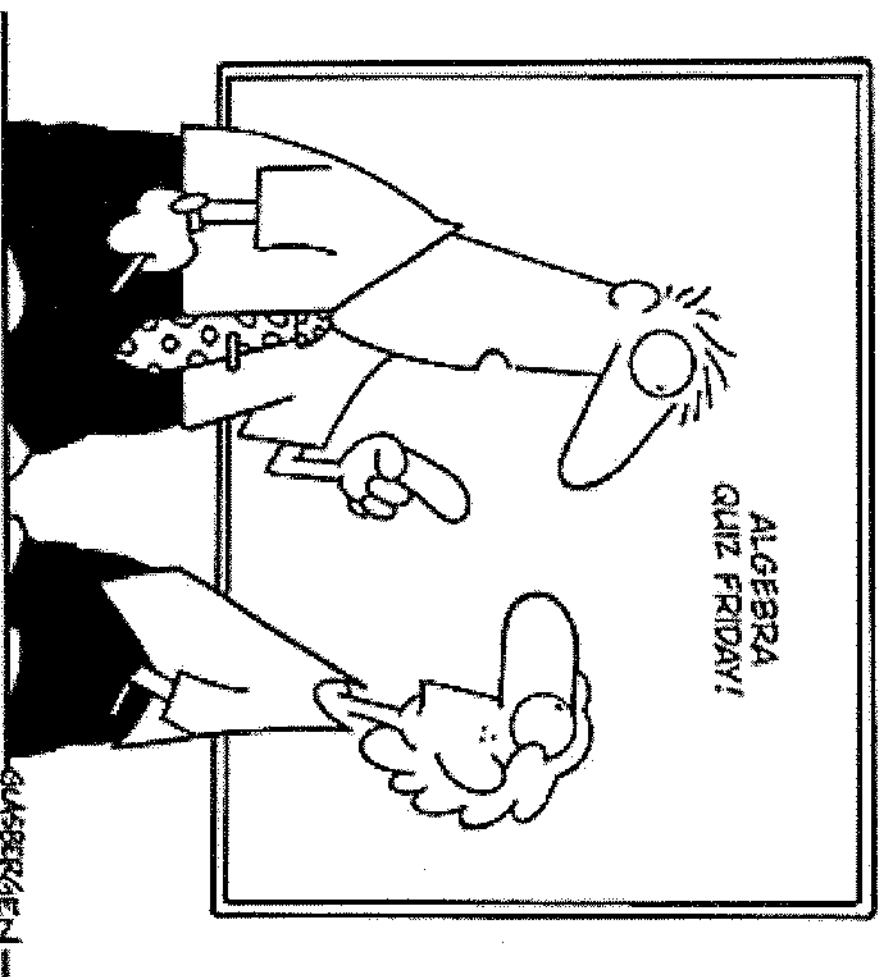


Function Operations & Compositions

Notes

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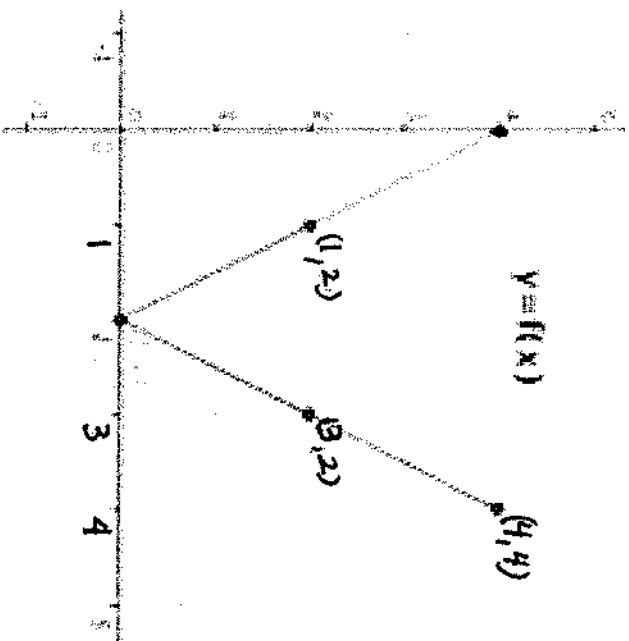


"It's important to learn math because someday you might accidentally buy a phone without a calculator."

