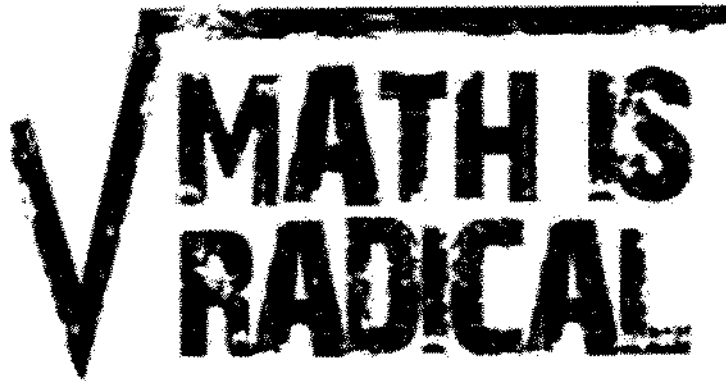


# Notes

*Unit 6: nth roots day 1*

*simplifying radicals/adding and subtracting radicals*



ex: Fill in the Chart...FAST!

$x$	$x^2$	$x^3$	$x^4$	$x^5$	$x^6$
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	-----
4	16	64	256	-----	-----
5	25	125	625	-----	-----
6	36	216	-----	-----	-----

## nth Roots

$$\boxed{n\sqrt{a}}$$


Where:

- a is called the radicand

- n is called the index

\*Square Roots:  $\sqrt{a}$ ,  $n = \underline{2}$

## Domain of nth Roots

$$\sqrt[n]{a}$$

If:

- n is even:  $a \geq 0$

- n is odd:  $a \in \mathbb{R}$  (all reals)

ex: Evaluate. If no real value exists, write "nonreal."

index = 2

a)  $\sqrt{25} = \boxed{5}$

$$\begin{array}{r} 5 \overline{) 25} \\ \underline{5} \phantom{0} \\ 0 \end{array}$$

~~$\sqrt{5^2}$~~

cube root

b)  $\sqrt[3]{8} = \boxed{2}$

$$\begin{array}{r} 2 \overline{) 8} \\ \underline{4} \phantom{0} \\ 4 \phantom{0} \\ \underline{4} \\ 0 \end{array}$$

~~$\sqrt[3]{2^3}$~~

odd

c)  $\sqrt[3]{-125} = \sqrt[3]{-1 \cdot 125}$

negative  $= \overset{\text{odd}}{\sqrt[3]{-1}} \cdot \sqrt[3]{125}$

$(-1)^3 =$

$(-1)(-1)(-1)$

$\checkmark \quad \checkmark$   
 $(1)(-1)$

$= \boxed{-1}$

~~$\sqrt[3]{(-5)^3}$~~   
 $= -1 \cdot 5$   
 $= \boxed{-5}$

$$\begin{array}{r} 5 \overline{) 125} \\ \underline{5} \phantom{00} \\ 0 \phantom{0} \\ 5 \overline{) 25} \\ \underline{5} \phantom{0} \\ 0 \end{array}$$

ex: Evaluate. If no real value exists, write "nonreal."

$$d) \sqrt[5]{243} = \boxed{3}$$

$$\sqrt[5]{3^5}$$

$$e) \sqrt[4]{-16}$$

even  
negative  $a \geq 0$   
even

$\boxed{\text{nonreal}}$

$$f) \sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}} = \boxed{\frac{1}{3}}$$

$$\textcircled{3} \overline{)243}$$

$$\textcircled{3} \overline{)81}$$

$$\textcircled{3} \overline{)27}$$

$$\textcircled{3} \overline{)9}$$

$$\textcircled{3}$$

$$3 \overline{)243}$$

$$3 \overline{)81}$$

$$3 \overline{)9}$$

$$3 \overline{)3}$$

$$\textcircled{3} \overline{)9}$$

$$\textcircled{3}$$

ex: Evaluate. If no real value exists, write "nonreal."

