A2 Notes: Vertex Form of a Quadratic (Parabola)

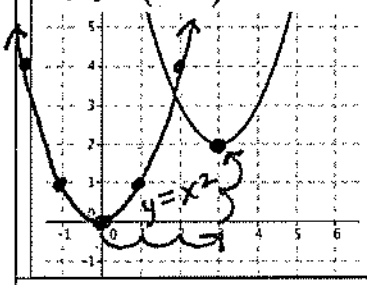
For each example,

a) Use the graph to determine the coordinates of the vertex.

b) Compare the graph of the functions below with their parent function, $y=x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

a) $y=(x-3)^2+2$



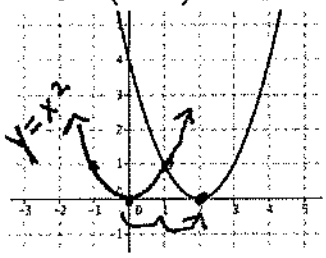
$x-3=0$

$x=3$

V: (3, 2)

Right 3, Up 2

b) $y=(x-2)^2+0$



$x-2=0$

$x=2$

V: (2, 0)

Right 2

For each example state the coordinate of the vertex without the use of the graph.

a) $y=(x+9)^2+5$ $x+9=0$, $x=-9$

V: (-9, 5)

b) $y=2(x-1)^2+10$ $x-1=0$, $x=1$

V: (1, 10)

c) $y=-(x+3)^2+7$ $x+3=0$, $x=-3$

V: (-3, 7)

Factor. Perfect square trinomials (LAST TERM is always positive).

a) $x^2-10x+25$

$(x-5)(x-5)$ Foil to check

$=(x-5)^2$

b) $x^2+12x+36$

$(x+6)(x+6)$

$=(x+6)^2$

c) x^2-2x+1

$(x-1)(x-1)$

$=(x-1)^2$

Rewriting from Standard: $y=ax^2+bx+c$ to Vertex Form: $y=a(x-h)^2+k$;Vertex: (h,k)

Write the quadratic function in vertex form.

a) $y=x^2+16x-33$ standard form (SF)

$a=1$, $b=16$, $c=-33$

$x=\frac{-b}{2a}=\frac{-(16)}{2(1)}=-8$

$4,16$, $\times 8$, 128

$y=(-8)^2+16(-8)-33$

$=64-128-33$

$=-64-33=-97$

V: (-8, -97) (h, k)

$y=a(x-h)^2+k$

$y=1(x-(-8))^2+(-97)$

$y=(x+8)^2-97$ vertex form (VF)

b) $y=x^2-6x+19$ SF

$a=1$, $b=-6$, $c=19$

$x=\frac{-b}{2a}=\frac{-(-6)}{2(1)}=\frac{6}{2}=3$

$y=(3)^2-6(3)+19$

$=9-18+19$

$=-9+19=10$

V: (3, 10) (h, k)

$y=a(x-h)^2+k$

$y=1(x-3)^2+10$

$y=(x-3)^2+10$ VF

c) $y=x^2+24x+100$ SF

$a=1$, $b=24$, $c=100$

$x=\frac{-b}{2a}=\frac{-(24)}{2(1)}=-12$

$y=(-12)^2+24(-12)+100$

$=144-288+100$

$=-144+100$

$=-44$

V: (-12, -44) (h, k)