Function Operations

Addition	$f(x) + g(x) = (f + g)(x)$
Subtraction	$f(x) - g(x) = (f - g)(x)$
Multiplication	$f(x)g(x) = (fg)(x)$
Division	$\frac{f(x)}{g(x)} = \left(\frac{f}{g}\right)(x)$

ex: Evaluate.



$$\text{a) } (f + g)(3) = f(3) + g(3)$$

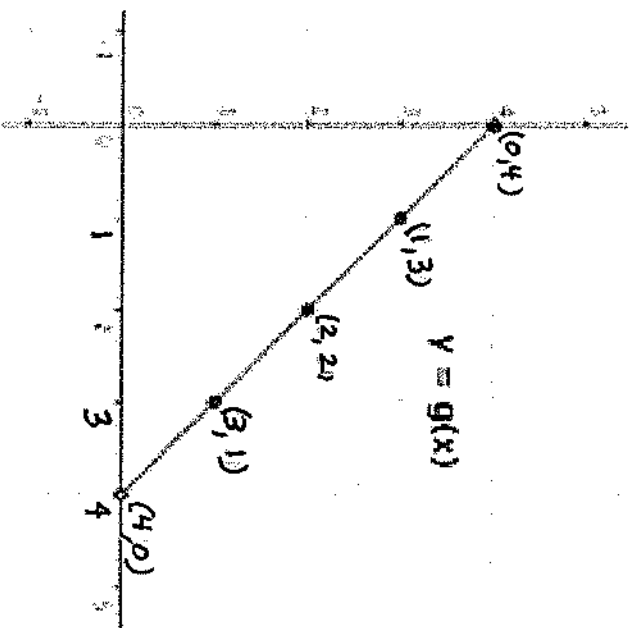
$$= 2 + 1$$

$$= \boxed{3}$$

$$\text{b) } (f - g)(1) = f(1) - g(1)$$

$$= 2 - 3$$

$$= \boxed{-1}$$

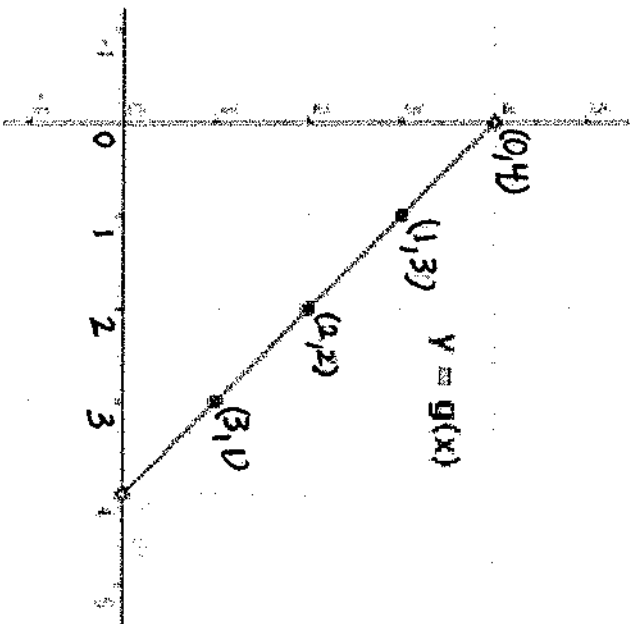
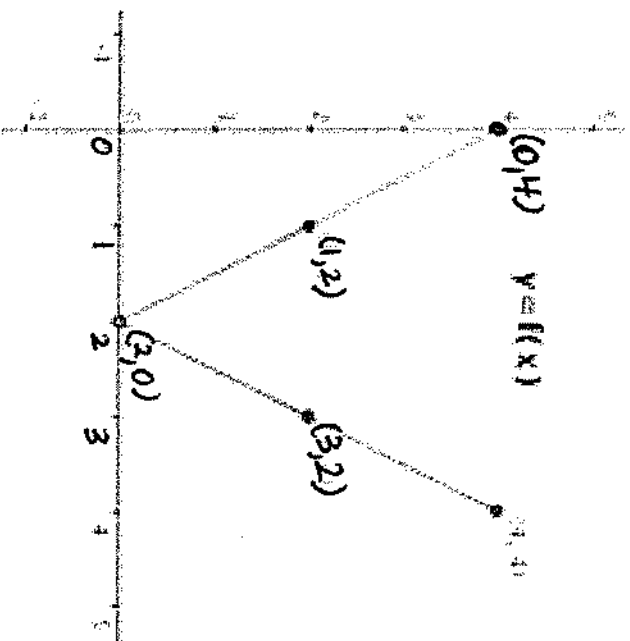