$$g) \sqrt[5]{32} = \boxed{2}$$

$$\begin{array}{r} \textcircled{2} \overline{) 32} \\ \underline{\textcircled{2} 16} \\ \textcircled{2} 16 \\ \underline{\textcircled{2} 8} \\ \textcircled{2} 14 \\ \underline{\textcircled{2}} \end{array} \quad \cancel{5 \overline{) 25}}$$

$$h) \sqrt[3]{\frac{125}{8}} = \frac{\sqrt[3]{125}}{\sqrt[3]{8}} = \boxed{\frac{5}{2}}$$

$$\begin{array}{r} \textcircled{2} \overline{) 8} \\ \underline{\textcircled{2} 4} \\ \textcircled{2} \end{array} \quad \begin{array}{r} \textcircled{5} \overline{) 125} \\ \underline{\textcircled{5} 25} \\ \textcircled{5} \end{array}$$

$$i) -5\sqrt[4]{16}$$

$$-5 \cdot \sqrt[4]{16}$$

$$-5 \cdot 2$$

$$= \boxed{-10}$$

$$\begin{array}{r} \textcircled{2} \overline{) 16} \\ \underline{\textcircled{2} 8} \\ \textcircled{2} 4 \\ \underline{\textcircled{2}} \end{array}$$

ex: Evaluate. If no real value exists, write "nonreal."

$$D) \sqrt[4]{1} = \boxed{1}$$

$$\sqrt[9]{1} = \boxed{1}$$

$$\sqrt[24]{1} = \boxed{1}$$

$$\sqrt{1} = \boxed{1}$$

$$K) \sqrt{27} = \boxed{3\sqrt{3}}$$

$$\begin{array}{r} \textcircled{3} \overline{)27} \\ \underline{\textcircled{3} \phantom{0}9} \\ \phantom{0}3 \end{array}$$

Side note  
 even  $\sqrt[6]{-1} = \boxed{\text{nonreal}}$   
 odd  $\sqrt[7]{-1} = \boxed{-1}$

$$D) \sqrt[4]{162}$$

$$= \boxed{3\sqrt[4]{2}}$$

$$\begin{array}{r} 2 \overline{)162} \\ = \textcircled{3} \overline{)81} \\ \textcircled{3} \overline{)27} \\ \textcircled{3} \overline{)9} \\ \phantom{0}3 \end{array}$$



ex: Evaluate. If no real value exists, write "nonreal."

m)  $\sqrt[5]{-96}$  =  $\sqrt[5]{-1 \cdot 96}$   
 $\uparrow$   
 odd  
 negative

=  $\sqrt[5]{-1} \cdot \sqrt[5]{96}$   
 =  $-1 \cdot 2\sqrt[5]{3}$

$$\begin{array}{r} 2 \overline{)96} \\ 2 \overline{)48} \\ 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ \underline{\quad} \\ 3 \end{array}$$

n)  $\sqrt[3]{32}$   
 =  $2\sqrt[3]{2 \cdot 2}$   
 =  $2\sqrt[3]{4}$

$$\begin{array}{r} 2 \overline{)32} \\ 2 \overline{)16} \\ 2 \overline{)8} \\ \underline{\quad} \\ 2 \overline{)4} \\ \underline{\quad} \\ 2 \end{array}$$

=  $\boxed{-2\sqrt[5]{3}}$

o)  $\sqrt{-64}$   
 $\uparrow$   
 even  
 root negative

$\boxed{\text{nonreal}}$

$a \geq 0$   
 even

## Operations with nth Roots

index / radicand

- Addition/Subtraction - the radicals must be "like radicals"

- Always simplify radicands.

ex: Perform the indicated operation.

$$a) 7\sqrt[5]{8} + 1\sqrt[5]{8} = \boxed{8\sqrt[5]{8}}$$

$$b) \boxed{4\sqrt[3]{2} - 6\sqrt[5]{2}}$$

• not same index,  
so not like radicals

- same index  
and
- same radicand

$$8 = 2^3$$

2	8
2	4
2	2

ex: Perform the indicated operation.