24, $\times 12$, 48, 240, 288

$y=a(x-h)^2+k$

$y=1(x-(-12))^2+(-44)$

$y=(x+12)^2-44$ VF

d) $y=x^2+14x+27$ SF

$a=1$ $b=14$ $c=27$

$x=\frac{-b}{2a}=\frac{-(14)}{2(1)}=-7$

$y=(-7)^2+14(-7)+27$

$=49-98+27$

$=-49+27$

$=-22$

V: $(-7,-22)$ h K

$y=(x+7)^2-22$ VF

e) $y=x^2-12x+36$ SF

$a=1$ $b=-12$ $c=36$

$x=\frac{-b}{2a}=\frac{-(-12)}{2(1)}=\frac{12}{2}=6$

$y=(6)^2-12(6)+36$

$=36-72+36$

$=-36+36=0$

V: $(6,0)$ h K

$y=1(x-6)^2+0$

$y=(x-6)^2$ VF

f) $y=x^2+20x$ SF

$a=1$ $b=20$ $c=0$

$x=\frac{-b}{2a}=\frac{-(20)}{2(1)}=-10$

$y=(-10)^2+20(-10)$

$=100-200$

$=-100$

V: $(-10,-100)$ h K

$y=(x+10)^2-100$ VF

g) $y=x^2+7x+2$ SF

$a=1$ $b=7$ $c=2$

$x=\frac{-b}{2a}=\frac{-(7)}{2(1)}=\frac{-7}{2}$

$y=(-\frac{7}{2})^2+\frac{7}{1}(-\frac{7}{2})+2$

V: $(-\frac{7}{2},\frac{-41}{4})$ h K

$(-\frac{7}{2})(-\frac{7}{2})$

$=\frac{49}{4}-\frac{49}{22}+\frac{2}{1}\cdot\frac{4}{4}$

$=\frac{49}{4}-\frac{98}{4}+\frac{8}{4}$

$=\frac{-49}{4}+\frac{8}{4}=\frac{-41}{4}$

VF: $y=(x+\frac{7}{2})^2-\frac{41}{4}$

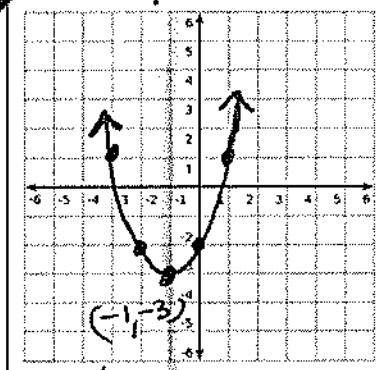
Sketching Quadratics In Vertex Form

*To sketch in vertex form, plot the vertex and at least two other points (on each side of the vertex).

Sketch the quadratic function. State the vertex, axis of symmetry, domain, and range. For a and b, answer in interval notation. For c and d, answer in set notation.

a) $y=(x+1)^2-3$ VF $x+1=0$, $x=-1$

$a=1$ up



Vertex $(-1,-3)$

AOS $x=-1$

Interval notation:

Domain $(-\infty,\infty)$

Range $[-3,\infty)$

Max/min value -3

$y=(0+1)^2-3$

$=(1)^2-3$

$=1-3$

$=-2$

x	y
-3	1
-2	-2
V -1	-3
0	-2
1	1

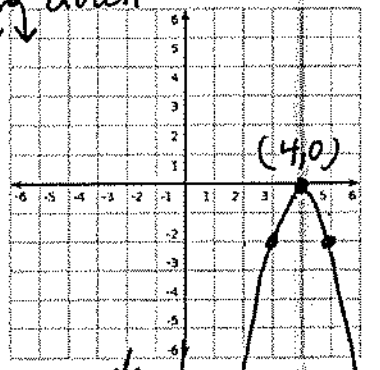
$y=(1+1)^2-3$

$=(2)^2-3$

$=4-3=1$

b) $y=-2(x-4)^2+0$ VF

$a=-2$ down



Vertex $(4,0)$

AOS $x=4$

Interval notation:

Domain $(-\infty,\infty)$

Range $(-\infty,0]$

Max/min value 0

$y=-2(6-4)^2$

$=-2(2)^2$

$=-2(4)$

$=-8$

x	y
2	-8
3	-2
V 4	0
5	-2
6	-8

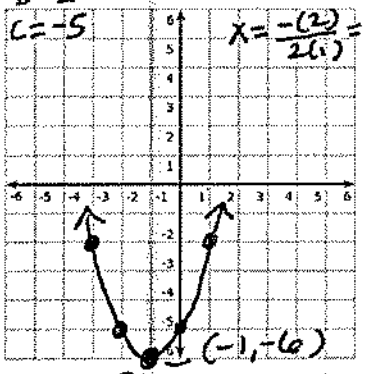
$y=-2(5-4)^2$

$=-2(1)^2$

$=-2(1)=-2$

c) $y = x^2 + 2x - 5$ SF

$a = 1$ up $\quad b = 2 \quad c = -5$ $\quad x = \frac{-b}{2a}$ $\quad x = \frac{-(2)}{2(1)} = -1$



$y = (-1)^2 + 2(-1) - 5$

$= 1 - 2 - 5$

$= -1 - 5 = -6$

VF: $y = (x+1)^2 - 6$

Vertex $(-1, -6)$

AOS $x = -1$

Set notation:

Domain $\{x \mid x \in \mathbb{R}\}$

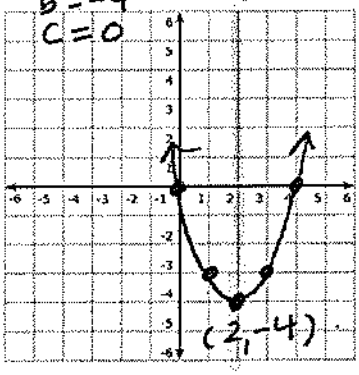
Range $\{y \mid y \geq -6\}$

Max/(min) value -6

x	y
-3	-2
-2	-5
V -1	-6
0	-5
1	-2

d) $y = x^2 - 4x$ SF $\quad x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

$a = 1$ up $\quad b = -4 \quad c = 0$



$y = (2)^2 - 4(2)$

$= 4 - 8$

$= -4$

VF: $y = (x-2)^2 - 4$

Vertex $(2, -4)$

AOS $x = 2$

Set notation:

Domain $\{x \mid x \in \mathbb{R}\}$

Range $\{y \mid y \geq -4\}$

Max/(min) value -4

x	y
0	0
1	-3
V 2	-4
3	-3
4	0

In general...

Vertex Form: $y = a(x-h)^2 + k$

a) When will the graph open up?	b) When will the graph open down?
$a > 0$ (positive)	$a < 0$ (negative)
c) What is the axis of symmetry?	d) What is the vertex?
$x = h$	(h, k)