$$\text{c) } (fg)(4) = f(4) \cdot g(4)$$

$$= (4)(0)$$

$$= \boxed{0}$$

ex: Evaluate.



$$d) (fg)(0) = f(0) \cdot g(0) = (4)(4)$$

$$= \boxed{16}$$

$$e) \left(\frac{g}{f} \right) (1) = \frac{g(1)}{f(1)} = \boxed{\frac{3}{2}}$$

$$f) \left(\frac{f}{g} \right) (3) = \frac{f(3)}{g(3)} = \frac{2}{1} = \boxed{2}$$

$$g) \overset{\star}{-5} \left(fg \right) (2) = \overset{\star}{-5} \cdot [f(2) \cdot g(2)] = -5 [(0) \cdot (2)] = -5 [0] = \boxed{0}$$

ex: Evaluate each expression given the functions below.

$$f(x) = x^{2/3}$$

$$g(x) = \sqrt[4]{x}$$

$$h(x) = x + 5$$

$$a) (f+h)(27) = f(27) + h(27)$$

$$= 9 + 32$$

$$= \boxed{41}$$

Label each:

$$f(27) = 27^{2/3}$$

$$= (\sqrt[3]{27})^2$$

$$= (3)^2$$

$$= 9$$

$$h(27) = 27 + 5$$

$$= 32$$

ex: Evaluate each expression given the functions below.

$$f(x) = x^{2/3}$$

$$g(x) = \sqrt[4]{x}$$

$$h(x) = x + 5$$

$$\begin{aligned} \text{b) } (fg)(1) &= f(1) \cdot g(1) \\ &= (1)(1) \\ &= \boxed{1} \end{aligned}$$

$$\begin{aligned} f(1) &= 1^{2/3} \\ &= (\sqrt[3]{1})^2 \\ &= (1)^2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} g(1) &= \sqrt[4]{1} \\ &= 1 \end{aligned}$$

ex: Evaluate each expression given the functions below.

$$f(x) = x^{2/3}$$

$$g(x) = \sqrt[4]{x}$$

$$h(x) = x + 5$$

$$c) \left(\frac{h}{g} \right) (64)$$

$$= \frac{h(64)}{g(64)}$$

$$h(64) = 64 + 5$$

$$= 69$$

$$= \frac{69}{\sqrt[4]{64}}$$

$$g(64) = \sqrt[4]{64}$$

$$\begin{array}{r} 2 \sqrt{64} \\ 2 \sqrt{32} \\ 2 \sqrt{16} \\ 2 \sqrt{8} \\ 2 \sqrt{4} \\ 2 \end{array}$$

$$= \frac{69}{2 \sqrt[4]{2^2}} \cdot \frac{\sqrt[4]{2^2}}{\sqrt[4]{2^2}}$$

$$= \frac{69 \sqrt[4]{2^2}}{2 \sqrt[4]{2^2}}$$

$$= \boxed{\frac{69 \sqrt[4]{4}}{4}} \quad \text{Good}$$

$$\text{or } \boxed{\frac{69 \sqrt{2}}{4}} \quad \text{Better}$$

ex: perform the indicated operation.