$$c) \sqrt[4]{2} - \sqrt[3]{2}$$

• not like radicals

$$d) \sqrt[3]{54} - \sqrt[3]{2}$$

↑  
simplify ↓

$$3\sqrt[3]{2} - 1\sqrt[3]{2}$$

$$= \boxed{2\sqrt[3]{2}}$$

$$\begin{array}{r} 2 \overline{) 54} \\ \underline{3} \phantom{0} \\ 3 \overline{) 27} \\ \underline{9} \phantom{0} \\ 3 \overline{) 9} \\ \underline{3} \phantom{0} \\ 0 \end{array}$$

Perform the indicated operation.

e)  $2\sqrt[3]{32} + \sqrt[3]{32} + \sqrt[3]{16}$

$$3\sqrt[3]{32} + \sqrt[3]{16}$$

$$3 \cdot 2\sqrt[3]{2 \cdot 2} \quad \downarrow$$

$$\boxed{6\sqrt[3]{4} + 2\sqrt[3]{2}}$$

$$\begin{array}{r} 2 \overline{) 32} \\ \underline{20} \phantom{0} \\ 12 \phantom{0} \\ \underline{12} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 16} \\ \underline{14} \phantom{0} \\ 2 \phantom{0} \\ \underline{2} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 16} \\ \underline{14} \phantom{0} \\ 2 \phantom{0} \\ \underline{2} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 8} \\ \underline{6} \phantom{0} \\ 2 \phantom{0} \\ \underline{2} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 8} \\ \underline{6} \phantom{0} \\ 2 \phantom{0} \\ \underline{2} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 4} \\ \underline{2} \phantom{0} \\ 2 \phantom{0} \\ \underline{2} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 4} \\ \underline{2} \phantom{0} \\ 2 \phantom{0} \\ \underline{2} \phantom{0} \\ 0 \phantom{0} \end{array}$$

• must be simplified radicands

f)  $4\sqrt{54} - 3\sqrt{6} + 2\sqrt{3}$

$$4 \cdot 3\sqrt{2 \cdot 3} \quad \downarrow \quad \downarrow$$

$$12\sqrt{6} - 3\sqrt{6} + 2\sqrt{3}$$

$$\boxed{9\sqrt{6} + 2\sqrt{3}}$$

$$\begin{array}{r} 2 \overline{) 54} \\ \underline{40} \phantom{0} \\ 14 \phantom{0} \\ \underline{12} \phantom{0} \\ 2 \phantom{0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 6} \\ \underline{6} \\ 0 \end{array}$$

$$\begin{array}{r} 3 \overline{) 27} \\ \underline{24} \phantom{0} \\ 3 \phantom{0} \end{array}$$

$$\begin{array}{r} 3 \overline{) 9} \\ \underline{9} \\ 0 \end{array}$$

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Between which two consecutive integers does the expression lie?

a)  $\sqrt{10}$   
Square root  
index = 2

$$\sqrt{9} < \sqrt{10} < \sqrt{16}$$

$$3 < \sqrt{10} < 4$$

$$\boxed{\text{Btwn } 3 \text{ \& } 4}$$

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

b)  $\sqrt[5]{40}$   
index = 5

$$\sqrt[5]{32} < \sqrt[5]{40} < \sqrt[5]{243}$$

$$\sqrt[5]{2} < \sqrt[5]{40} < \sqrt[5]{3}$$

$$2 < \sqrt[5]{40} < 3$$

$$\boxed{\text{Btwn } 2 \text{ \& } 3}$$

$$1^5 = 1$$

$$2^5 = 32$$

$$3^5 = 243$$

c)  $\sqrt[4]{350}$   
index = 4

$$\sqrt[4]{256} < \sqrt[4]{350} < \sqrt[4]{625}$$

$$4 < \sqrt[4]{350} < 5$$

$$\boxed{\text{Btwn } 4 \text{ \& } 5}$$

$$1^4 = 1$$

$$2^4 = 16$$

$$3^4 = 81$$

$$4^4 = 256$$

$$5^4 = 625$$