Given:

$$f(x) = 7x^2 - 2x + 3$$

$$g(x) = 5x + 2$$

$$h(x) = -2x$$

$$a) (g - f)(x) = g(x) - f(x) \quad \star$$

$$= 5x + 2 - (7x^2 - 2x + 3)$$

need to use ()

$$= 5x + 2 - 7x^2 + 2x - 3$$

$$= \boxed{-7x^2 + 7x - 1}$$

ex: perform the indicated operation.

$$f(x) = 7x^2 - 2x + 3 \quad g(x) = 5x + 2$$

$$h(x) = -2x$$

$$b) (g+h)(x) = g(x)h(x)$$

$$= (5x+2)(-2x)$$

$$= \boxed{-10x^2 - 4x}$$

ex: perform the indicated operation.

$$f(x) = 7x^2 - 2x + 3 \quad g(x) = 5x + 2$$

$$h(x) = -2x$$

$$c) \left(\frac{h}{f}\right)(x) = \frac{h(x)}{f(x)}$$

$$\boxed{\frac{-2x}{7x^2 - 2x + 3}}$$

ex: perform the indicated operation.

$$f(x) = 7x^2 - 2x + 3 \quad g(x) = 5x + 2$$

$$h(x) = -2x$$

$$d) (f+g)(x) = f(x) + g(x)$$

$$= (7x^2 - 2x + 3) + (5x + 2)$$

$$= 35x^3 - 10x^2 + 15x + 14x^2 - 4x + 6$$

$$= \boxed{35x^3 + 4x^2 + 11x + 6}$$

Function Compositions

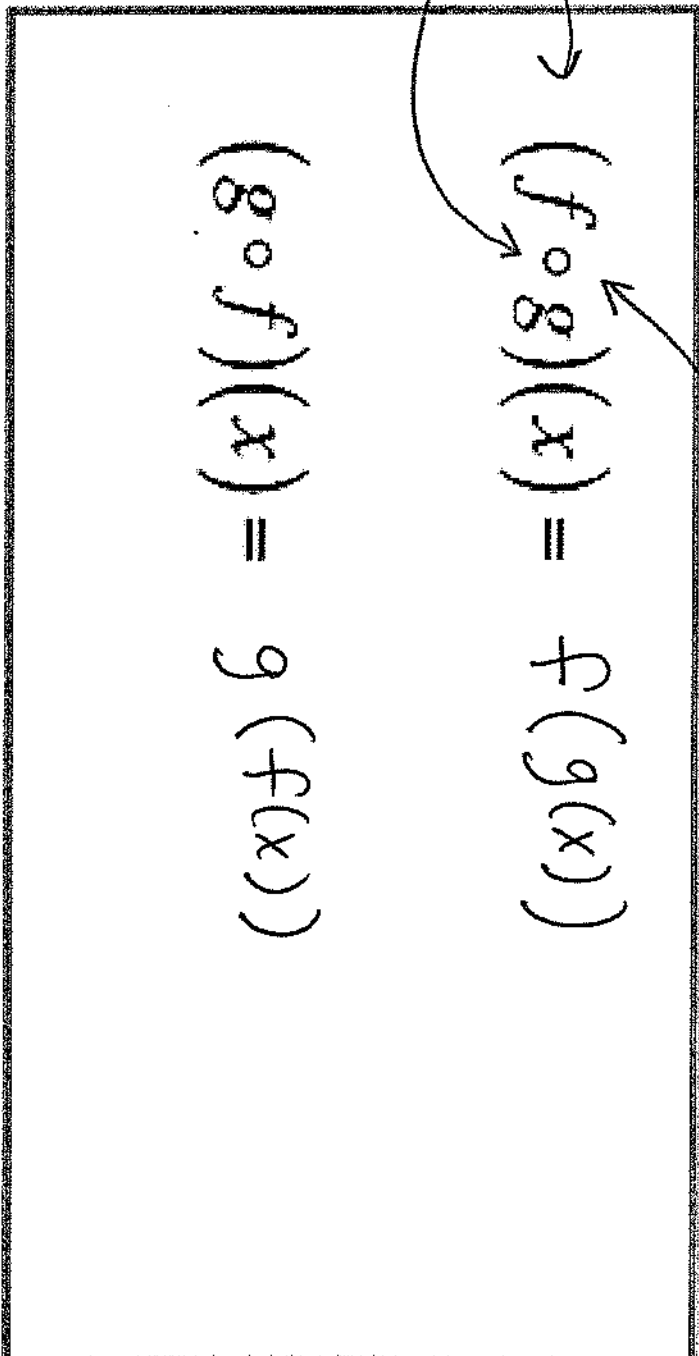
Not a product (not multiplication)

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

Read as "f of g of x"

Open
Circle



ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition, if possible.

$$a) (f \circ g)(1) = f(\underline{g(1)})$$

$$= f(\underline{1})$$

$$f(x) = \sqrt{x} \quad \leftarrow$$

$$= \sqrt{1}$$

$$= 1$$

Inside:

$$g(1) = \sqrt[3]{1} = 1$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition, if possible.

$$b) (n \circ m)(0) = n(\underline{\underline{m(0)}})$$

$$= n(\underline{\underline{0}})$$

Inside:

$$m(0) = (0)^2 = 0$$

$$h(x) = \textcircled{X} + 5 = 0 + 5$$

$$= \underline{\underline{5}}$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition, if possible.

$$c) (f \circ h) \left(\frac{1}{32} \right) = f \left(h \left(\frac{1}{32} \right) \right)$$

$$= f(32)$$

$$= \sqrt{32}$$

$$= 2 \cdot 2 \sqrt{2}$$

$$= \boxed{4\sqrt{2}}$$

$$\left. \begin{array}{l} 2 \sqrt{32} \\ 2 \sqrt{16} \\ 2 \sqrt{18} \\ 2 \sqrt{4} \end{array} \right\}$$

Inside:

$$h\left(\frac{1}{32}\right) = \frac{1}{32}$$

Keep change flip

$$= 1 \cdot \frac{32}{1}$$

$$= 32$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[5]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition, if possible.

$$d) (f \circ m)(-6) = f(\underline{m(-6)})$$

$$= f(-1)$$

$$f(x) = \sqrt{\otimes} \xrightarrow{\substack{\text{arg} \\ \text{root} - 1}}$$

$$= \boxed{\text{nonreal}}$$

Inside:

$$\left\{ \begin{array}{l} m(-6) = -6 + 5 \\ = -1 \end{array} \right.$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition, if possible.

$$e) (p \circ h)(0) = p(\underline{h(0)})$$

$$= \boxed{\text{undefined}}$$

$$\left. \begin{array}{l} \text{Inside:} \\ h(0) = \frac{1}{0} \\ = \text{undefined} \end{array} \right\}$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition, if possible.

$$f) (p \circ p)(5) = p(p(5))$$

$$= p(0)$$

$$= \cancel{(0)^2} - 10\cancel{(0)} + 25$$

$$= \boxed{25}$$

Inside:

$$p(5) =$$

$$= (5)^2 - 10(5) + 25$$

$$= 25 - 50 + 25$$

$$= 0$$

$$g) (n \circ f \circ m)(3)$$

$$= n(f(\underline{m(3)}))$$

$$= n(\underline{f(9)})$$

$$= n(3)$$

$$= 3 + 5$$

$$= \boxed{8}$$

Inside:

$$m(3) = (3)^2 = 9$$

$$f(9) = \sqrt{9} = 3$$

$$n(x) = \textcircled{x} + 5$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition.

$$\begin{aligned} \text{a) } (f \circ n)(x) &= f(n(x)) \\ &= f(x+5) \end{aligned}$$

$$f(x) = \sqrt{\textcircled{x}} \quad \swarrow \quad = \boxed{\sqrt{x+5}}$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition.

$$\begin{aligned} \text{b) } (h \circ p)(x) &= h(p(x)) \\ &= h(x^2 - 10x + 25) \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{x} \\ &= \frac{1}{x^2 - 10x + 25} \end{aligned}$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition.

$$\begin{aligned} \star \star \star \\ \text{c) } (f \circ m)(x) &= f(m(x)) \\ &= f(x^2) \end{aligned}$$

$$= f(x^2)$$

$$f(x) = \sqrt{x} \quad \leftarrow \begin{aligned} &= \sqrt{x^2} \\ &= |x| \end{aligned}$$

$$= |x|$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition.

$$d) (p \circ n)(x) = p(n(x))$$

$$= p(x+5)$$

$$= (x+5)^2 - 10(x+5) + 25$$

$$= (x+5)(x+5) - 10x - 50 + 25$$

$$= x^2 + 10x + 25 - 10x - 50 + 25$$

$$= x^2$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition.

$$\begin{aligned} \text{e) } (h \circ n)(x) &= h(\underline{n(x)}) \\ &= h(x+5) \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{x} \\ &= \boxed{\frac{1}{x+5}} \end{aligned}$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition.

$$\begin{aligned} f) (m \circ n)(x) &= m(\underline{n(x)}) \\ &= m(x+5) \end{aligned}$$

$$m(x) = x^2 = (x+5)^2$$

$$\begin{aligned} &= (x+5)(x+5) \\ &= \boxed{x^2 + 10x + 25} \end{aligned}$$

ex: Let

$$f(x) = \sqrt{x}$$

$$m(x) = x^2$$

$$g(x) = \sqrt[3]{x}$$

$$n(x) = x + 5$$

$$h(x) = \frac{1}{x}$$

$$p(x) = x^2 - 10x + 25$$

Find the composition.

$$g) (m \circ f)(x) = m(f(x))$$

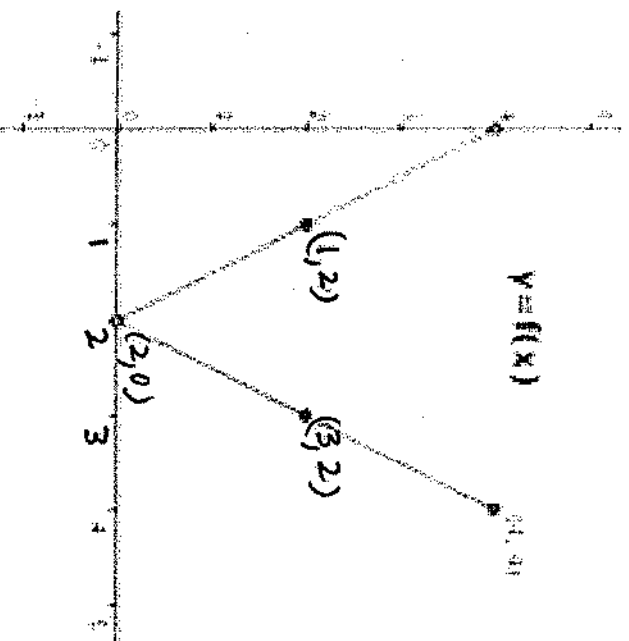
$$= m(\sqrt{x})$$

$$m(x) = x^2$$

$$= (\sqrt{x})^2$$

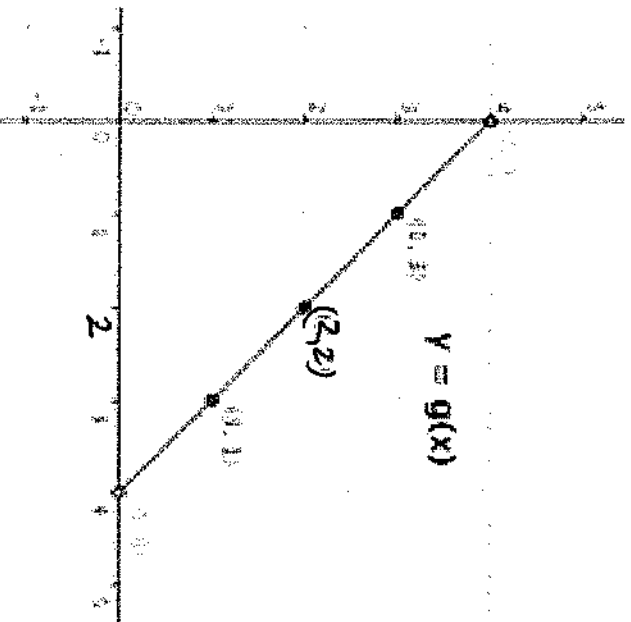
$$= \boxed{x}$$

ex: Evaluate.



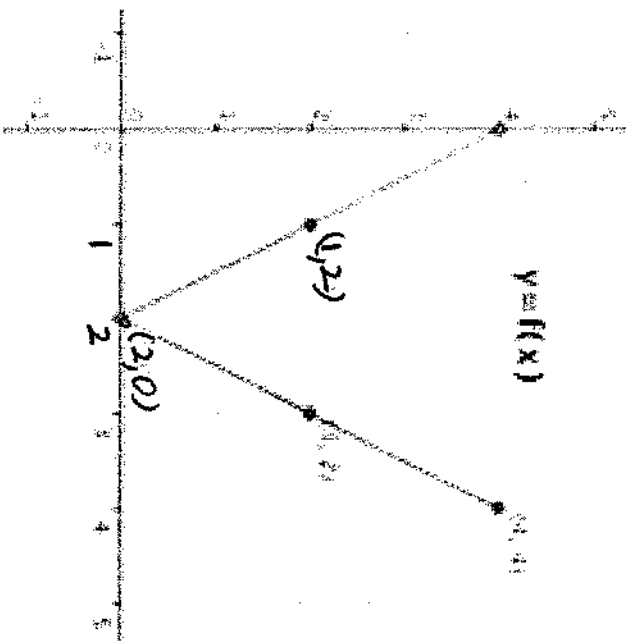
$$\begin{aligned} \text{a) } (f \circ g)(2) &= f(g(2)) \\ &= f(2) \\ &= \boxed{0} \end{aligned}$$

$$\begin{aligned} \text{b) } (g \circ f)(3) &= g(f(3)) \\ &= g(2) \\ &= \boxed{2} \end{aligned}$$

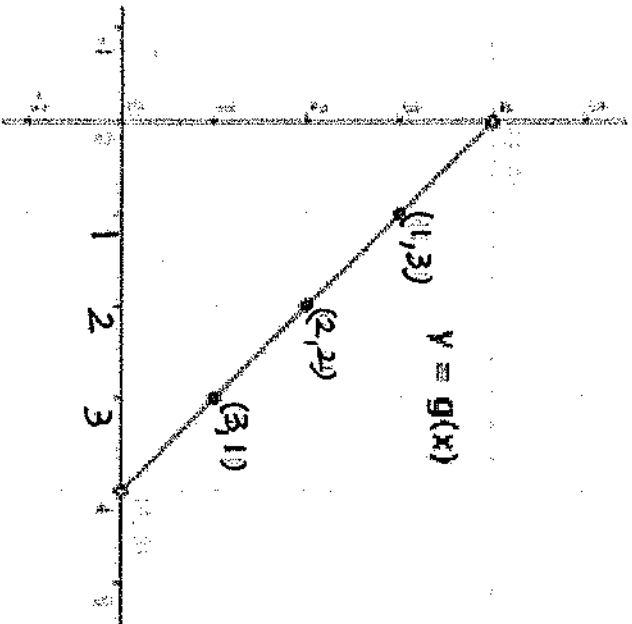


$$\begin{aligned} \text{c) } (f \circ f)(1) &= f(f(1)) \\ &= f(2) \\ &= \boxed{0} \end{aligned}$$

3.3 Notes - WKST



$$\begin{aligned} d) (g \circ g)(3) &= g(\underline{g(3)}) \\ &= g(1) \\ &= \boxed{3} \end{aligned}$$



$$\begin{aligned} e) (f \circ g \circ f)(1) \\ &= f(g(\underline{f(1)})) \\ &= f(\underline{g(2)}) \\ &= f(2) \\ &= \boxed{0} \end{aligned}$$